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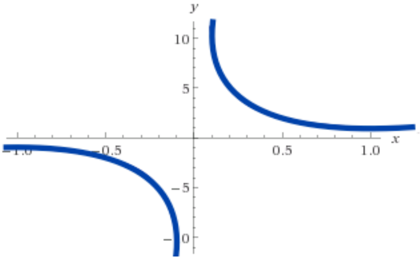

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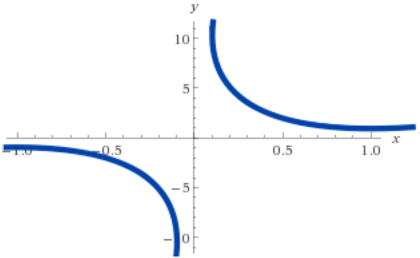


$\lim\limits_{x\to\infty} \dfrac{1}{x} = 0 = \lim\limits_{x\to-\infty} \dfrac{1}{x}$. Note that for a positive number k,

$$\lim_{x\to\infty} \frac{1}{x^k} = 0 = \lim_{x\to-\infty} \frac{1}{x^k}.$$

Mathematical Definition

For $f(x)$ a real function, the limit of f as x approaches infinity is L, denoted

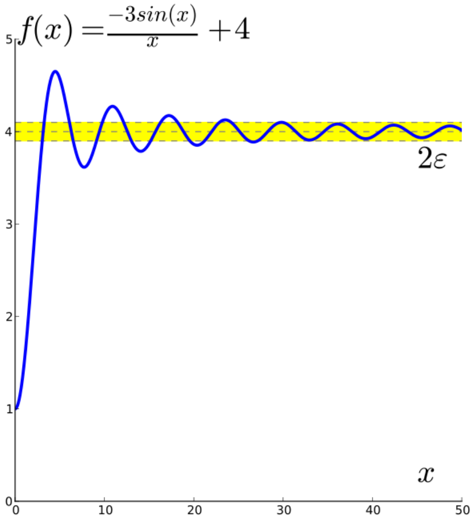
$$\lim_{x \to \infty} f(x) = L,$$

means that for all $\varepsilon > 0$, there exists c such that $|f(x) - L| < \varepsilon$ whenever $x > c$. Similarly, the limit of f as x approaches negative infinity is L, denoted

$$\lim_{x \to -\infty} f(x) = L,$$

means that for all $\varepsilon > 0$ there exists c such that $|f(x) - L| < \varepsilon$ whenever $x < c$. For example

$$\lim_{x \to -\infty} e^{x} = 0.$$


$f(x) = \frac{-3sin(x)}{x} + 4$
5
4
3
2
1
0
2ε
x
0
10
20
30
40
50

An application: Finding the Area of a Circle

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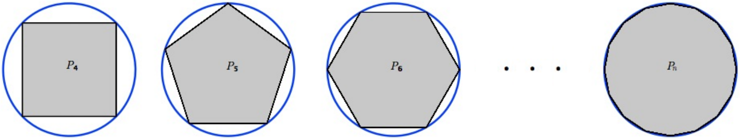

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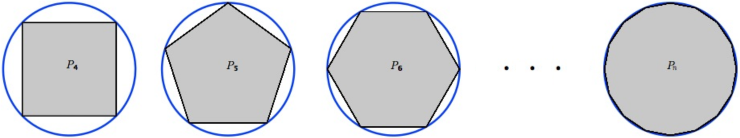

Let $f(n)$ be the area of the n-gon inscribed in a circle of radius r.

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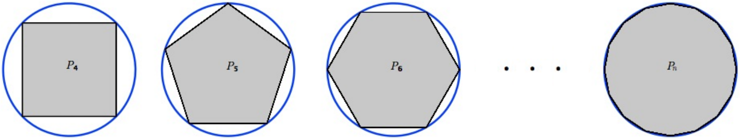
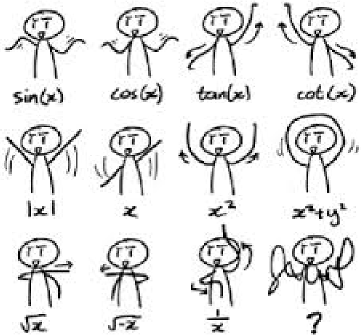


Let $f(n)$ be the area of the n-gon inscribed in a circle of radius r. Then, Area of a circle with radius r is $\lim_{n\to\infty} f(n) = \pi r^2$

Math Dance Moves


sin(x)
cos(x)
tan(x)
cot(x)
|x|
x
x^2
x^2+y^2
√x
√-x
1/x
?

Infinite Limits

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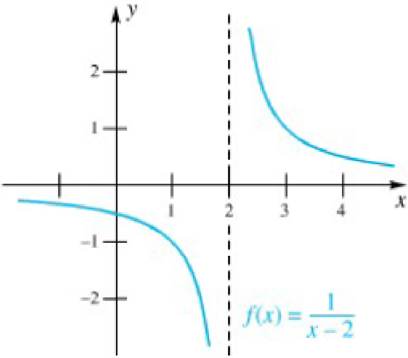

Infinite Limits

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YOU WANT PROOF? I'LL GIVE YOU PROOF!

Vertical and Horizontal Asymptotes

Definition

Line $x = c$ is a vertical asymptote of the graph of $y = f(x)$. If any of the following is satisfied:

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Definition

Line $y = b$ is a horizontal asymptote of the graph of $y = f(x)$. If any of the following is satisfied:

- $\lim_{x \to \infty} f(x) = b$
- $\lim_{x \to -\infty} f(x) = b$