Calculus
Lecture 7

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Integration

Definition

Antiderivative
A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example

An antiderivative of $f(x) = x$ is $F(x) = \frac{x^2}{2}$.

Theorem

Let G be an antiderivative of a function f. Then, every antiderivative F of f must be of the form $F(x) = G(x) + C$, where C is a constant.

The Indefinite Integral

The process of finding all antiderivatives of a function is called antidifferentiation, or integration. We use the symbol $\int$, called an integral sign, to indicate that the operation of integration is to be performed on some function f. Thus,

$$\int f(x)\ dx = F(x) + C$$

Example

$$\int x\ dx = \frac{x^2}{2} + C$$

Rules for Integration

- $\int k \, dx = kx + C$
- $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C \quad n \neq -1$
- $\int kf(x) \, dx = k \int f(x) \, dx$
- $\int f(x) \pm g(x) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$
- $\int e^x \, dx = e^x + C$
- $\int x^{-1} \, dx = \ln |x| + C$

Initial Value Problems

Example

Find the function f *if it is known that* $f'(x) = 3x^2 - 4x + 8$ *and* $f(1) = 9$.

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Example

In a test run of a maglev along a straight elevated monorail track, data obtained from reading its speedometer indicate that the velocity of the maglev at time t can be described by the velocity function

$$V(t) = 8t \quad (0 \leq t \leq 30).$$

Find the position function of the maglev. Assume that initially the maglev is located at the origin of a coordinate line.

Integration by Substitution

Step 1 Let $u = g(x)$, where $g(x)$ is part of the integrand, usually the inside function of the composite function $f(g(x))$.

Step 2 Find $du = g'(x)\ dx$.

Step 3 Use the substitution $u = g(x)$ and $du = g'(x)\ dx$ to convert the entire integral into one involving only u.

Step 4 Evaluate the resulting integral.

Step 5 Replace u by $g(x)$ to obtain the final solution as a function of x.

Integration by Substitution

Example

Find

$$\int 3x^2(x^3+1)^4 \, dx.$$

Integration by Substitution

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Example

A study prepared by the marketing department of Universal Instruments forecasts that, after its new line of Galaxy Home Computers is introduced into the market, sales will grow at the rate of

$$2000 - 1500e^{-0.05t} \quad (0 \leq t \leq 60)$$

units per month. Find an expression that gives the total number of computers that will sell t months after they become available on the market. How many computers will Universal sell in the first year they are on the market?

Integration by Parts

$$D_x(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$$

or

$$u(x)v'(x) = D_x(u(x)v(x)) - u'(x)v(x)$$

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By integrating both sides

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since $dv = v'(x)dx$ and $du = u'(x)dx$ we get

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or

$$u(x)v'(x) = D_x(u(x)v(x)) - u'(x)v(x)$$

By integrating both sides

$$\int u(x)v'(x)\ dx = u(x)v(x) - \int u'(x)v(x)\ dx$$

since $dv = v'(x)dx$ and $du = u'(x)dx$ we get

$$\int u\ dv = uv - \int v\ du$$

Example

Find $\int x \cos(x)\ dx$.

Example

Find $\int \ln(x)\ dx$.

Example

Find $\int e^x \sin(x)\ dx$.