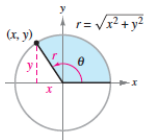
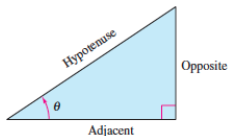


Calculus Lecture 1

Oktay Ölmez, Murat Şahin and Serhan Varma

Some special functions: Trigonometric Function



Definitions of the Trigonometric Functions

Right Triangle Definition: $0 < \theta < \frac{\pi}{2}$. (See Figure 8.11.)

$$\sin \theta = \frac{\text{opp.}}{\text{hyp.}} \quad \csc \theta = \frac{\text{hyp.}}{\text{opp.}}$$

$$\cos \theta = \frac{\text{adj.}}{\text{hyp.}} \quad \sec \theta = \frac{\text{hyp.}}{\text{adj.}}$$

$$\tan \theta = \frac{\text{opp.}}{\text{adj.}} \quad \cot \theta = \frac{\text{adj.}}{\text{opp.}}$$

Circular Function Definition: θ is any angle in standard position and (x, y) is a point on the terminal ray of the angle. (See Figure 8.12.)

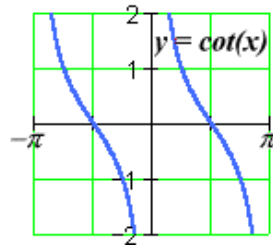
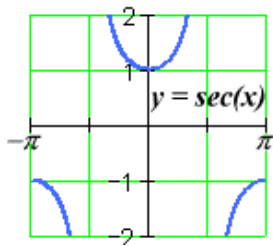
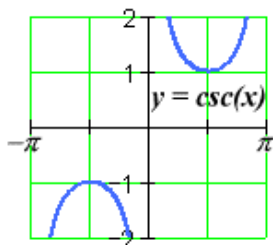
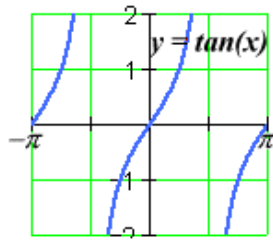
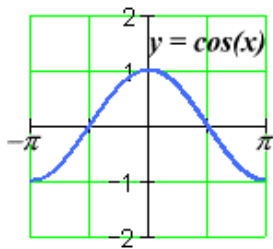
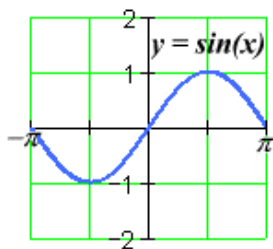
$$\sin \theta = \frac{y}{r} \quad \csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r} \quad \sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$$

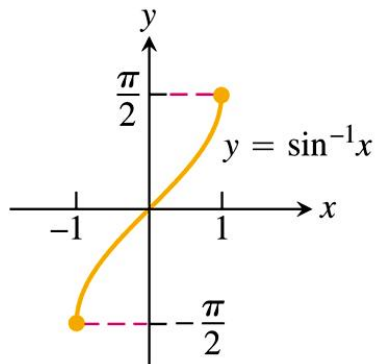
The full names of the trigonometric functions are **sine**, **cosecant**, **cosine**, **secant**, **tangent**, and **cotangent**.

Some special functions: Trigonometric Function



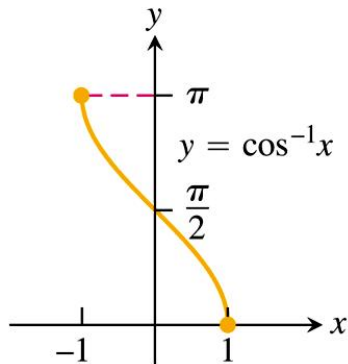
Some special functions

Domain: $-1 \leq x \leq 1$
Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



(a)

Domain: $-1 \leq x \leq 1$
Range: $0 \leq y \leq \pi$

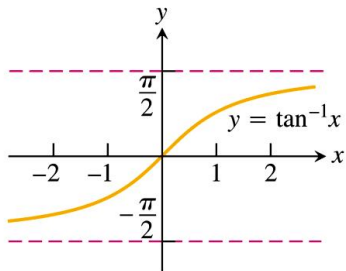


(b)

Some special functions

Domain: $-\infty < x < \infty$

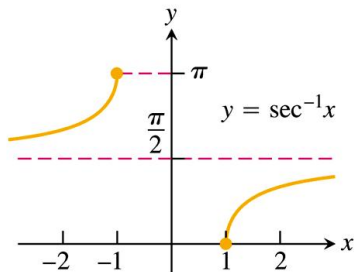
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



(c)

Domain: $x \leq -1$ or $x \geq 1$

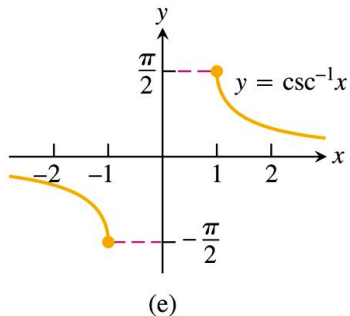
Range: $0 \leq y \leq \pi, y \neq \frac{\pi}{2}$



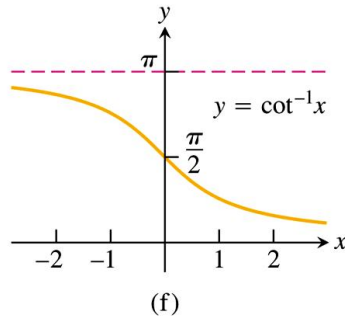
(d)

Some special functions

Domain: $x \leq -1$ or $x \geq 1$
Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$



Domain: $-\infty < x < \infty$
Range: $0 < y < \pi$



Trigonometric identities

θ (degrees)	0	30°	45°	60°	90°	180°	270°
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined	0	Undefined

Trigonometric identities

Trigonometric Identities

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Sum or Difference of Two Angles

$$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$$

$$\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

$$\tan(\theta \pm \phi) = \frac{\tan \theta \pm \tan \phi}{1 \mp \tan \theta \tan \phi}$$

Double Angle

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

Reduction Formulas

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sin \theta = -\sin(\theta - \pi)$$

$$\cos \theta = -\cos(\theta - \pi)$$

$$\tan \theta = \tan(\theta - \pi)$$

Half Angle

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

Example

The population P of a predator at time t (in months) is modeled by

$$P(t) = 10000 + 3000 \sin\left(\frac{2\pi t}{24}\right) \quad t \geq 0$$

and the population p of its primary food source (its prey) is modeled by

$$p(t) = 15000 + 5000 \cos\left(\frac{2\pi t}{24}\right) \quad t \geq 0.$$

Sketch the graph both models on the same set of axes and explain the oscillations in the size of each population.

Model

