MAGNETIC FORCES, MATERIALS AND DEVICES

Force on a Charged Particle

According to earlier information, the electric force $\mathbf{F_e}$, on a stationary or moving electric charge *Q* in an electric field is given by Coulornb's experimental law and is related to the electric field intensity E as

$$F_e = QE \qquad (2.1)$$

This shows that if *Q* is Positive, F_e and E have the same direction.

A magnetic field can exert force only on a moving charge. From experiments, it is found that the magnetic force F_m experienced by a charge *Q* moving with a velocity **u** in a magnetic field **B** is

$$F_m = Qu \times B \qquad (2.2)$$

This clearly shows that F_m is perpendicular to both u and **B.**

From eqs. (2.1) and (2.2), a comparison between the electric force F_e and the magnetic force F_m can be made. F_e is independent of the velocity of the charge and can perform work on the charge and change its kinetic energy. Unlike F_e, F_m depends on the charge velocity and is normal to it. F_m cannot perform work because it is at right angles to the direction of motion, of the charge ($F_m.dl = 0$); it does not cause an increase in kinetic energy of the charge. The magnitude of F_m is generally small compared to F_e except at high velocities.

For a moving charge Q in the Presence of both electric and magnetic fields, the total force on the charge is given by

$$\mathbf{F} = \boldsymbol{F_e} + \mathbf{F_m}$$

or

$$\mathbf{F} = \mathbf{Q}\ (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \qquad (2.3)$$

This is known as the *Lorentz force equation.* It relates mechanical force to electrical force. If the mass of the charged Particle moving in **E** and **B** fields is m, by Newton's second law of motion.

$$F = m\frac{du}{dt} = Q(E + u \times B) \qquad (2.4)$$

The solution to this equation is important in determining the motion of charged particles in E and B fields. We should bear in mind that in such fields, energy transfer can be only by means of the electric field. A summary on the force exerted on a charged particle is given in table 2.1.

TABLE 2.1: Force on a Charged Particle

State of Particle	**E Field**	**B Field**	**Combined E and B Fields**
Stationary	*QE*	-	*QE*

Moving	*QE*	*Qu x* B	*Q(E* + u x B)

The magnetic field B is defined as the force per unit current element

Alternatively, **B** may be defined from eq. (2.2) as the vector which satisfies $F_m / q = u \times \mathbf{B}$ just as we defined electric field E as the force per unit charge, F_e / q.

Force between Two Current Elements

Let us now consider the force between two elements $I_1\, dl_1$ *and* $I_2\, dl_2$. According to Biot-Savart's law, both current elements produce magnetic fields. So we may find the force $d(dF_1)$ on element $I_1\, dl_1$ due to the field $d\mathbf{B}_2$ produced by element $I_2\, dl_2$ as shown in Figure 2.1.

As per equation

$$d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B}_2$$

$$d(d\mathbf{F}_1) = I_1\, d\mathbf{l}_1 \times d\mathbf{B}_2 \qquad (2.5)$$

But from Biot-Savart's law,

$$dB_2 = \frac{\mu_0 I_2 dl_2 \times a_{R_{21}}}{4\pi R_{21}^2} \qquad (2.6)$$

Hence,

$$d(dF_1) = \frac{\mu_0 I_1 dl_1 \times (I_2 dl_2 \times a_{R_{21}})}{4\pi R_{21}^2} \tag{2.7}$$

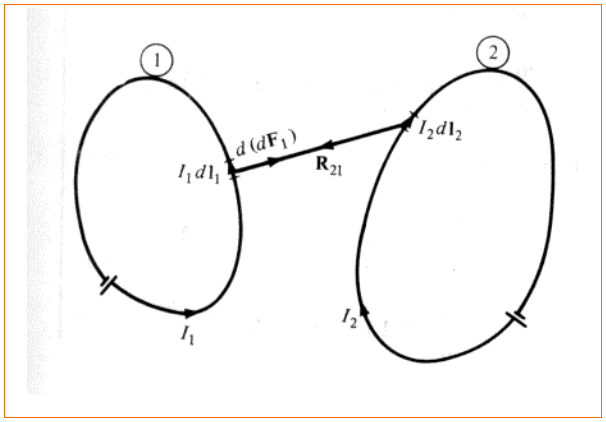

Figure 2.1: Force between two current loops.

This equation is essentially the law of force between two current elements and is analogous to Coulomb's law, which expresses the force between two stationary charges. From eq. (2.7), we obtain the total force $\mathbf{F_1}$ on current loop 1 due to current loop 2 shown Figure 2.1 as

$$F_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L2} \frac{dl_1 \times (dl_2 \times a_{R_{21}})}{R_{21}^2} \tag{2.8}$$

Although this equation appears complicated, we should remember that it is based on eq. (2.5). It is eq. (8. 10) that is of fundamental importance.

The force $\mathbf{F_2}$ on loop 2 due to the magnetic field $\mathbf{B_1}$ from loop 1 is obtained from eq. (2.8) by interchanging subscripts 1 and 2. It can be shown that $\mathbf{F_2 = - F_1}$; thus F_1 and F_2 obey Newton's third law that action and reaction are equal and opposite. It is worthwhile to mention that eq. (2.8) was experimentally established by Qersted and Ampete; Biot and Savart (Ampere's colleagues) actually based their law on it.

MAGNETIC TORQUE AND MOMENT

Now that we have considered the force on a current loop in a magnetic field, we can determine the torque on it. The concept of a current loop experiencing a torque in a magnetic field is of paramount importance in understanding the behavior of orbiting charged particles, d.c. motors, and generators. If the loop is placed parallel to a magnetic field, it experiences a force that tends to rotate it.

The torque T (or mechanical moment of force) on the loop is the, vector product of the force F and the moment arm r.

That is,

$$\mathbf{T} = \mathbf{r} \times \mathbf{F} \tag{2.9}$$

and its units are Newton-meters.

Let us apply this to a rectangular loop of length l and width w placed in a uniform magnetic field **B** as shown in Figure 8.5(a). From this figure, we notice that dl is parallel to **B** along sides 12 and 34 of the loop and no force is exerted on those sides. Thus

$$F = I\int_2^3 dl \times B + I\int_4^1 dl \times B$$

$$= I\int_2^l dz\ a_z \times B + I\int_l^0 dz\ a_z \times B$$

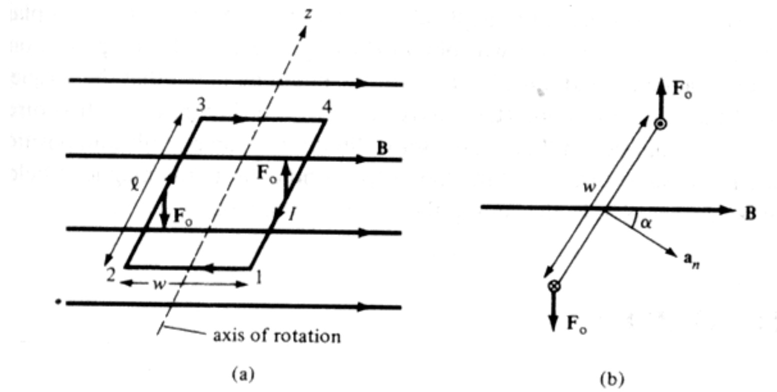

Figure 2.2: Rectangular planar loop in a uniform magnetic field.

or

$$\mathbf{F} = \mathbf{F_0} - \mathbf{F_0} = 0 \qquad (2.10)$$

Where, $|\mathbf{F_0}| = \boldsymbol{I}\ \mathbf{Bl}$ because **B** is uniform. Thus, no force is exerted on the loop as a whole. However, $\mathbf{F_0}$ and $-\mathbf{F_0}$ act at different points on the loop, thereby creating a couple. If the normal to the plane of the loop makes an angle α with **B**, as shown in the cross-sectional view of Figure 2.2(b), the torque on the loop is

$$|\mathbf{T}| = |\mathbf{F_0}|\ w \sin \alpha$$

or

$$T = B\ I\ l\ w \sin \alpha \qquad (2.11)$$

But lw = S, the area of the loop. Hence,

$$T = BIS \sin \alpha \qquad (2.12)$$

We define the quantity

$$\mathbf{m} = I\,S\,a_n \qquad (2.13)$$

as the *magnetic dipole moment* (in A/M^2) of the loop. In eq. (2.13), $\mathbf{a_n}$ is a unit normal vector to the plane of the loop and its direction is determined by the right-hand rule: fingers in the direction of current Hand thumb along $\mathbf{a_n}$.

The magnetic dipole moment is the product of current and area of the loop; its reaction is normal to the loop.

Introducing eq. (2.13) in eq. (2.12), we obtain

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \qquad (2.14)$$

3.0 STOKE'S THEOREM

Stoke's Theorem relates a line integral to the surface integral and vice-versa, that is

$$\oint_C H \cdot dL = \int_S (\nabla \times H) \cdot dS \qquad (3.1)$$

FORCE ON A MOVING CHARGE DUE TO ELECTRIC AND MAGNETIC FIELDS

If there is a charge or a moving charge, Q in an electric field, E, there exists a force on the charge. This force is given by

$$F_E = QE \qquad (3.2)$$

If a charge, Q moving with a velocity, **V** is placed in a magnetic field, **B (=μH)**, then there exists a force on the charge (Fig. 3.1). This force is given by

$$F_H = Q(V \text{ x } B) \qquad (3.3)$$

B = magnetic flux density, (wb/m^2)

V = velocity of the charge, m/s

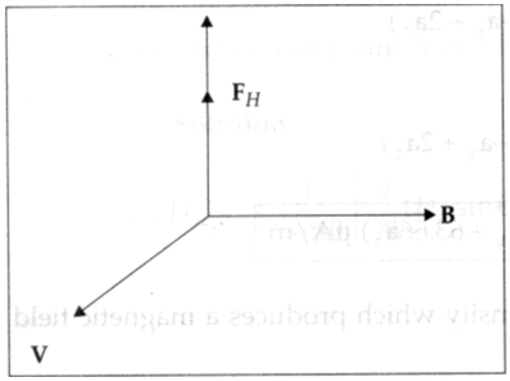

Fig. 3.1: Direction of field, velocity and force

If the charge, Q is placed in both electric and magnetic fields, then the force on the charge is

$$\mathbf{F} = Q\,(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \qquad (3.4)$$

This equation is known as **Lorentz force equation.**

Problem 1: A charge of 12 C has velocity of $\mathbf{5a_x} + \mathbf{2a_y} - 3a_z$ m/s. Determine **F** on the charge in the field of (a) $E=18a_x, +5a_y +10a_z$ V/m

(b) $\mathbf{B} = 4a_x + 4a_y + 3a_z$ wb/m^2.

Solution:

(a) The force, **F** on the charge, Q due to E is

$$F = QE = 12\,(18a_x + 5a_y + 10a_z)$$

$= 216a_x + 60a_y + 120a_z$

or, $F = Q\,|E| = 12\sqrt{18^2 + 5^2 + 10^2}$

$F = 254.27$ N

(b) The force F on the charge due to B is

$F = Q[V \times B)$

Here $V = 5a_x + 2a_y - 3a_z$ m/s

$B = 4\,a_x + 4\,a_y + 3\,a_z$ wb / m^2

$F = 12\,[18a_x - 27a_y + 12a_z]$

or, $F = 12\sqrt{(324 + 729 + 144)}$

$F = 415.17$ N

FORCE ON A CURRENT ELEMENT IN A MAGNETIC FIELD

The force on a current element when placed in a magnetic field, B is

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B} \tag{3.5}$$

or,

$$F = I\,L\,B\,\text{Sin}\,\theta \quad \text{Newton} \tag{3.6}$$

where θ is the angle between the direction of the current element and the direction of magnetic flux density

$\mathbf{B}$ = magnetic flux density, wb/m^2

$I\mathbf{L}$ = current element, Amp-m

Proof: Consider a differential charge, dQ to be moving with a velocity, V in a magnetic field, $\mathbf{H} = (\mathbf{B}/\mu)$. Then the differential force on the charge is given by

$$d\mathbf{F} = dQ\,(\mathbf{V} \times \mathbf{B}) \tag{3.7}$$

But

$$dQ = \rho_\upsilon\, d\upsilon$$

$$dF = \rho_\upsilon\, d\upsilon\,(V \times B)$$

$$= (\rho_\upsilon\, V \times B)\, d\upsilon$$

But $\rho_\upsilon\, V = J$

$$d\mathbf{F} = \mathbf{J}\, d\upsilon \times \mathbf{B}$$

$\mathbf{J}d\upsilon$ is nothing but $Id\mathbf{L}$,

$$d\mathbf{F} = Id\mathbf{L} \times \mathbf{B}$$

or, $$\mathbf{F} = I\mathbf{L} \times \mathbf{B}, \text{ Newton} \tag{3.8}$$

Problem 2: A current element 4 cm long is along y-axis with a current of 10 mA flowing in y-direction. Determine the force on the current element due to the magnetic field if the magnetic field $H = (5a_x/\mu)$ A/m.

Solution:

The force on a current element under the influence of magnetic field is

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

Here, $$I\mathbf{L} = 10 \times 10^{-3} \times 0.04\mathbf{a}_y$$

$$= 4 \times 10^{-4}\, \mathbf{a}_y$$

$$H = (5\mathbf{a}_x/\mu) \text{ A/m}$$

$$B = 5\mathbf{a}_x \text{ wb/m}^2$$

$$F = 4 \times 10^{-4}\, \mathbf{a}_y \times 5\mathbf{a}_x$$

or $$F = (0.4\mathbf{a}_y \times 5\mathbf{a}_x) \times 10^{-3}$$

$$F = -2.0\mathbf{a}_z \text{ mN}$$

FORCE AND TORQUE ON A LOOP OR COIL

Consider Fig. 3.5 in which a rectangular loop is placed under a uniform magnetic flux density, **B**.

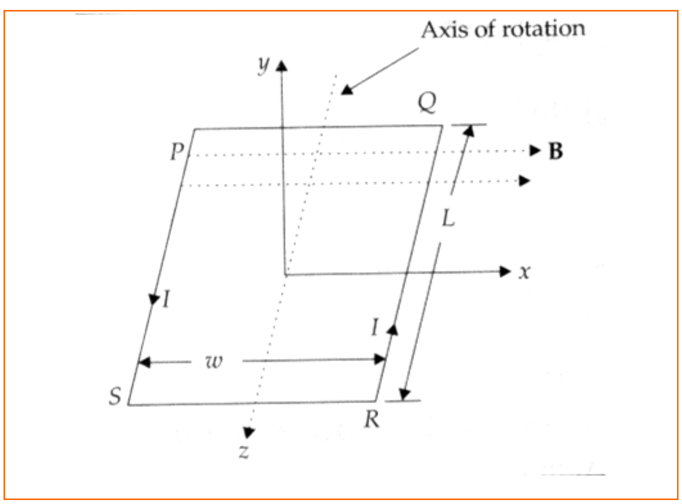

Fig. 3.5: Rectangular conductor loop in *x-z* plane

From Fig. 3.5, the force on QR due to **B** is

$$F_1 = IL \times B = -IL\mathbf{a}_z \times B\mathbf{a}_x \quad (3.25)$$

$$F_1 = -IL\mathbf{B}\mathbf{a}_y \quad (3.26)$$

that is, the force, F_1 on QR moves it downwards. Now the force on *PS* is

$$F_2 = I\mathbf{L} \times \mathbf{B} = -IL\mathbf{a}_z \times B\mathbf{a}_x \quad (3.27)$$

$$F_2 = -IL\mathbf{B}\mathbf{a}_y \quad (3.28)$$

Force, F_2 on PS moves it upwards. It may be noted that the sides PQ and SR will not experience force as they are parallel to the field, **B**.

The forces on QR *and* PS exert a torque. This torque tends to rotate the coil about its axis.

The torque, **T** is nothing but a mechanical moment of force. The torque on the loop is defined as the vector product of moment arm and force,

that is,

$$\mathbf{T} \equiv r \times \mathbf{F}, \text{ N-m} \quad (3.29)$$

where r = moment arm

F = force

Applying this definition to the loop considered above, the expression for torque is given by

$$T = r_1 \text{ x } F_1 + r_2 \text{ x } F_2 \tag{3.30}$$

$$= \frac{w}{2}a_x \times (-ILBa_y) + \left(-\frac{w}{2}a_x\right) \times (ILBa_y) \tag{3.31}$$

$$= -BILw\mathbf{a}_z$$

or, $$T = -BIS\boldsymbol{a}_z \tag{3.32}$$

where $S = wL$ = area of the loop

The torque in terms of magnetic dipole moment, **m** is

$$\mathbf{T} = \mathbf{m} \text{ x } \mathbf{B}, \text{ N-m} \tag{3.34}$$

where $\boldsymbol{m} = I\,l\,w\,\mathbf{a}_y$

$$= I\,S\,\mathbf{a}_y$$

Problem 5:

A rectangular coil is placed in a field of $\mathbf{B} = (2\mathbf{a}_x + \mathbf{a}_y)$ wb/m^2. The coil is in y-z plane and has dimensions of 2 m x 2 m. It carries a current of 1 A. Find the torque about the z-axis.

Solution:

$$m = IS\,\mathbf{a}_n = 1 \times 4\mathbf{a}_x$$

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} = 4\mathbf{a}_x \times (2\mathbf{a}_x + \mathbf{a}_y)$$

$$\mathbf{T} = \mathbf{4a}_z, \text{ N-m}$$