Again from slide 13,

$$\vec{H}_{\perp i} = \left(\frac{E_{\perp i}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\right)\left(-\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z}\right)$$

$$\vec{H}_{\perp r} = \left(\frac{E_{\perp r}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\right)\left(\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z}\right)$$

$$\vec{H}_{\perp t} = \left(\frac{\boldsymbol{E}_{\perp t}}{\eta_2}\right)\left(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\right)\left(-\cos\theta_t\,\hat{x} + \sin\theta_t\,\hat{z}\right)$$

Lossless $H_{\perp}$

New At the boundary (*z*=0 / on *xy*-plane), the tangential component (x- and y-component) is

$$\vec{H}_{\perp r,tangential} = \left(\frac{E_{\perp r}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_r - 0\cos\theta_r)}\right)\left(\cos\theta_r\,\hat{x}\right)$$

$$\vec{H}_{\perp r,tangential} = \left(\frac{E_{\perp r}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_r)}\right)\left(\cos\theta_r\,\hat{x}\right)$$

z=0 (on the boundary surface, *xy*-plane), Lossless, $H_{\perp}$

Again from slide 13,

$$\vec{H}_{\perp i} = \left(\frac{E_{\perp i}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\right)\left(-\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z}\right)$$

$$\vec{H}_{\perp r} = \left(\frac{E_{\perp r}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\right)\left(\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z}\right)$$

$$\vec{H}_{\perp t} = \left(\frac{\boldsymbol{E}_{\perp t}}{\eta_2}\right)\left(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\right)\left(-\cos\theta_t\,\hat{x} + \sin\theta_t\,\hat{z}\right)$$

Lossless $H_{\perp}$

New At the boundary (z=0 / on xy-plane), the tangential component (x- and y-component) is

$$\vec{H}_{\perp t,tangential} = \left(\frac{E_{\perp t}}{\eta_2}\right)\left(e^{-j\beta_2(x\sin\theta_t + 0\cos\theta_r)}\right)\left(-\cos\theta_t\,\hat{x}\right)$$

$$\vec{H}_{\perp t,tangential} = \left(\frac{E_{\perp t}}{\eta_2}\right)\left(e^{-j\beta_2(x\sin\theta_t)}\right)\left(-\cos\theta_t\,\hat{x}\right)$$

z=0 (on the boundary surface, xy-plane), Lossless, $H_{\perp}$

Summary of the tangential components of the magnetic field vectors at the boundary which is the *xy*-plane or equivalently *z*=0

$$\vec{H}_{\perp i,tangential} = (\frac{E_{\perp i}}{\eta_1})(e^{-j\beta_1(x\sin\theta_i)})(-\cos\theta_i\,\hat{x})$$

$$\vec{H}_{\perp r,tangential} = (\frac{E_{\perp r}}{\eta_1})(e^{-j\beta_1(x\sin\theta_r)})(\cos\theta_r\,\hat{x})$$

$$\vec{H}_{\perp t,tangential} = (\frac{E_{\perp t}}{\eta_2})(e^{-j\beta_2(x\sin\theta_t)})(-\cos\theta_t\,\hat{x})$$

z=0 (on the boundary surface, *xy*-plane), Lossless, $H_{\perp}$

In general,

$$\vec{H}_{\perp 1} = \vec{H}_{\perp i} + \vec{H}_{\perp r}$$

$$\vec{H}_{\perp 2} = \vec{H}_{\perp t}$$

Boundary Condition

$$\hat{n}_2 \times (\hat{H}_1 - \hat{H}_2) = \vec{J}_S$$

Boundary Condition for lossless case:

$\vec{J}_S = 0$ *(No conduction current at the boundary)*

$$\vec{H}_{\perp 1,tangential} = \vec{H}_{\perp 2,tangential}$$

Using BC at the boundary (z=0)

$$\vec{H}_{\perp i,tangential} + \vec{H}_{\perp r,tangential} = \vec{H}_{\perp t,tangential}$$

$$\left(\frac{E_{\perp i}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_i)}\right)\left(-\cos\theta_i\ \hat{x}\right) + \left(\frac{E_{\perp r}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_r)}\right)\left(\cos\theta_r\ \hat{x}\right) = \left(\frac{E_{\perp t}}{\eta_2}\right)\left(e^{-j\beta_2(x\sin\theta_t)}\right)\left(-\cos\theta_t\ \hat{x}\right)$$

In the equation above, we have used the tangential magnetic field vector epression given in slide 23

Starting equation which is found in slide 24 and repeated below is different from the equation in slide 16

$$\left(\frac{E_{\perp i}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_i)}\right)(-\cos\theta_i\ \hat{x}) + \left(\frac{E_{\perp r}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_r)}\right)(\cos\theta_r\ \hat{x}) = \left(\frac{E_{\perp t}}{\eta_2}\right)\left(e^{-j\beta_2(x\sin\theta_t)}\right)(-\cos\theta_t\ \hat{x})$$

BUT THE Conlusions ARE THE SAME, AS EXPECTED

Conclusion 1)

$$\beta_1(x\sin\theta_i)=\beta_1(x\sin\theta_r)=\beta_2(x\sin\theta_t)$$

$$\theta_i=\theta_r$$

Phase is preserved at the boundary

Conclusion 2)

$$\beta_1(x\sin\theta_i)=\beta_2(x\sin\theta_t)$$

$$\omega\sqrt{\varepsilon_1\mu_1}(x\sin\theta_i)=\omega\sqrt{\varepsilon_2\mu_2}(x\sin\theta_t)$$

$$\omega\sqrt{\varepsilon_{r1}\varepsilon_0\mu_{r1}\mu_0}(x\sin\theta_i)=\omega\sqrt{\varepsilon_{r2}\varepsilon_0\mu_{r2}\mu_0}(x\sin\theta_t)$$

$$\omega n_1(x\sin\theta_i)=\omega n_2(x\sin\theta_t)$$

$$\omega\sqrt{\varepsilon_{r1}\mu_{r1}}(x\sin\theta_i)=\omega\sqrt{\varepsilon_{r2}\mu_{r2}}(x\sin\theta_t)$$

$$n_1(\sin\theta_i)=n_2(\sin\theta_t)$$

Snell's Law

n_1: *refractive index of the* 1^{st} *medium*

n_2: *refractive index of the* 2^{nd} *medium*

$$\left(\frac{E_{\perp i}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_i)}\right)(-\cos\theta_i\,\hat{x}) + \left(\frac{E_{\perp r}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_r)}\right)(\cos\theta_r\,\hat{x}) = \left(\frac{E_{\perp t}}{\eta_2}\right)\left(e^{-j\beta_2(x\sin\theta_t)}\right)(-\cos\theta_t\,\hat{x})$$

Using the starting equation found in slide 24 (and rewritten above for convenience), we obtain the second equation which comes from the boundary condition for the tangential components of the magnetic field vectors. This second equation, together with the first equation found in slide 18 ($E_{\perp i} + E_{\perp r} = E_{\perp t}$), will be used to express the unknown quantities $E_{\perp r}$ and $E_{\perp t}$ in terms of the known quantities like $E_{\perp i}$, θ_i and θ_t:

The equation at the top can be simplied as the following using the Conclusion 1 in slide 25

$$\left(\frac{E_{\perp i}}{\eta_1}\right)(-\cos\theta_i\,\hat{x}) + \left(\frac{E_{\perp r}}{\eta_1}\right)(\cos\theta_r\,\hat{x}) = \left(\frac{E_{\perp t}}{\eta_2}\right)(-\cos\theta_t\,\hat{x})$$

Dropping the $\hat{x}$ on both sides we obtain the equality below which is the 2^{nd} Equation

$$\left(\frac{E_{\perp i}}{\eta_1}\right)\cos\theta_i - \left(\frac{E_{\perp r}}{\eta_1}\right)\cos\theta_r = \left(\frac{E_{\perp t}}{\eta_2}\right)\cos\theta_t$$

a21

Again

$$\left(\frac{E_{\perp i}}{\eta_1}\right)\cos\theta_i - \left(\frac{E_{\perp r}}{\eta_1}\right)\cos\theta_r = \left(\frac{E_{\perp t}}{\eta_2}\right)\cos\theta_t \quad \text{a21}$$

New

$$E_{\perp i} + E_{\perp r} = E_{\perp t} \quad \text{a20}$$

$\theta_i = \theta_r$ (found before)

$\Rightarrow \cos(\theta_i) = \cos(\theta_r) \Rightarrow$

$$\left(\frac{E_{\perp i}}{\eta_1} - \frac{E_{\perp r}}{\eta_1}\right)\cos(\theta_i) = \frac{E_{\perp t}}{\eta_2}\cos(\theta_t) \quad \text{a21}$$

Again

Enlarge a20 with $\eta_1 \cos(\theta_t)$	=>	$E_{\perp i}(\eta_1\cos(\theta_t)) + E_{\perp r}(\eta_1\cos(\theta_t)) = E_{\perp t}(\eta_1\cos(\theta_t))$	a22
Enlarge a21 with $\eta_1\eta_{c2}$	=>	$E_{\perp i}(\eta_2\cos(\theta_i)) - E_{\perp r}(\eta_2\cos(\theta_i)) = E_{\perp t}(\eta_1\cos(\theta_t))$	a23
Enlarge a20 with $\eta_2 \cos(\theta_i)$	=>	$E_{\perp i}(\eta_2\cos(\theta_i)) + E_{\perp r}(\eta_2\cos(\theta_i)) = E_{\perp t}(\eta_2\cos(\theta_i))$	a24

Subtract a22 from a23 and write

New

$$\Gamma_{\perp} = \frac{E_{\perp r}}{E_{\perp i}} = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

Add a23 to a24 and write

New

$$\tau_{\perp} = \frac{E_{\perp t}}{E_{\perp i}} = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Again

Subtract a22 from a23 and write

$$\Gamma_{\perp} = \frac{E_{\perp r}}{E_{\perp i}} = \frac{\eta_2 \cos(\theta_i) - \eta_1 \cos(\theta_t)}{\eta_2 \cos(\theta_i) + \eta_1 \cos(\theta_t)}$$

Again

Add a23 to a24 and write

$$\tau_{\perp} = \frac{E_{\perp t}}{E_{\perp i}} = \frac{2\eta_2 \cos(\theta_i)}{\eta_2 \cos(\theta_t) + \eta_1 \cos(\theta_i)}$$

New

Check $(1 + \Gamma_{\perp}) = \tau_{\perp}$

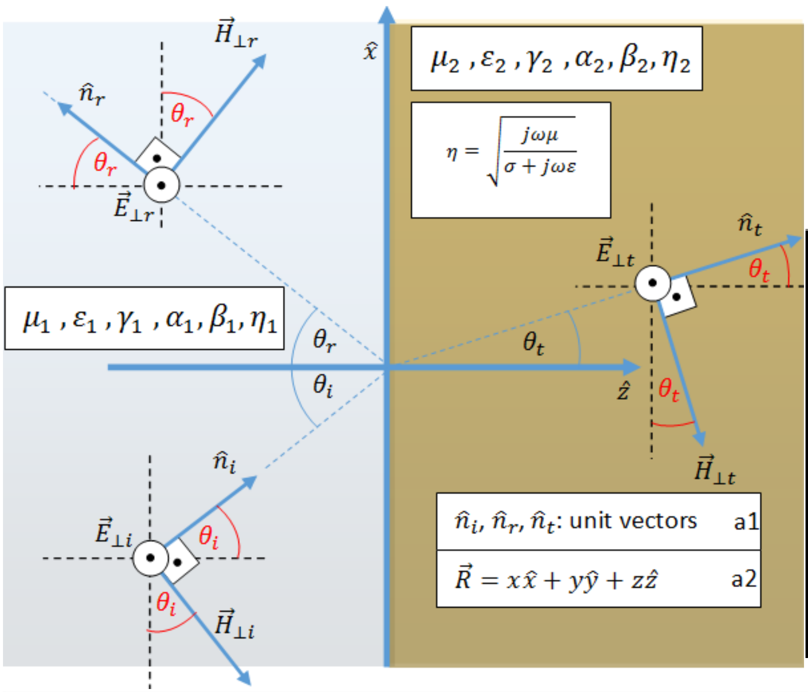
Summary, ⊥ Polarization

From Slide 17 or Slide 24

$$\theta_i = \theta_r$$

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$



$$\vec{E}_{\perp i} = E_{\perp i}\left(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\right)\hat{y}$$

$$\vec{E}_{\perp r} = E_{\perp r}\left(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\right)\hat{y}$$

$$\vec{E}_{\perp t} = E_{\perp t}\left(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\right)\hat{y}$$

From Slide 12 Lossless $E_\perp$

$$\vec{H}_{\perp i} = \left(\frac{E_{\perp i}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\right)\left(-\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z}\right)$$

$$\vec{H}_{\perp r} = \left(\frac{E_{\perp r}}{\eta_1}\right)\left(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\right)\left(\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z}\right)$$

$$\vec{H}_{\perp t} = \left(\frac{\boldsymbol{E}_{\perp t}}{\eta_2}\right)\left(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\right)\left(-\cos\theta_t\,\hat{x} + \sin\theta_t\,\hat{z}\right)$$

From Slide 13 Lossless $H_\perp$

$$\Gamma_\perp = \frac{E_{\perp r}}{E_{\perp i}} = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\tau_\perp = \frac{E_{\perp t}}{E_{\perp i}} = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

$$(1 + \Gamma_\perp) = \tau_\perp$$