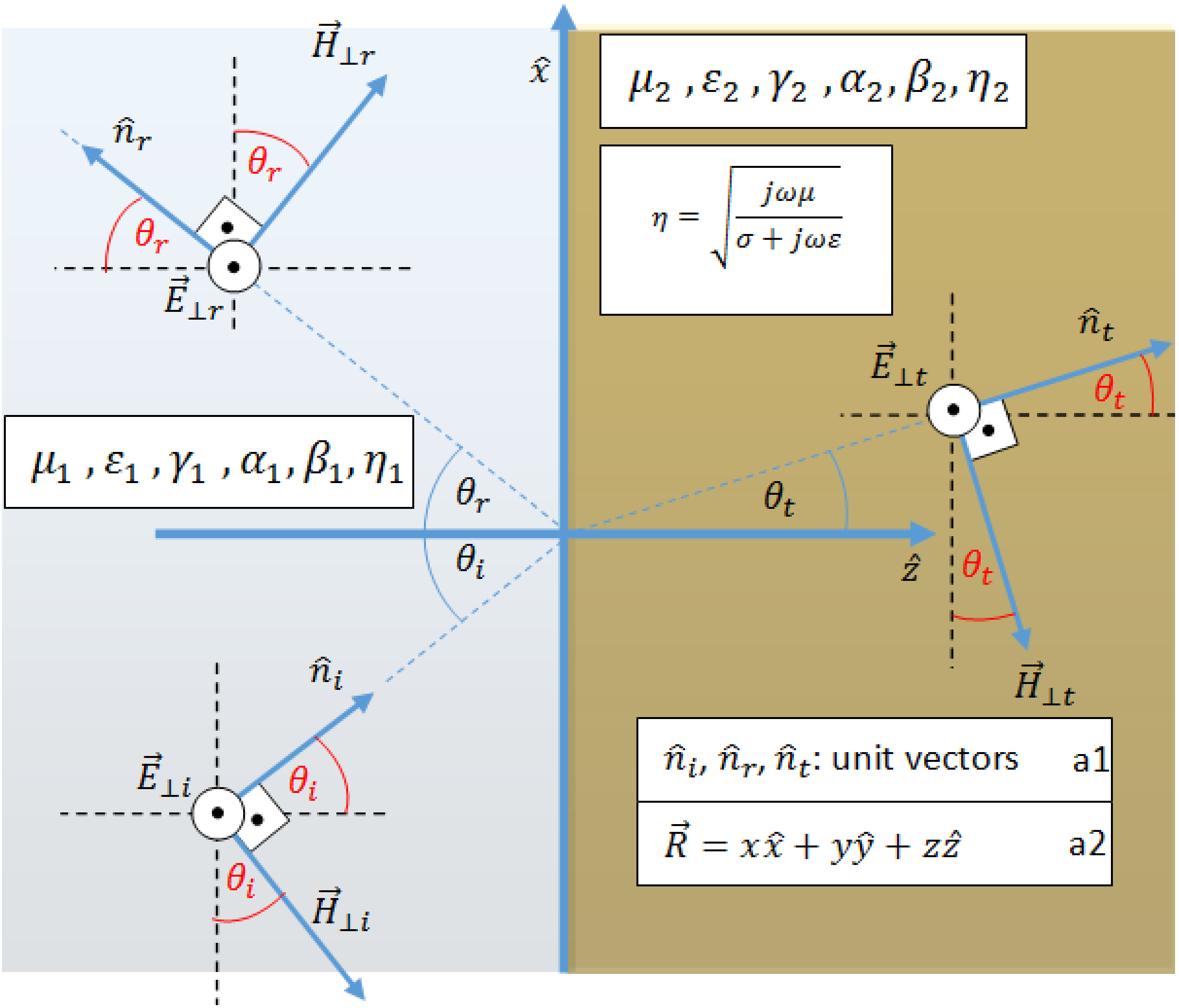
# Perpendicular Polarization ( $\vec{E}_\perp$ )

Electric field is **perpendicular** to the propagation plane (here, *xz*-plane)




$$\hat{n}_i = \sin\theta_i\,\hat{x} + \cos\theta_i\,\hat{z}$$

$$\hat{n}_r = \sin\theta_r\,\hat{x} - \cos\theta_r\,\hat{z}$$

$$\hat{n}_t = \sin\theta_t\,\hat{x} + \cos\theta_t\,\hat{z}$$

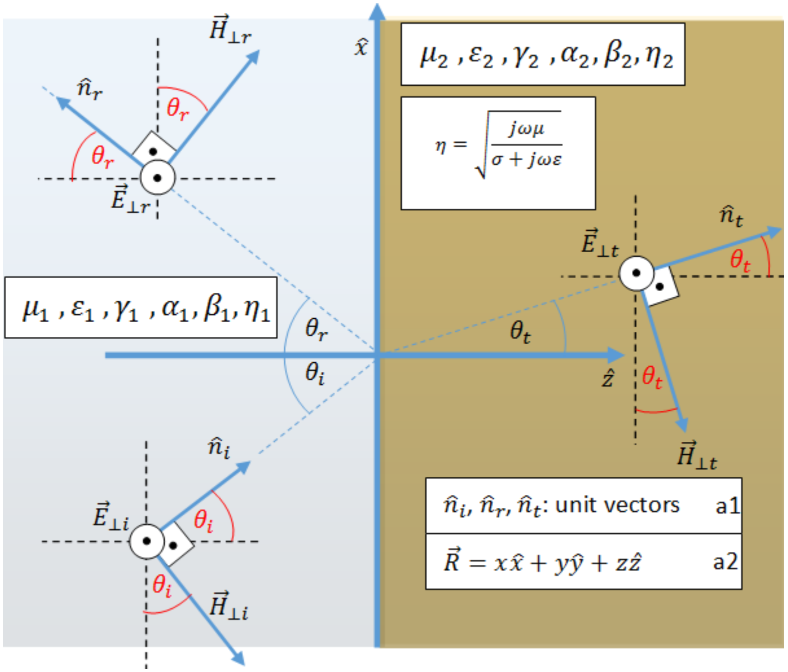
a4

$$\hat{n}_i \cdot \vec{R} = x\sin\theta_i + z\cos\theta_i$$

$$\hat{n}_r \cdot \vec{R} = x\sin\theta_r - z\cos\theta_r$$

$$\hat{n}_t \cdot \vec{R} = x\sin\theta_t + z\cos\theta_t$$

a2,a4->a5




1 2 3 4

$\vec{H}_{\perp i}$ , $\boldsymbol{H}_{\perp i}$ , $\hat{H}_{\perp i}$ , $H_{\perp i}$ ?

1

$$\left.\begin{aligned} \vec{H}_{\perp i} &= \boldsymbol{H}_{\perp i}\, \hat{H}_{\perp i} \\ \vec{H}_{\perp r} &= \boldsymbol{H}_{\perp r}\, \hat{H}_{\perp r} \\ \vec{H}_{\perp t} &= \boldsymbol{H}_{\perp t}\, \hat{H}_{\perp t} \end{aligned}\right\} \Rightarrow$$

a6

4

$$H_{\perp i} = |H_{\perp i}|(e^{j\theta_{H\perp i}})$$
$$H_{\perp r} = |H_{\perp r}|(e^{j\theta_{H\perp r}})$$
$$H_{\perp t} = |H_{\perp t}|(e^{j\theta_{H\perp t}})$$

3

$$\hat{H}_{\perp i} = -\cos\theta_i\, \hat{x} + \sin\theta_i\, \hat{z}$$
$$\hat{H}_{\perp r} = \cos\theta_r\, \hat{x} + \sin\theta_r\, \hat{z}$$
$$\hat{H}_{\perp t} = -\cos\theta_t\, \hat{x} + \sin\theta_t\, \hat{z}$$

a7

again

$\vec{H}_{\perp i}$ (1), $\boldsymbol{H}_{\perp i}$ (2), $\hat{H}_{\perp i}$ (3), $H_{\perp i}$ (4) ?

1

$$\left.\begin{aligned}\vec{H}_{\perp i} &= \boldsymbol{H}_{\perp i}\,\hat{H}_{\perp i}\\ \vec{H}_{\perp r} &= \boldsymbol{H}_{\perp r}\,\hat{H}_{\perp r}\\ \vec{H}_{\perp t} &= \boldsymbol{H}_{\perp t}\,\hat{H}_{\perp t}\end{aligned}\right\} \Rightarrow$$

a6

$$\left.\begin{aligned}\vec{H}_{\perp i} &= \boldsymbol{H}_{\perp i}\,\hat{H}_{\perp i}\\ \vec{H}_{\perp r} &= \boldsymbol{H}_{\perp r}\,\hat{H}_{\perp r}\\ \vec{H}_{\perp t} &= \boldsymbol{H}_{\perp t}\,\hat{H}_{\perp t}\end{aligned}\right\} \Rightarrow$$

a6

new

2

$$\boldsymbol{H}_{\perp i} = H_{\perp i}\left(e^{-\gamma_1 \hat{n}_i\cdot\vec{R}}\right) = |H_{\perp i}|\left(e^{j\theta_{H\perp i}}\right)\left(e^{-\gamma_1 \hat{n}_i\cdot\vec{R}}\right)$$

$$\boldsymbol{H}_{\perp r} = H_{\perp r}\left(e^{-\gamma_1 \hat{n}_r\cdot\vec{R}}\right) = |H_{\perp r}|\left(e^{j\theta_{H\perp r}}\right)\left(e^{-\gamma_1 \hat{n}_r\cdot\vec{R}}\right)$$

$$\boldsymbol{H}_{\perp t} = H_{\perp t}\left(e^{-\gamma_2 \hat{n}_t\cdot\vec{R}}\right) = |H_{\perp t}|\left(e^{j\theta_{H\perp t}}\right)\left(e^{-\gamma_2 \hat{n}_t\cdot\vec{R}}\right)$$

a8

**IMPORTANT**

again

$$\boldsymbol{H}_{\perp i} = H_{\perp i}\,(e^{-\gamma_1 \hat{n}_i \cdot \vec{R}}) = |H_{\perp i}|(e^{j\theta_{H\perp i}})(e^{-\gamma_1 \hat{n}_i \cdot \vec{R}}) \qquad 2$$

$$\boldsymbol{H}_{\perp r} = H_{\perp r}\,(e^{-\gamma_1 \hat{n}_r \cdot \vec{R}}) = |H_{\perp r}|(e^{j\theta_{H\perp r}})(e^{-\gamma_1 \hat{n}_r \cdot \vec{R}})$$

$$\boldsymbol{H}_{\perp t} = H_{\perp t}\,(e^{-\gamma_2 \hat{n}_t \cdot \vec{R}}) = |H_{\perp t}|(e^{j\theta_{H\perp t}})(e^{-\gamma_2 \hat{n}_t \cdot \vec{R}})$$

a8

$\gamma_1$ **is used for Incident and Reflected Waves since they propagate in Medium 1**

$\gamma_2$ **is used for Transmitted Wave since it propagates in Medium 2**

Again,

Observe the same routine marked for the incident wave also for reflected and transmitted waves

$$\boldsymbol{H}_{\perp i} = H_{\perp i}\left(e^{-\gamma_1 \hat{n}_i\cdot\vec{R}}\right) = |H_{\perp i}|\left(e^{j\theta_{H\perp i}}\right)\left(e^{-\gamma_1 \hat{n}_i\cdot\vec{R}}\right) \quad 2$$

$$\boldsymbol{H}_{\perp r} = H_{\perp r}\left(e^{-\gamma_1 \hat{n}_r\cdot\vec{R}}\right) = |H_{\perp r}|\left(e^{j\theta_{H\perp r}}\right)\left(e^{-\gamma_1 \hat{n}_r\cdot\vec{R}}\right)$$

$$\boldsymbol{H}_{\perp t} = H_{\perp t}\left(e^{-\gamma_2 \hat{n}_t\cdot\vec{R}}\right) = |H_{\perp t}|\left(e^{j\theta_{H\perp t}}\right)\left(e^{-\gamma_2 \hat{n}_t\cdot\vec{R}}\right)$$

a8

new 2

$$\boldsymbol{H}_{\perp i} = H_{\perp i}\left(e^{-(\alpha_1+j\beta_1)\hat{n}_i\cdot\vec{R}}\right) = |H_{\perp i}|\left(e^{j\theta_{H\perp i}}\right)\left(e^{-(\alpha_1+j\beta_1)\hat{n}_i\cdot\vec{R}}\right)$$

$$\boldsymbol{H}_{\perp r} = H_{\perp r}\left(e^{-(\alpha_1+j\beta_1)\hat{n}_r\cdot\vec{R}}\right) = |H_{\perp r}|\left(e^{j\theta_{H\perp r}}\right)\left(e^{-(\alpha_1+j\beta_1)\hat{n}_r\cdot\vec{R}}\right)$$

$$\boldsymbol{H}_{\perp t} = H_{\perp t}\left(e^{-(\alpha_2+j\beta_2)\hat{n}_t\cdot\vec{R}}\right) = |H_{\perp t}|\left(e^{j\theta_{H\perp t}}\right)\left(e^{-(\alpha_2+j\beta_2)\hat{n}_t\cdot\vec{R}}\right)$$

modified a9

Again,

Observe the same routine marked for the incident wave also for reflected and transmitted waves

$$\boldsymbol{H}_{\perp i} = H_{\perp i}(e^{-(\alpha_1+j\beta_1)\hat{n}_i\cdot\vec{R}}) = |H_{\perp i}|(e^{j\theta_{H\perp i}})(e^{-(\alpha_1+j\beta_1)\hat{n}_i\cdot\vec{R}})$$

$$\boldsymbol{H}_{\perp r} = H_{\perp r}(e^{-(\alpha_1+j\beta_1)\hat{n}_r\cdot\vec{R}}) = |H_{\perp r}|(e^{j\theta_{H\perp r}})(e^{-(\alpha_1+j\beta_1)\hat{n}_r\cdot\vec{R}})$$

$$\boldsymbol{H}_{\perp t} = H_{\perp t}(e^{-(\alpha_2+j\beta_2)\hat{n}_t\cdot\vec{R}}) = |H_{\perp t}|(e^{j\theta_{H\perp t}})(e^{-(\alpha_2+j\beta_2)\hat{n}_t\cdot\vec{R}})$$

modified a9

New, using a5 in 'modified a9', 'modified a10' is obtained

$$\boldsymbol{H}_{\perp i} = |H_{\perp i}|(e^{j\theta_{H\perp i}})(e^{-\alpha_1(x\sin\theta_i+z\cos\theta_i)})(e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)})$$

$$\boldsymbol{H}_{\perp r} = |H_{\perp r}|(e^{j\theta_{H\perp r}})(e^{-\alpha_1(x\sin\theta_r-z\cos\theta_r)})(e^{-j\beta_1(x\sin\theta_r-z\cos\theta_r)})$$

$$\boldsymbol{H}_{\perp t} = |H_{\perp t}|(e^{j\theta_{H\perp t}})(e^{-\alpha_2(x\sin\theta_t+z\cos\theta_t)})(e^{-j\beta_2(x\sin\theta_t+z\cos\theta_t)})$$

modified a10

Again

$$\boldsymbol{H}_{\perp i} = |H_{\perp i}|(e^{j\theta_{H\perp i}})(e^{-\alpha_1(x\sin\theta_i + z\cos\theta_i)})(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})$$

$$\boldsymbol{H}_{\perp r} = |H_{\perp r}|(e^{j\theta_{H\perp r}})(e^{-\alpha_1(x\sin\theta_r - z\cos\theta_r)})(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})$$

$$\boldsymbol{H}_{\perp t} = |H_{\perp t}|(e^{j\theta_{H\perp t}})(e^{-\alpha_2(x\sin\theta_t + z\cos\theta_t)})(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})$$

modified a10

New,

Using ‘modified 10’,

Assume $\alpha_1 = \alpha_2 = 0$ (lossless medium 1 and lossless medium 2) =>

$$e^{-0(x\sin\theta_i + z\cos\theta_i)} = e^{-0(x\sin\theta_r - z\cos\theta_r)} = e^{-0(x\sin\theta_t + z\cos\theta_t)} = 1 \Rightarrow$$

$$\boldsymbol{H}_{\perp i} = |H_{\perp i}|(e^{j\theta_{H\perp i}})(1)(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}) = H_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})$$

$$\boldsymbol{H}_{\perp r} = |H_{\perp r}|(e^{j\theta_{H\perp r}})(1)(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}) = H_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})$$

$$\boldsymbol{H}_{\perp t} = |H_{\perp t}|(e^{j\theta_{H\perp t}})(1)(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}) = H_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})$$

modified a10, lossless case

Again,

$$\boldsymbol{H}_{\perp i} = |H_{\perp i}|(e^{j\theta_{H\perp i}})(1)(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}) = H_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})$$

$$\boldsymbol{H}_{\perp r} = |H_{\perp r}|(e^{j\theta_{H\perp r}})(1)(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}) = H_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})$$

$$\boldsymbol{H}_{\perp t} = |H_{\perp t}|(e^{j\theta_{H\perp t}})(1)(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}) = H_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})$$

modified a10, lossless case

$$\left.\begin{aligned} \vec{H}_{\perp i} &= \boldsymbol{H}_{\perp i}\,\hat{H}_{\perp i} \\ \vec{H}_{\perp r} &= \boldsymbol{H}_{\perp r}\,\hat{H}_{\perp r} \\ \vec{H}_{\perp t} &= \boldsymbol{H}_{\perp t}\,\hat{H}_{\perp t} \end{aligned}\right\} \Rightarrow$$

a6

$$\hat{H}_{\perp i} = -\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z}$$

$$\hat{H}_{\perp r} = \cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z}$$

$$\hat{H}_{\perp t} = -\cos\theta_t\,\hat{x} + \sin\theta_t\,\hat{z}$$

a7

New,

$$\vec{H}_{\perp i} = H_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})(-\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z})$$

$$\vec{H}_{\perp r} = H_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})(\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z})$$

$$\vec{H}_{\perp t} = H_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})(-\cos\theta_t\,\hat{x} + \sin\theta_t\,\hat{z})$$

Lossless $H_{\perp}$

New,

$$\vec{E}_{\perp i} = \eta_1(\vec{H}_{\perp i} \times \hat{n}_i) = \eta_1(\boldsymbol{H}_{\perp i}\, \hat{H}_{\perp i} \times \hat{n}_i) = (\,\eta_1 \boldsymbol{H}_{\perp i}\,)(\hat{H}_{\perp i} \times \hat{n}_i)$$

$$= (\eta_1 \boldsymbol{H}_{\perp i}\,)(-\cos\theta_i\, \hat{x} + \sin\theta_i\, \hat{z}\,) \times (\sin\theta_i\, \hat{x} + \cos\theta_i \hat{z})$$

$$= \, (\eta_1 \boldsymbol{H}_{\perp i}\,)(-\cos\theta_i).(\cos\theta_i)(-\hat{y}) + (\sin\theta_i).(\sin\theta_i)\hat{y}$$

$$= (\eta_1 \boldsymbol{H}_{\perp i})\, [\cos^2\theta_i(+\hat{y}) + \sin^2\theta_i(+\hat{y})] = (\eta_1 \boldsymbol{H}_{\perp i})(+1)\hat{y}$$

modified a14

$$\vec{E}_{\perp i} = (\eta_1 \boldsymbol{H}_{\perp i})\, \hat{E}_{\perp i} = (\,\eta_1 \boldsymbol{H}_{\perp i}\,)(+\hat{y}) = \boldsymbol{E}_{\perp i}\, \hat{E}_{\perp i}$$

$$\eta_1 \boldsymbol{H}_{\perp i} = \, \boldsymbol{E}_{\perp i} = \eta_1 H_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i\,)})$$

$$\vec{E}_{\perp i} = \eta_1 H_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i\,)})\hat{y}$$

$$\vec{E}_{\perp i} = E_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i\,)})\, \hat{y}$$

modified a15

New,

$$\vec{E}_{\perp r} = \eta_1(\vec{H}_{\perp r} \times \hat{n}_r) = \eta_1(\boldsymbol{H}_{\perp r}\,\hat{H}_{\perp r} \times \hat{n}_r) = (\,\eta_1 \boldsymbol{H}_{\perp r}\,)(\hat{H}_{\perp r} \times \hat{n}_r)$$

$$= (\eta_1 \boldsymbol{H}_{\perp r}\,)(\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z}\,) \times (\sin\theta_r\,\hat{x} - \cos\theta_r \hat{z})$$

$$= \ (\eta_1 \boldsymbol{H}_{\perp r}\,)(\cos\theta_r).(-\cos\theta_r)(-\hat{y}) + (\sin\theta_r).(\sin\theta_r)\hat{y}$$

$$= (\eta_1 \boldsymbol{H}_{\perp r})\,[\cos^2\theta_r(+\hat{y}) + \sin^2\theta_r(+\hat{y})] = (\eta_1 \boldsymbol{H}_{\perp r})(+1)\hat{y}$$

modified a16

$$\vec{E}_{\perp r} = (\eta_1 \boldsymbol{H}_{\perp r}\,)\,\hat{E}_{\perp r} = (\,\eta_1 \boldsymbol{H}_{\perp r}\,)(+\hat{y}) = \boldsymbol{E}_{\perp r}\,\hat{E}_{\perp r}$$

$$\eta_1 \boldsymbol{H}_{\perp r} = \ \boldsymbol{E}_{\perp r} = \eta_1 H_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r\,)})$$

$$\vec{E}_{\perp r} = \eta_1 H_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r\,)})\hat{y}$$

$$\vec{E}_{\perp r} = E_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r\,)})\,\hat{y}$$

modified a17