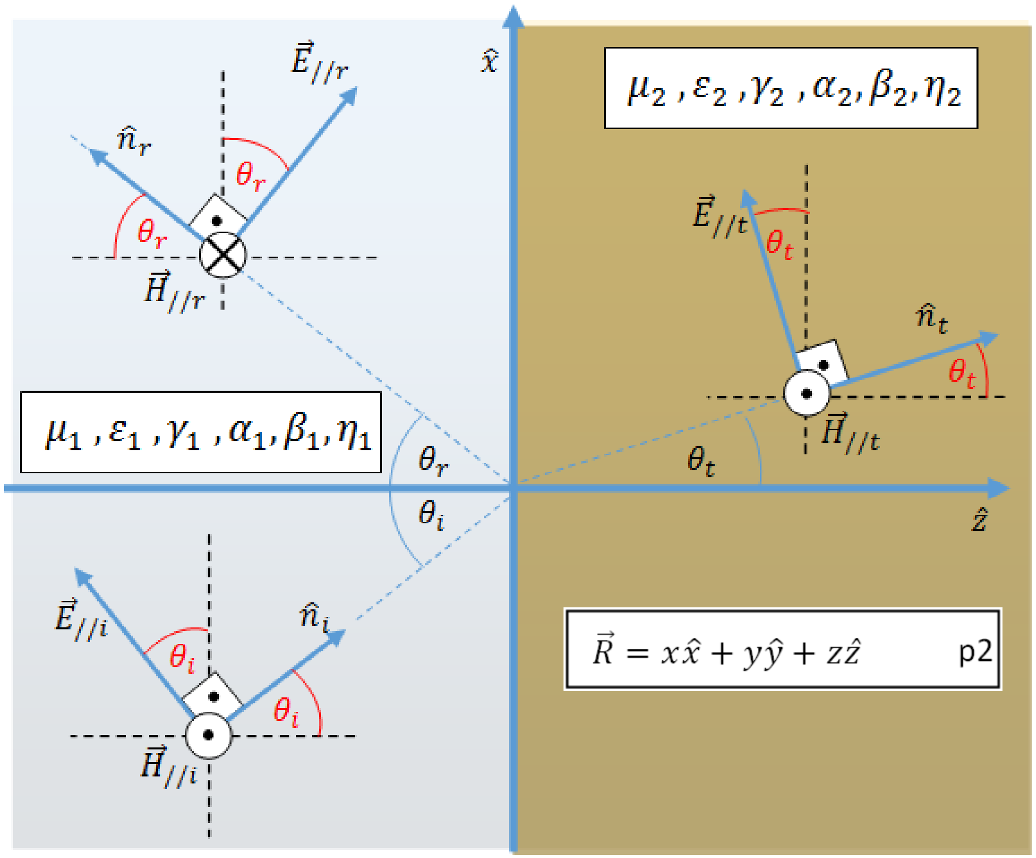
Parallel Polarization ($\vec{E}_{//}$)

Electric field is **parallel** to the propagation plane (here, *xz*-plane)



$$\hat{n}_i = \sin\theta_i\, \hat{x} + \cos\theta_i\, \hat{z}$$
$$\hat{n}_r = \sin\theta_r\, \hat{x} - \cos\theta_r\, \hat{z}$$
$$\hat{n}_t = \sin\theta_t\, \hat{x} + \cos\theta_t\, \hat{z}$$

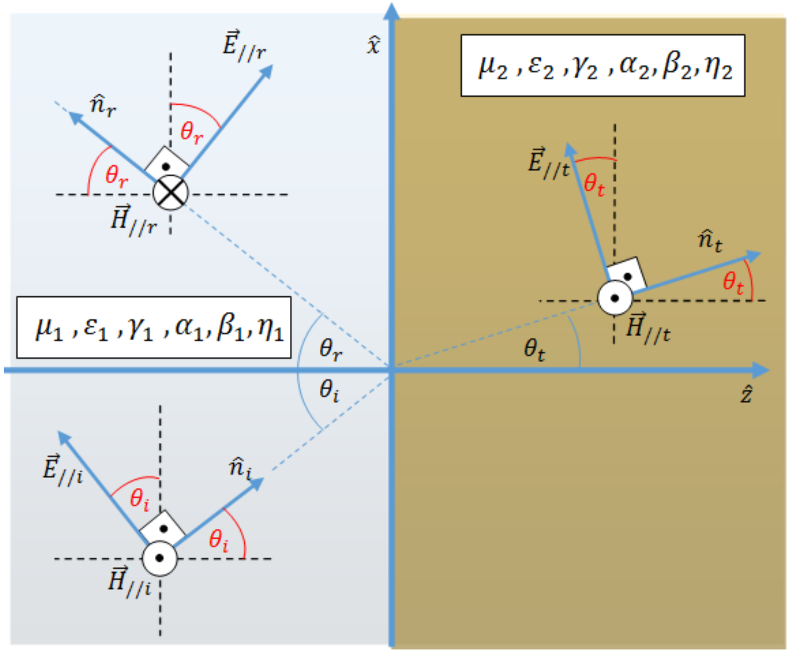
p4

$$\hat{n}_i \cdot \vec{R} = x\sin\theta_i + z\cos\theta_i$$
$$\hat{n}_r \cdot \vec{R} = x\sin\theta_r - z\cos\theta_r$$
$$\hat{n}_t \cdot \vec{R} = x\sin\theta_t + z\cos\theta_t$$

p2,p4->p5

$$\gamma_1 = \alpha_1 + j\beta_1$$
$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon_1}} = \sqrt{\frac{j\omega\mu_{r1}\mu_0}{\sigma_1 + j\omega\varepsilon_{r1}\varepsilon_0}}$$

$$\gamma_2 = \alpha_2 + j\beta_2$$
$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}} = \sqrt{\frac{j\omega\mu_{r2}\mu_0}{\sigma_2 + j\omega\varepsilon_{r2}\varepsilon_0}}$$



1	2	3	4
$\vec{E}_{//}$	$\boldsymbol{E}_{//i}$	$\hat{E}_{//i}$	$E_{//i}$**?**

1

$$\vec{E}_{//i} = \boldsymbol{E}_{//i}\,\hat{E}_{//i}$$
$$\vec{E}_{//r} = \boldsymbol{E}_{//r}\,\hat{E}_{//r}$$
$$\vec{E}_{//t} = \boldsymbol{E}_{//t}\,\hat{E}_{//t}$$

=>

p6

4

$$E_{//i} = |E_{//i}|(e^{j\theta_{E//i}})$$
$$E_{//r} = |E_{//r}|(e^{j\theta_{E//r}})$$
$$E_{//t} = |E_{//t}|(e^{j\theta_{E//t}})$$

3

$$\hat{E}_{//i} = \cos\theta_i\,\hat{x} - \sin\theta_i\,\hat{z}$$
$$\hat{E}_{//r} = \cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z}$$
$$\hat{E}_{//t} = \cos\theta_t\,\hat{x} - \sin\theta_t\,\hat{z}$$

p7

again

1 $\vec{E}_{//}$ 2 $\boldsymbol{E}_{//i}$ 3 $\hat{E}_{//i}$ 4 $E_{//i}$?

$$\left.\begin{aligned}\vec{E}_{//i} &= \boldsymbol{E}_{//i}\,\hat{E}_{//i}\\ \vec{E}_{//r} &= \boldsymbol{E}_{//r}\,\hat{E}_{//r}\\ \vec{E}_{//t} &= \boldsymbol{E}_{//t}\,\hat{E}_{//t}\end{aligned}\right\} =>$$

1

p6

again

$$\left.\begin{aligned}\vec{E}_{//i} &= \boldsymbol{E}_{//i}\,\hat{E}_{//i}\\ \vec{E}_{//r} &= \boldsymbol{E}_{//r}\,\hat{E}_{//r}\\ \vec{E}_{//t} &= \boldsymbol{E}_{//t}\,\hat{E}_{//t}\end{aligned}\right\} =>$$

p6

new

$$\boldsymbol{E}_{//i} = E_{//i}\left(e^{-\gamma_1 \hat{n}_i\cdot\vec{R}}\right) = \left|E_{//i}\right|\left(e^{j\theta_{E//i}}\right)\left(e^{-\gamma_1 \hat{n}_i\cdot\vec{R}}\right)$$

2

$$\boldsymbol{E}_{//r} = E_{//r}\left(e^{-\gamma_1 \hat{n}_r\cdot\vec{R}}\right) = \left|E_{//r}\right|\left(e^{j\theta_{E//r}}\right)\left(e^{-\gamma_1 \hat{n}_r\cdot\vec{R}}\right)$$

$$\boldsymbol{E}_{//t} = E_{//t}\left(e^{-\gamma_2 \hat{n}_t\cdot\vec{R}}\right) = \left|E_{//t}\right|\left(e^{j\theta_{E//t}}\right)\left(e^{-\gamma_2 \hat{n}_t\cdot\vec{R}}\right)$$

p8

IMPORTANT

again

$$\boldsymbol{E}_{//i} = E_{//i}\,(e^{-\gamma_1 \hat{n}_i \cdot \vec{R}}) = |E_{//i}|(e^{j\theta_{E//i}})(e^{-\gamma_1 \hat{n}_i \cdot \vec{R}}) \quad 2$$

$$\boldsymbol{E}_{//r} = E_{//r}\,(e^{-\gamma_1 \hat{n}_r \cdot \vec{R}}) = |E_{//r}|(e^{j\theta_{E//r}})(e^{-\gamma_1 \hat{n}_r \cdot \vec{R}})$$

$$\boldsymbol{E}_{//t} = E_{//t}\,(e^{-\gamma_2 \hat{n}_t \cdot \vec{R}}) = |E_{//t}|(e^{j\theta_{E//t}})(e^{-\gamma_2 \hat{n}_t \cdot \vec{R}})$$

p8

γ_1 **is used for Incident and Reflected Waves since they propagate in Medium 1**

γ_2 **is used for Transmitted Wave since it propagates in Medium 2**

Again,

Observe the same routine marked for the incident wave also for reflected and transmitted waves

$$\boldsymbol{E}_{//i} = E_{//i}\left(e^{-\gamma_1 \hat{n}_i \cdot \vec{R}}\right) = |E_{//i}|\left(e^{j\theta_{E//i}}\right)\left(e^{-\gamma_1 \hat{n}_i \cdot \vec{R}}\right)$$

$$\boldsymbol{E}_{//r} = E_{//r}\left(e^{-\gamma_1 \hat{n}_r \cdot \vec{R}}\right) = |E_{//r}|\left(e^{j\theta_{E//r}}\right)\left(e^{-\gamma_1 \hat{n}_r \cdot \vec{R}}\right)$$

$$\boldsymbol{E}_{//t} = E_{//t}\left(e^{-\gamma_2 \hat{n}_t \cdot \vec{R}}\right) = |E_{//t}|\left(e^{j\theta_{E//t}}\right)\left(e^{-\gamma_2 \hat{n}_t \cdot \vec{R}}\right)$$

p8

new 2

$$\boldsymbol{E}_{//i} = E_{//i}\left(e^{-(\alpha_1 + j\beta_1)\hat{n}_i \cdot \vec{R}}\right) = |E_{//i}|\left(e^{j\theta_{E//i}}\right)\left(e^{-(\alpha_1 + j\beta_1)\hat{n}_i \cdot \vec{R}}\right)$$

$$\boldsymbol{E}_{//r} = E_{//r}\left(e^{-(\alpha_1 + j\beta_1)\hat{n}_r \cdot \vec{R}}\right) = |E_{//r}|\left(e^{j\theta_{E//r}}\right)\left(e^{-(\alpha_1 + j\beta_1)\hat{n}_r \cdot \vec{R}}\right)$$

$$\boldsymbol{E}_{//t} = E_{//t}\left(e^{-(\alpha_2 + j\beta_2)\hat{n}_t \cdot \vec{R}}\right) = |E_{//t}|\left(e^{j\theta_{E//t}}\right)\left(e^{-(\alpha_2 + j\beta_2)\hat{n}_t \cdot \vec{R}}\right)$$

Modified p9

Again,

Observe the same routine marked for the incident wave also for reflected and transmitted waves

2

$$\boldsymbol{E}_{//i} = E_{//i}\left(e^{-(\alpha_1+j\beta_1)\hat{n}_i\cdot\vec{R}}\right) = \left|E_{//i}\right|\left(e^{j\theta_{E//i}}\right)\left(e^{-(\alpha_1+j\beta_1)\hat{n}_i\cdot\vec{R}}\right)$$

$$\boldsymbol{E}_{//r} = E_{//r}\left(e^{-(\alpha_1+j\beta_1)\hat{n}_r\cdot\vec{R}}\right) = \left|E_{//r}\right|\left(e^{j\theta_{E//r}}\right)\left(e^{-(\alpha_1+j\beta_1)\hat{n}_r\cdot\vec{R}}\right)$$

$$\boldsymbol{E}_{//t} = E_{//t}\left(e^{-(\alpha_2+j\beta_2)\hat{n}_t\cdot\vec{R}}\right) = \left|E_{//t}\right|\left(e^{j\theta_{E//t}}\right)\left(e^{-(\alpha_2+j\beta_2)\hat{n}_t\cdot\vec{R}}\right)$$

Modified p9

New, using p5 in 'modified p9', 'modified p10' is obtained

$$\boldsymbol{E}_{//i} = \left|E_{//i}\right|\left(e^{j\theta_{E//i}}\right)\left(e^{-\alpha_1(x\sin\theta_i+z\cos\theta_i)}\right)\left(e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)}\right)$$

$$\boldsymbol{E}_{//r} = \left|E_{//r}\right|\left(e^{j\theta_{E//r}}\right)\left(e^{-\alpha_1(x\sin\theta_r-z\cos\theta_r)}\right)\left(e^{-j\beta_1(x\sin\theta_r-z\cos\theta_r)}\right)$$

$$\boldsymbol{E}_{//t} = \left|E_{//t}\right|\left(e^{j\theta_{E//t}}\right)\left(e^{-\alpha_2(x\sin\theta_t+z\cos\theta_t)}\right)\left(e^{-j\beta_2(x\sin\theta_t+z\cos\theta_t)}\right)$$

modified p10

Again

$$\boldsymbol{E}_{//i} = |E_{//i}|(e^{j\theta_{E//i}})(e^{-\alpha_1(x\sin\theta_i + z\cos\theta_i)})(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})$$

$$\boldsymbol{E}_{//r} = |E_{//r}|(e^{j\theta_{E//r}})(e^{-\alpha_1(x\sin\theta_r - z\cos\theta_r)})(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})$$

$$\boldsymbol{E}_{//t} = |E_{//t}|(e^{j\theta_{E//t}})(e^{-\alpha_2(x\sin\theta_t + z\cos\theta_t)})(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})$$

modified p10

New,

Using 'modified 10',

Assume $\alpha_1 = \alpha_2 = 0$ (lossless medium 1 and lossless medium 2) =>

$$e^{-0(x\sin\theta_i + z\cos\theta_i)} = e^{-0(x\sin\theta_r - z\cos\theta_r)} = e^{-0(x\sin\theta_t + z\cos\theta_t)} = 1 \Rightarrow$$

$$\boldsymbol{E}_{//i} = |E_{//i}|(e^{j\theta_{E//i}})(1)(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}) = E_{//i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})$$

$$\boldsymbol{E}_{//r} = |E_{//r}|(e^{j\theta_{E//r}})(1)(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}) = E_{//r}(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)})$$

$$\boldsymbol{E}_{//t} = |E_{//t}|(e^{j\theta_{E//t}})(1)(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}) = E_{//t}(e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)})$$

modified p10, lossless case

Again,

$$\boldsymbol{E}_{//i} = |E_{//i}|(e^{j\theta_{E//i}})(1)(e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)}) = E_{//i}(e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)})$$

$$\boldsymbol{E}_{//r} = |E_{//r}|(e^{j\theta_{E//r}})(1)(e^{-j\beta_1(x\sin\theta_r-z\cos\theta_r)}) = E_{//r}(e^{-j\beta_1(x\sin\theta_r-z\cos\theta_r)})$$

$$\boldsymbol{E}_{//t} = |E_{//t}|(e^{j\theta_{E//t}})(1)(e^{-j\beta_2(x\sin\theta_t+z\cos\theta_t)}) = E_{//t}(e^{-j\beta_2(x\sin\theta_t+z\cos\theta_t)})$$

modified p10, lossless case

$$\hat{E}_{//i} = \cos\theta_i\,\hat{x} - \sin\theta_i\,\hat{z}$$

$$\hat{E}_{//r} = \cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z}$$

$$\hat{E}_{//t} = \cos\theta_t\,\hat{x} - \sin\theta_t\,\hat{z}$$

p7

$$\left.\begin{aligned} \vec{E}_{//i} &= \boldsymbol{E}_{//i}\,\hat{E}_{//i} \\ \vec{E}_{//r} &= \boldsymbol{E}_{//r}\,\hat{E}_{//r} \\ \vec{E}_{//t} &= \boldsymbol{E}_{//t}\,\hat{E}_{//t} \end{aligned}\right\} \Rightarrow$$

p6

New,

$$\vec{E}_{//i} = E_{//i}(e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)})(\cos\theta_i\,\hat{x} - \sin\theta_i\,\hat{z})$$

$$\vec{E}_{//r} = E_{//r}(e^{-j\beta_1(x\sin\theta_r-z\cos\theta_r)})(\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z})$$

$$\vec{E}_{//t} = E_{//t}(e^{-j\beta_2(x\sin\theta_t+z\cos\theta_t)})(\cos\theta_t\,\hat{x} - \sin\theta_t\,\hat{z})$$

Lossless $E_{//}$

New,

$$\vec{H}_{//i} = (\frac{\hat{n}_i \times \vec{E}_{//i}}{\eta_1}) = (\frac{\hat{n}_i \times \boldsymbol{E}_{//i}\hat{E}_{//i}}{\eta_1}) = (\frac{\boldsymbol{E}_{//i}}{\eta_1})(\hat{n}_i \times \hat{E}_{//i})$$

$$= (\frac{\boldsymbol{E}_{//i}}{\eta_1})(\sin\theta_i\,\hat{x} + \cos\theta_i\hat{z}) \times (\cos\theta_i\,\hat{x} - \sin\theta_i\,\hat{z})$$

$$= (\frac{\boldsymbol{E}_{//i}}{\eta_1})\,[(\sin\theta_i).(-\sin\theta_i)(-\hat{y}) + (\cos\theta_i).(\cos\theta_i)\hat{y}]$$

$$= (\frac{\boldsymbol{E}_{//i}}{\eta_1})\,[\sin^2\theta_i(+\hat{y}) + \cos^2\theta_i(+\hat{y})] = (\frac{\boldsymbol{E}_{//i}}{\eta_1})(+1)\hat{y}$$

modified p14

New,

$$\vec{H}_{//i} = (\frac{\boldsymbol{E}_{//i}}{\eta_1})\,\hat{H}_{//i} = (\frac{\boldsymbol{E}_{//i}}{\eta_1})(+\hat{y}) = \boldsymbol{H}_{//i}\,\hat{H}_{//i}$$

$$\frac{\boldsymbol{E}_{//i}}{\eta_1} = \boldsymbol{H}_{//i} = \frac{E_{//i}}{\eta_1}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})$$

$$\vec{H}_{//i} = \frac{E_{//i}}{\eta_1}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})\hat{y}$$

$$\vec{H}_{//i} = H_{//i}(e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)})\,\hat{y} = \boldsymbol{H}_{//i}\,\hat{H}_{//i}$$

modified p15

New,

$$\vec{H}_{//r} = \left(\frac{\hat{n}_r \times \vec{E}_{//r}}{\eta_1}\right) = \left(\frac{\hat{n}_r \times \boldsymbol{E}_{//r}\hat{E}_{//r}}{\eta_1}\right) = \left(\frac{\boldsymbol{E}_{//r}}{\eta_1}\right)(\hat{n}_r \times \hat{E}_{//r})$$

$$= \left(\frac{\boldsymbol{E}_{//r}}{\eta_1}\right)(\sin\theta_r\,\hat{x} - \cos\theta_r\,\hat{z}) \times (\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z})$$

$$= \left(\frac{\boldsymbol{E}_{//r}}{\eta_1}\right)[(\sin\theta_r).(\sin\theta_r)(-\hat{y}) + (-\cos\theta_r).(\cos\theta_r)\hat{y}]$$

$$= \left(\frac{\boldsymbol{E}_{//r}}{\eta_1}\right)[\sin^2\theta_r(-\hat{y}) + \cos^2\theta_r(-\hat{y})] = \left(\frac{\boldsymbol{E}_{//r}}{\eta_1}\right)(-1)\hat{y}$$

modified p16

New,

$$\vec{H}_{//r} = \left(\frac{\boldsymbol{E}_{//r}}{\eta_1}\right)\hat{H}_{//r} = \left(\frac{\boldsymbol{E}_{//r}}{\eta_1}\right)(-\hat{y}) = \boldsymbol{H}_{//r}\,\hat{H}_{//r}$$

$$\frac{\boldsymbol{E}_{//r}}{\eta_1} = \boldsymbol{H}_{//r} = \frac{E_{//r}}{\eta_1}\left(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\right)$$

$$\vec{H}_{//r} = \frac{E_{//r}}{\eta_1}\left(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\right)(-\hat{y})$$

$$\vec{H}_{//r} = H_{//r}\left(e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\right)(-\hat{y}) = \boldsymbol{H}_{//r}\,\hat{H}_{//r}$$

modified p17