New,

$$\vec{E}_{\perp t} = \eta_2(\vec{H}_{\perp t} \times \hat{n}_t) = \eta_2(\boldsymbol{H}_{\perp t}\, \hat{H}_{\perp t} \times \hat{n}_t) = (\eta_2 \boldsymbol{H}_{\perp t})(\hat{H}_{\perp t} \times \hat{n}_t)$$

$$= (\eta_2 \boldsymbol{H}_{\perp t})(-\cos\theta_t\, \hat{x} + \sin\theta_t\, \hat{z}) \times (\sin\theta_t\, \hat{x} + \cos\theta_t \hat{z})$$

$$= (\eta_2 \boldsymbol{H}_{\perp t})(-\cos\theta_t).(\cos\theta_t)(-\hat{y}) + (\sin\theta_t).(\sin\theta_t)\hat{y}$$

$$= (\eta_2 \boldsymbol{H}_{\perp t})\,[\cos^2\theta_t(+\hat{y}) + \sin^2\theta_t(+\hat{y})] = (\eta_2 \boldsymbol{H}_{\perp t})(+1)\hat{y}$$

modified a18

$$\vec{E}_{\perp t} = (\eta_2 \boldsymbol{H}_{\perp t})\,\hat{E}_{\perp t} = (\eta_2 \boldsymbol{H}_{\perp t})(+\hat{y}) = \boldsymbol{E}_{\perp t}\,\hat{E}_{\perp t}$$

$$\eta_2 \boldsymbol{H}_{\perp t} = \boldsymbol{E}_{\perp t} = \eta_2 H_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})$$

$$\vec{E}_{\perp t} = \eta_2 H_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})\hat{y}$$

$$\vec{E}_{\perp t} = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})\,\hat{y}$$

modified a19

Summary of electric field vectors from Slide 9, 10 , 11 for perpendicular polarization

$$\vec{E}_{\perp i} = E_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})\ \hat{y}$$

$$\vec{E}_{\perp r} = E_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})\ \hat{y}$$

$$\vec{E}_{\perp t} = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})\ \hat{y}$$

Lossless $E_{\perp}$

Summary of magnetic field vectors from Slide 8, for perpendicular polarization

$$\vec{H}_{\perp i} = H_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})(-\cos\theta_i\ \hat{x} + \sin\theta_i\ \hat{z})$$

$$\vec{H}_{\perp r} = H_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})(\cos\theta_r\ \hat{x} + \sin\theta_r\ \hat{z})$$

$$\vec{H}_{\perp t} = H_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})(-\cos\theta_t\ \hat{x} + \sin\theta_t\ \hat{z})$$

Lossless $H_{\perp}$

Again,

$$\vec{H}_{\perp i} = H_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})(-\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z})$$

$$\vec{H}_{\perp r} = H_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})(\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z})$$

$$\vec{H}_{\perp t} = H_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})(-\cos\theta_t\,\hat{x} + \sin\theta_t\,\hat{z})$$

Lossless $H_{\perp}$

New, we can pass from the box above to the box below using H=E/eta, for incident (i), reflected (r) and transmitted (t) vectors

$$\vec{H}_{\perp i} = (\frac{E_{\perp r}}{\eta_1})(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})(-\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z})$$

$$\vec{H}_{\perp i} = (\frac{E_{\perp r}}{\eta_1})(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})(\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z})$$

$$\vec{H}_{\perp t} = (\frac{\boldsymbol{E}_{\perp t}}{\eta_2})(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})(-\cos\theta_t\,\hat{x} + \sin\theta_t\,\hat{z})$$

Lossless $H_{\perp}$

Boundary Condition for Electric Field Intensity

$$\vec{E}_{1,tangential} = \vec{E}_{2,tangential}$$

(This is the 1st Equation for Perpendicular Polarization, Equation 1)

Boundary Condition for Magnetic Field Intensity

$$\hat{n}_2 \times \left(\hat{H}_1 - \hat{H}_2\right) = \vec{J}_S$$

$\vec{J}_S = 0$ (Medium 1 and Medium 2 are perfect dielectric)

Since, $\hat{n}_2 = -\hat{z}$, we can not have any ***z***-component in $\hat{n}_2 \times \left(\hat{H}_1 - \hat{H}_2\right)$

$\hat{n}_2 \times \left(\hat{H}_1 - \hat{H}_2\right)$ can include only ***x***-component and/or ***y***-component:

$$=> \ \vec{H}_{1,tangential} = \vec{H}_{2,tangential}$$

(This is the 2nd Equation for Perpendicular Polarization, Equation 2)

$$\vec{E}_{\perp i} = E_{\perp i}(e^{-j\beta_1(x\sin\theta_i + 0\cos\theta_i)})\ \hat{y}$$

$$\vec{E}_{\perp r} = E_{\perp r}(e^{-j\beta_1(x\sin\theta_r - 0\cos\theta_r)})\ \hat{y}$$

$$\vec{E}_{\perp t} = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t + 0\cos\theta_t)})\ \hat{y}$$

Lossless $E_{\perp}$

At the boundary (z=0), the tangential component can involve a combination of $\hat{x}$ or , $\hat{y}$ since the boundary surface is the *xy*-plane. But, Electric field has only *y*-component. So we will consider this *y*-component while examining the boundary condition for the electric field (i.e., equality of tangential electric field components).

$$\vec{E}_{\perp i} = E_{\perp i}(e^{-j\beta_1(x\sin\theta_i)})\ \hat{y}$$

$$\vec{E}_{\perp r} = E_{\perp r}(e^{-j\beta_1(x\sin\theta_r)})\ \hat{y}$$

$$\vec{E}_{\perp t} = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t)})\ \hat{y}$$

z=0, Lossless, $E_{\perp}$

Tangential components of the **Electric field** (Tangential components which are **tangential to** or which are **parallel to** the **boundary surface**, i.e., ***xy*-plane** or equivalently ***z*=0**) again,

$$\vec{E}_{\perp i} = E_{\perp i}(e^{-j\beta_1(x\sin\theta_i)})\ \hat{y}$$

$$\vec{E}_{\perp r} = E_{\perp r}(e^{-j\beta_1(x\sin\theta_r)})\ \hat{y}$$

$$\vec{E}_{\perp t} = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t)})\ \hat{y}$$

z=0, Lossless, $E_\perp$

In general,

$$\vec{E}_{\perp 1} = \vec{E}_{\perp i} + \vec{E}_{\perp r}$$

$$\vec{E}_{\perp 2} = \vec{E}_{\perp t}$$

Boundary Condition

$$\vec{E}_{\perp 1,tangential} = \vec{E}_{\perp 2,tangential}$$

$$\vec{E}_{\perp i,tangential} + \vec{E}_{\perp r,tangential} = \vec{E}_{\perp t,tangential}$$

New, Using BC at the boundary (z=0)

$$E_{\perp i}(e^{-j\beta_1(x\sin\theta_i)})\ \hat{y} + E_{\perp r}(e^{-j\beta_1(x\sin\theta_r)})\ \hat{y} = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t)})\ \hat{y}$$

z=0, Lossless $E_\perp$

$$E_{\perp i}(e^{-j\beta_1(x\sin\theta_i)}) + E_{\perp r}(e^{-j\beta_1(x\sin\theta_r)}) = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t)})$$

z=0, Lossless $E_\perp$

Again from the previous slide

$$E_{\perp i}(e^{-j\beta_1(x\sin\theta_i)}) + E_{\perp r}(e^{-j\beta_1(x\sin\theta_r)}) = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t)})$$

z=0, Lossless $E_{\perp}$

Conlusions

Conclusion 1)

$$\beta_1(x\sin\theta_i)=\beta_1(x\sin\theta_r)=\beta_2(x\sin\theta_t)$$

$$\theta_i=\theta_r$$

Phase is preserved at the boundary

Conclusion 2)

$$\beta_1(x\sin\theta_i)=\beta_2(x\sin\theta_t)$$

$$\omega\sqrt{\varepsilon_1\mu_1}(x\sin\theta_i)=\omega\sqrt{\varepsilon_2\mu_2}(x\sin\theta_t)$$

$$\omega\sqrt{\varepsilon_{r1}\varepsilon_0\mu_{r1}\mu_0}(x\sin\theta_i)=\omega\sqrt{\varepsilon_{r2}\varepsilon_0\mu_{r2}\mu_0}(x\sin\theta_t)$$

$$\omega n_1(x\sin\theta_i)=\omega n_2(x\sin\theta_t)$$

$$\omega\sqrt{\varepsilon_{r1}\mu_{r1}}(x\sin\theta_i)=\omega\sqrt{\varepsilon_{r2}\mu_{r2}}(x\sin\theta_t)$$

$$n_1(\sin\theta_i)=n_2(\sin\theta_t)$$

Snell's Law

n_1: *refractive index of the* 1^{st} *medium*

n_2: *refractive index of the* 2^{nd} *medium*

Again from the previous slide, using the conclusions in the equality below

$$E_{\perp i}(e^{-j\beta_1(x\sin\theta_i)}) + E_{\perp r}(e^{-j\beta_1(x\sin\theta_r)}) = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t)})$$

z=0, Lossless $E_{\perp}$

Again $\beta_1(x\sin\theta_i)=\beta_1(x\sin\theta_r)=\beta_2(x\sin\theta_t)$

Again $\theta_i=\theta_r$

(New) we find

$$E_{\perp i} + E_{\perp r} = E_{\perp t}$$

a20

This is the first equation, Equation 1

Remember, from slide 12, the meanings of **complex scalar** parameters in **a20** as seen in **the box on the right**

$$\vec{E}_{\perp i} = E_{\perp i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})\ \hat{y}$$

$$\vec{E}_{\perp r} = E_{\perp r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})\ \hat{y}$$

$$\vec{E}_{\perp t} = E_{\perp t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})\ \hat{y}$$

again Lossless $E_{\perp}$

We have obtained the boundary condition for electric field in slide 17 and obtained a relation between $E_{\perp i}, E_{\perp r}, E_{\perp t}$, which is the first equation.

Note that, θ_i and $E_{\perp i}$ are known, which are the inputs.

We can find θ_t (the angle of propagation in the second medium, i.e., the refraction angle) using Snell's law as found in slide 16. Thus, θ_t is, indeed, a known quantity.

$E_{\perp r}$ and $E_{\perp t}$ are unknowns which are to be found in terms of θ_i , θ_t and $E_{\perp i}$. This is our aim. We have two unkowns, as a result we need a second equation to solve for $E_{\perp r}$ and $E_{\perp t}$.

Next, using the tangential components of magnetic field vectors, we will find the boundary condition for the magnetic field vectors which will be the second equation we are looking for.

Again from slide 13,

$$\vec{H}_{\perp i} = (\frac{E_{\perp i}}{\eta_1})(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})(-\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z})$$

$$\vec{H}_{\perp r} = (\frac{E_{\perp r}}{\eta_1})(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})(\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z})$$

$$\vec{H}_{\perp t} = (\frac{\boldsymbol{E}_{\perp t}}{\eta_2})(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})(-\cos\theta_t\,\hat{x} + \sin\theta_t\,\hat{z})$$

Lossless $H_{\perp}$

New At the boundary (*z*=0 / on *xy*-plane), the tangential component is

$$\vec{H}_{\perp i,tangential} = (\frac{E_{\perp i}}{\eta_1})(e^{-j\beta_1(x\sin\theta_i + 0\cos\theta_i)})(-\cos\theta_i\,\hat{x})$$

$$\vec{H}_{\perp i,tangential} = (\frac{E_{\perp i}}{\eta_1})(e^{-j\beta_1(x\sin\theta_i)})(-\cos\theta_i\,\hat{x})$$

z=0 (on the boundary surface, *xy*-plane), Lossless, $H_{\perp}$