Plane Electromagnetic Waves

Wavefront: The geometrical view a wave according to an observer who stays outside of the wave.

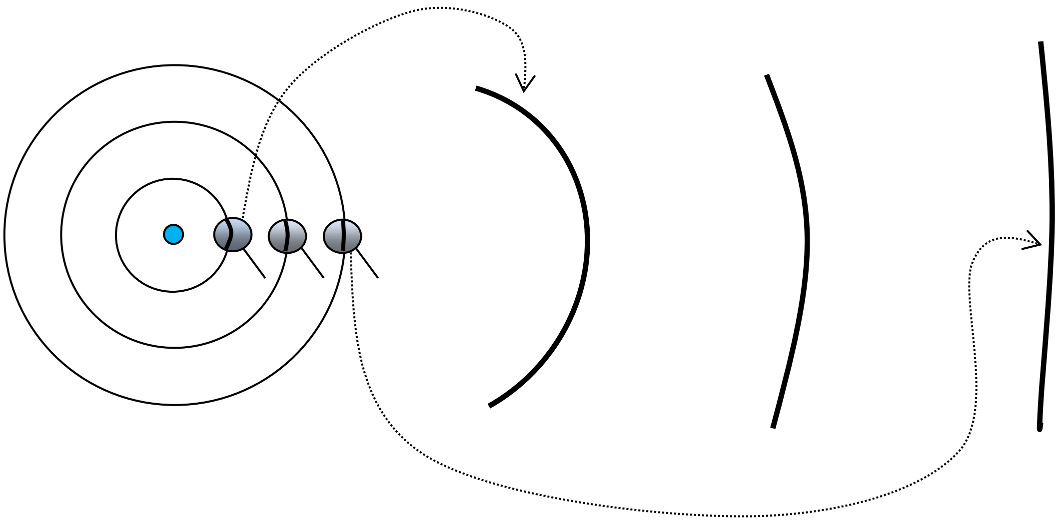
As an example, consider surface of steady water in a pole without any wind. Assume that surface of water is tipped by a pen at a constant frequency. An observer will see circular wavefronts getting away from the contact point of the pen on water.

Considering one of those circular rings on water surface, all points of equal distance from the contact point on the ring will have the same height (i.e., the same phase value)

Basic Idea:

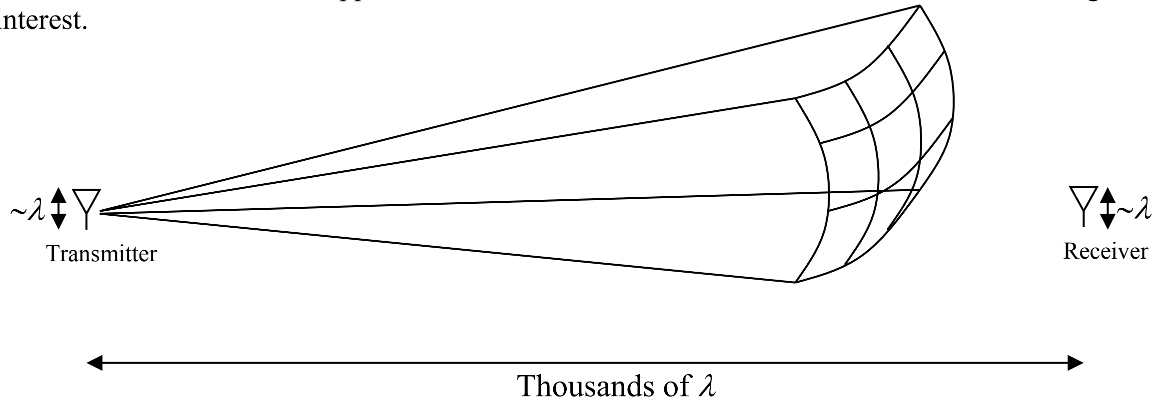
- In 2 dimensional space, we can consider a point source which radiates to all directions. Thus, the wavefronts of the radiated waves are circles. As the radius of the ring increases, any portion of the ring approaches to a line,

- In 3 dimensional space, we can consider a point source which radiates to all directions. Thus, the wavefronts of the radiated waves are spheres. As the radius of the wave inreases, the portion of the surface of the wavefront approaches to a plane.



Since Maxwell's equations are scalable, distances and other dimesions are expressed in terms of wavelength instead of meters or centimeters or so.

In wireless communication applications, field values at a distance of thousands of wavelength are of interest.

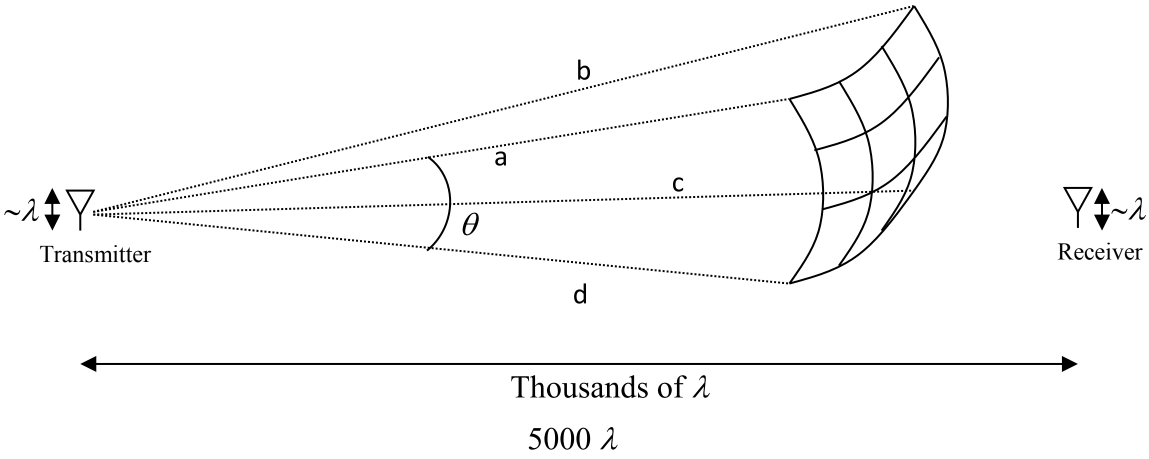


For example, a radio transmitter which broadcasts at 100MHz can reach to 40-50 km without a repeater. For such a case, assuming a receiver antenna 15 km away, the wavelength at 100MHz:

$$\lambda f = c \;\Rightarrow\; \lambda\, 100.10^{6} = 3.10^{8} \;\Rightarrow\; \lambda = 3\text{m}$$

In thşs case, the distance between the transmitter and receiver becomes 5000λ.

Assuming the receiver antenna size is approximately 1λ, the region under examination corresponds to a small angular size:

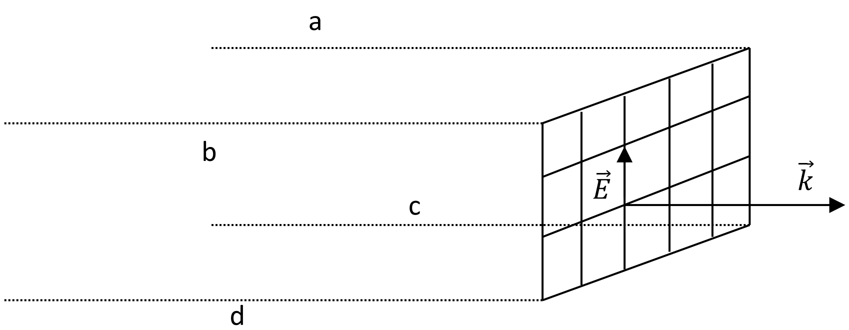


$$\theta = \lambda / 5000\lambda = 2\text{x}10^{-4} \text{ radians.}$$

Since 2π radians corresponds to 360°, $\theta = 2\text{x}5{,}73.10^{-3\circ} = 0{,}01146°$

This shows how small is the angle which spans the portion under examination.

For plane wave assumption, ‘a’, ‘b’, ‘c’, ‘d’ are in parallel.



A uniform plane wave is a particular solution of Maxwell’s equation assuming electric field (and magnetic field) has same magnitude and phase in infinite planes perpendicular to the direction of propagation. It may be noted that in the strict sense a uniform plane wave doesn’t exist in practice as creation of such waves are possible with sources of infinite extent. However, at large distances from the source, the wavefront or the surface of the constant phase becomes almost spherical and a small portion of this large sphere can be considered to plane. The characteristics of plane waves are simple and useful for studying many practical scenarios.

In a lossless medium, ε and μ are real numbers so k is real.

$$\nabla^2\vec{E} + k^2\vec{E}=0$$

For each component $E_x, E_y \& E_z$ or $H_x, H_y \& H_z$ in the Hemholtz Equation abo ve, we obtain a scalar equation:

For example if we consider E_x component we can write

$$\frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}+k^2 E_x=0$$

Let us consider a plane wave which has only E_x component and propagating along z. Since the plane wave will have no variation along the plane perpendicular to z i.e., xy plane, $\frac{\partial E_x}{\partial x}=\frac{\partial E_x}{\partial y}=0$. The Helmholtz's equation reduces to,

$$\frac{d^2 E_x}{dz^2}+k^2 E_x=0$$

The solution to this equation can be written as

$$E_x(z) = E_x^+(z) + E_x^-(z)$$

$$E_x(z) = E_0^+ e^{-jkz} + E_0^- e^{+jkz}$$

$E_0^+ \ \& E_0^-$ are the amplitude constants (can be determined from boundary conditions).

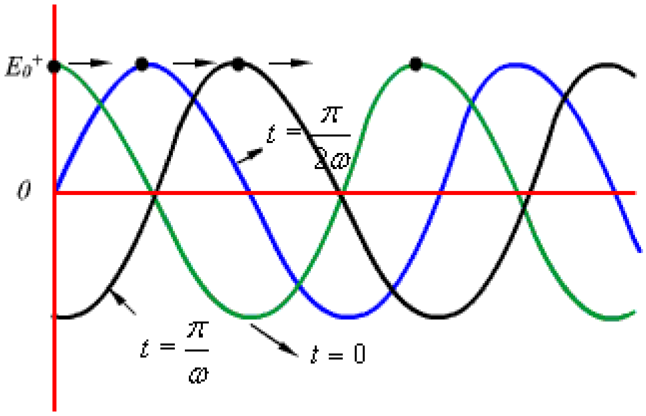
In the time domain,

$$E_x(z,t) = Re\{E_x(z)e^{j\omega t}\}$$

$$E_x(z,t) = E_0^+ \cos(\omega t - kz) + E_0^- \cos(\omega t + kz)$$

assuming $E_0^+ \ \& E_0^-$ are real constants.

Here, $E_x^+(z,t) = E_0^+ \cos(\omega t - kz)$ represents the forward traveling wave:



As can be seen from the figure, as time progresses, the wave propagates in the +z direction.

Considering the phase on the wave

$$\omega t - kz = \text{constant}$$

Then it is seen that as t is increases from t to $t+\Delta t$, z increases from z to $z+\Delta z$ so that

$$\omega(t+\Delta t) - k(z+\Delta z) = \text{constant} = \omega t - \beta z$$

$$\omega \Delta t = k \Delta z$$
$$\frac{\Delta z}{\Delta t} = \frac{\omega}{k}$$

When $\Delta t \rightarrow 0$

$$\lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t} = \frac{dz}{dt} = v_P \text{ (phase velocity)}$$

$$v_p = \frac{\omega}{k}$$

If the medium in which the wave is propagating is free space i.e., $\varepsilon = t_0$, $\mu = \mu_0$

$$\text{then } v_P = \frac{\omega}{\omega\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = C$$

Where 'C' is the speed of light. That is plane EM wave travels in free space with this speed of light.