

# Calculus

## Lecture 7

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# Some Trigonometric Integrals-Type A

$\int \sin^n(x) dx$  or  $\int \cos^n(x) dx$ .

## Example ( $n$ odd)

Find

$$\int \sin^5(x) dx.$$

Use *Pythagorean Identities*.

## Example ( $n$ even)

Find

$$\int \sin^2(x) dx.$$

Use *Half-angle Identities*.

# Some Trigonometric Integrals-Type B

$$\int \sin^n(x) \cos^m(x) dx.$$

Example ( $n$  or  $m$  odd)

Find

$$\int \sin^3(x) \cos^2(x) dx.$$

Use *Pythagorean Identities*.

Example (Both  $m$  and  $n$  even)

Find

$$\int \cos^2(x) \sin^2(x) dx.$$

Use *Half-angle Identities*.

# Some Trigonometric Integrals-Type C

$\int \sin(nx) \cos(mx) dx$ ,  $\int \sin(nx) \sin(mx) dx$  or  $\int \cos(nx) \cos(mx) dx$

## Example

*Find*

$$\int \sin(2x) \cos(3x) dx.$$

*Use Product Identities.*

# Some Trigonometric Integrals-Type D

For  $\int \tan^n(x) dx$  and  $\int \cot^n(x) dx$  use the identities  $\tan^2(x) = \sec^2(x) - 1$  and  $\cot^2(x) = \csc^2(x) - 1$

## Example

*Find*

$$\int \cot^4(x) dx.$$

# Rationalizing-Type A

For integrals involved with  $\sqrt[n]{ax + b}$  use the substitution  $u^n = ax + b$ .

## Example

*Find*

$$\int x\sqrt[3]{x-4} dx.$$

# Rationalizing-Type B

For integrals involved with  $\sqrt{a^2 - x^2}$  use the substitution  $u = a \sin(t)$ .

## Example

*Find*

$$\int \frac{\sqrt{4 - x^2}}{x} dx.$$

# Rationalizing-Type C

For integrals involved with  $\sqrt{a^2 + x^2}$  use the substitution  $u = a \tan(t)$ .

## Example

*Find*

$$\int \frac{1}{(x^2 + 4)^{3/2}} dx.$$



# Rationalizing-Type D

For integrals involved with  $\sqrt{x^2 - a^2}$  use the substitution  $u = a \sec(t)$ .

## Example

*Find*

$$\int \frac{\sqrt{x^2 - 1}}{x^3} dx.$$

# Partial Fractions-Type A

Distinct linear factors.

## Example

*Find*

$$\int \frac{5x + 3}{x^3 - 2x^2 - 3x} dx.$$

# Partial Fractions-Type B

Repeated linear factors.

## Example

*Find*

$$\int \frac{x}{(x-3)^2} dx.$$

# Partial Fractions-Type C

Some distinct, some repeated linear factors.

## Example

*Find*

$$\int \frac{3x^2 - 8x + 13}{(x - 1)^2(x + 3)} dx.$$

# Partial Fractions-Type D

Single quadratic factor.

## Example

*Find*

$$\int \frac{6x^2 - 3x + 11}{(4x + 1)(x^2 + 1)} dx.$$

# Partial Fractions-Type E

Repeated quadratic factors.

## Example

*Find*

$$\int \frac{6x^2 - 15x + 22}{(x + 3)(x^2 + 2)^2} dx.$$