Calculus
Lecture 5

Oktay Ölmez, Murat Şahin and Serhan Varma

Implicit Differentiation

- The equation $y = x^2$ defines y explicitly.

Implicit Differentiation

- The equation $y = x^2$ defines y explicitly.
- The equation $y^3 + 7y = x^3$ defines y implicitly.

Implicit Differentiation

Implicit Differentiation

- Assume y is a function of x. Denote this function by $y(x)$.

Implicit Differentiation

- Assume y is a function of x. Denote this function by $y(x)$.
- We can find a relation between $x, y(x)$ and $y'(x)$.

Implicit Differentiation

- Assume y is a function of x. Denote this function by $y(x)$.
- We can find a relation between $x, y(x)$ and $y'(x)$.
- $\frac{d}{dx}(y^3) + \frac{d}{dx}(7y) = \frac{d}{dx}(x^3)$

Implicit Differentiation

- Assume y is a function of x. Denote this function by $y(x)$.
- We can find a relation between $x, y(x)$ and $y'(x)$.
- $\frac{d}{dx}(y^3) + \frac{d}{dx}(7y) = \frac{d}{dx}(x^3)$
- $\frac{dy}{dx} = \frac{3x^2}{3y^2 + 7}$.

Implicit Differentiation

- Assume y is a function of x. Denote this function by $y(x)$.
- We can find a relation between $x, y(x)$ and $y'(x)$.
- $\frac{d}{dx}(y^3) + \frac{d}{dx}(7y) = \frac{d}{dx}(x^3)$
- $\frac{dy}{dx} = \frac{3x^2}{3y^2 + 7}$.
- The method of finding $\frac{dy}{dx}$ without solving the given equation explicitly for y in terms of x is called implicit differentiation.

Implicit Differentiation

Example

Find the equation of the tangent line to

$$x^2 + y^2 = 9$$

at the point $(2, \sqrt{5})$.

Implicit Differentiation

Example

Consider the equation

$$x^2 + xy + y^2 = 1.$$

Find equations for y' and y'' in terms of x and y only.

Related rates

- If a variable y depends on time t, then its derivative dy/dt is called a rate of change.

Related rates

- If a variable y depends on time t, then its derivative dy/dt is called a rate of change.
- Consider the case we do not know y explicitly in terms of t but we know a relationship that connects y and another variable x, and we also know something about the derivative dx/dt.

Related rates

- If a variable y depends on time t, then its derivative dy/dt is called a rate of change.
- Consider the case we do not know y explicitly in terms of t but we know a relationship that connects y and another variable x, and we also know something about the derivative dx/dt.
- In this case, we may still find dy/dt since dy/dt and dx/dt are related rates.

Example

Example

A small balloon is released at a point 150 feet away from an observer, who is on level ground. If the balloon goes straight up at a rate of 8 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 50 feet high?

Example

Example

Water is pouring into a conical tank at the rate of 8 cubic feet per minute. If the height of the tank is 12 feet and the radius of its circular opening is 6 feet, how fast is the water level rising when the water is 4 feet deep?

Procedure

- Draw a diagram.

Procedure

- Draw a diagram.
- State what is given and what information is wanted.

Procedure

- Draw a diagram.
- State what is given and what information is wanted.
- Relate the variables by writing an equation.

Procedure

- Draw a diagram.
- State what is given and what information is wanted.
- Relate the variables by writing an equation.
- Differentiate the equation implicitly.

Procedure

- Draw a diagram.
- State what is given and what information is wanted.
- Relate the variables by writing an equation.
- Differentiate the equation implicitly.
- Solve for the desired derivative

Example

Example

A woman standing on a cliff is watching a motorboat through a telescope as the boat approaches the shore lie directly below her. If the telescope is 250 feet above the water level and if the boat is approaching at 20 feet per second, at what rate is the angle of the telescope changing when the boat is 250 feet from the shore?