Genel olarak:

$$\left(\frac{\partial^2 u}{\partial x^2}\right)+\kappa\left(\frac{\partial^2 u}{\partial t^2}\right)=0$$

denklemine, “Dalga Denklemi” adı verilir. Bu denklemi sağlayan skaler veya vektörel “$u(x;t)$” fonksiyonu, bir dalgadır.

Maxwell Denklemleri sayesinde, zamanla değişen elektrik ve manyetik alanın dalga denklemini sağladığını gösterebiliriz.

Maxwell Denklemleri:

$$\overline{\nabla}\times\overline{\mathscr{E}}=-\frac{\partial\overline{\mathscr{B}}}{\partial t} \qquad \text{Faraday Yasası}$$

$$\overline{\nabla}\times\overline{\mathscr{H}}=\overline{\mathscr{J}}+\frac{\partial\overline{\mathscr{D}}}{\partial t} \qquad \text{Amperé Yasası}$$

$$\overline{\nabla}\cdot\overline{\mathscr{D}}=\rho_v \qquad \text{Gauss Yasası}$$

$$\overline{\nabla}\cdot\overline{\mathscr{B}}=0 \qquad \text{İzole Manyetik Bulunmama Yasası}$$

Faraday Yasası’nın buklesini alırsak:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{E}}=\overline{\nabla}\times\left(-\frac{\partial\overline{\mathscr{B}}}{\partial t}\right)=-\overline{\nabla}\times\left(\frac{\partial\overline{\mathscr{B}}}{\partial t}\right)=-\overline{\nabla}\times\left(\frac{\partial(\mu\overline{\mathscr{H}})}{\partial t}\right)$$

$$=-\mu\left(\overline{\nabla}\times\frac{\partial\overline{\mathscr{H}}}{\partial t}\right)=-\mu\left(\frac{\partial(\overline{\nabla}\times\overline{\mathscr{H}})}{\partial t}\right)$$

Deklemde, kaynaksız ortam (yani akım yoğunluğunun ve yük yoğunluğunun 0 olduğu ortam) için Amperé Yasası’nı kullanırsak:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{E}}=-\mu\left(\frac{\partial(\overline{\nabla}\times\overline{\mathscr{H}})}{\partial t}\right)=-\mu\left(\frac{\partial(\overline{\mathscr{J}}+\frac{\partial\overline{\mathscr{D}}}{\partial t})}{\partial t}\right)=-\mu\left(\frac{\partial(\frac{\partial\overline{\mathscr{D}}}{\partial t})}{\partial t}\right)$$

$$=-\mu\left(\frac{\partial^2\overline{\mathscr{D}}}{\partial t^2}\right)=-\mu\left(\frac{\partial^2(\varepsilon\overline{\mathscr{E}})}{\partial t^2}\right)=-\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2}\right)$$

$$\Rightarrow\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{E}}=-\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2}\right)$$

$$\Rightarrow\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{E}}+\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2}\right)=0$$

Herhangi bir vektörel alan için:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{A}}=\overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{A}})-\overline{\nabla}^2\overline{\mathscr{A}}$$

Dolayısıyla:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{E}}+\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2}\right)=\overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{E}})-\overline{\nabla}^2\overline{\mathscr{E}}+\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2}\right)=0$$

Kaynaksız ortamda Gauss Yasası'nı kullanırsak:

$$\overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{E}}) = \overline{\nabla}(\overline{\nabla}\cdot(\frac{1}{\varepsilon}\overline{\mathscr{D}})) = \frac{1}{\varepsilon}\overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{D}}) = \frac{1}{\varepsilon}\overline{\nabla}(\rho_v)$$

$$\rho_v = 0$$

$$\Rightarrow\ \overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{E}}) = \frac{1}{\varepsilon}\overline{\nabla}(\rho_v) = 0$$

Bu durumda:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{E}} + \mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2}\right) = \overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{E}}) - \overline{\nabla}^2\overline{\mathscr{E}} + \mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2}\right) = -\overline{\nabla}^2\overline{\mathscr{E}} + \mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2}\right) = 0$$

$$\Rightarrow\ \overline{\nabla}^2\overline{\mathscr{E}} - \mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2}\right) = 0$$

Daha açık bir şekilde yazarsak, Kartezyen koordinatlarda:

$$\overline{\nabla}^2 = \overline{\nabla}\cdot\overline{\nabla} = \left(\hat{a}_x\frac{\partial}{\partial x} + \hat{a}_y\frac{\partial}{\partial y} + \hat{a}_z\frac{\partial}{\partial z}\right)\cdot\left(\hat{a}_x\frac{\partial}{\partial x} + \hat{a}_y\frac{\partial}{\partial y} + \hat{a}_z\frac{\partial}{\partial z}\right) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right), \text{ve}$$

$$\overline{\mathscr{E}} = \overline{\mathscr{E}}(x,y,z;t) = \hat{a}_x\mathscr{E}_x(x,y,z;t) + \hat{a}_y\mathscr{E}_y(x,y,z;t) + \hat{a}_z\mathscr{E}_z(x,y,z;t)$$

$$\Rightarrow\ \overline{\nabla}^2\overline{\mathscr{E}} = \overline{\nabla}^2\overline{\mathscr{E}}(x,y,z;t) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(\hat{a}_x\mathscr{E}_x(x,y,z;t) + \hat{a}_y\mathscr{E}_y(x,y,z;t) + \hat{a}_z\mathscr{E}_z(x,y,z;t)\right)$$

$$= \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial x^2} + \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial x^2} + \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial x^2}$$

$$+ \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial y^2} + \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial y^2} + \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial y^2}$$

$$+ \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial z^2} + \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial z^2} + \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial z^2}$$

veya

$$\overline{\nabla}^2\overline{\mathscr{E}} = \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial x^2} + \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial y^2} + \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial z^2}$$

$$+ \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial x^2} + \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial y^2} + \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial z^2}$$

$$+ \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial x^2} + \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial y^2} + \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial z^2}$$

Bu durumda:

$$\overline{\nabla}^2\overline{\mathscr{E}} - \mu\varepsilon\frac{\partial^2\overline{\mathscr{E}}}{\partial t^2} =$$

$$\hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial x^2} + \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial y^2} + \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial z^2}$$

$$+\hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial x^2} + \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial y^2} + \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial z^2}$$

$$+\hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial x^2} + \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial y^2} + \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial z^2}$$

$$-\hat{a}_x\mu\varepsilon\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial t^2} - \hat{a}_y\mu\varepsilon\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial t^2} - \hat{a}_z\mu\varepsilon\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial t^2}$$

$$=\hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial x^2} + \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial y^2} + \hat{a}_x\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial z^2} - \hat{a}_x\mu\varepsilon\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial t^2}$$

$$+\hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial x^2} + \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial y^2} + \hat{a}_y\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial z^2} - \hat{a}_y\mu\varepsilon\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial t^2}$$

$$+\hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial x^2} + \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial y^2} + \hat{a}_z\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial z^2} - \hat{a}_z\mu\varepsilon\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial t^2} = 0$$

En son elde ettiğimiz ifadede, her bir satırdaki terimler farklı yönlerdedir; dolayısıyla her biri 0'a eşittir. Bir başka deyişle, yukarıdaki ifade 3 ayrı skaler denkleme ayrıştırılabilir:

$$\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial x^2} + \frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial y^2} + \frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial z^2} - \mu\varepsilon\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial t^2} = 0$$

$$\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial x^2} + \frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial y^2} + \frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial z^2} - \mu\varepsilon\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial t^2} = 0$$

$$\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial x^2} + \frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial y^2} + \frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial z^2} - \mu\varepsilon\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial t^2} = 0$$

Ya da, kısaca

$$\overline{\nabla}^2\mathscr{E}_x(x,y,z;t) - \mu\varepsilon\frac{\partial^2(\mathscr{E}_x(x,y,z;t))}{\partial t^2} = 0$$

$$\overline{\nabla}^2\mathscr{E}_y(x,y,z;t) - \mu\varepsilon\frac{\partial^2(\mathscr{E}_y(x,y,z;t))}{\partial t^2} = 0$$

$$\overline{\nabla}^2\mathscr{E}_z(x,y,z;t) - \mu\varepsilon\frac{\partial^2(\mathscr{E}_z(x,y,z;t))}{\partial t^2} = 0$$

Bu denklemler, en başta tanımladığımız

$$\left(\frac{\partial^2 u}{\partial x^2}\right) + \kappa\left(\frac{\partial^2 u}{\partial t^2}\right) = 0$$

dalga denklemine çok benzemektedir. Dolayısıyla, zamanla değişen elektrik alan, dalga denklemini sağlamaktadır; ya da bir başka deyişle, zamanla değişen elektrik alan bir dalgadır!...

Benzer şekilde, bu sefer kaynaksız ortam (yani akım yoğunluğunun ve yük yoğunluğunun 0 olduğu ortam) için Amperé Yasası'nın buklesini alırsak:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{H}}=\overline{\nabla}\times\left(\overline{\mathscr{J}}+\frac{\partial\overline{\mathscr{D}}}{\partial t}\right)=\overline{\nabla}\times\left(\frac{\partial\overline{\mathscr{D}}}{\partial t}\right)=\overline{\nabla}\times\left(\frac{\partial(\varepsilon\overline{\mathscr{E}})}{\partial t}\right)$$

$$=\varepsilon\overline{\nabla}\times\left(\frac{\partial\overline{\mathscr{E}}}{\partial t}\right)=\varepsilon\left(\frac{\partial(\overline{\nabla}\times\overline{\mathscr{E}})}{\partial t}\right)$$

Deklemde, Faraday Yasası'nı kullanırsak:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{H}}=\varepsilon\left(\frac{\partial(\overline{\nabla}\times\overline{\mathscr{E}})}{\partial t}\right)=\varepsilon\left(\frac{\partial(-\frac{\partial\overline{\mathscr{B}}}{\partial t})}{\partial t}\right)=-\varepsilon\left(\frac{\partial(\frac{\partial\overline{\mathscr{B}}}{\partial t})}{\partial t}\right)$$

$$=-\varepsilon\left(\frac{\partial^2\overline{\mathscr{B}}}{\partial t^2}\right)=-\varepsilon\left(\frac{\partial^2(\mu\overline{\mathscr{H}})}{\partial t^2}\right)=-\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2}\right)$$

$$\Rightarrow\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{H}}=-\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2}\right)$$

$$\Rightarrow\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{H}}+\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2}\right)=0$$

Herhangi bir vektörel alan için:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{A}}=\overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{A}})-\overline{\nabla}^2\overline{\mathscr{A}}$$

Dolayısıyla:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{H}}+\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2}\right)=\overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{H}})-\overline{\nabla}^2\overline{\mathscr{H}}+\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2}\right)=0$$

Doğada izole manyetik yük bulunmama yasasını kullanırsak:

$$\overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{H}})=\overline{\nabla}(\overline{\nabla}\cdot(\frac{1}{\mu}\overline{\mathscr{B}}))=\frac{1}{\mu}\overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{B}})=0$$

$$\Rightarrow\ \overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{H}})=0$$

Bu durumda:

$$\overline{\nabla}\times\overline{\nabla}\times\overline{\mathscr{H}}+\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2}\right)=\overline{\nabla}(\overline{\nabla}\cdot\overline{\mathscr{H}})-\overline{\nabla}^2\overline{\mathscr{H}}+\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2}\right)=-\overline{\nabla}^2\overline{\mathscr{H}}+\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2}\right)=0$$

$$\Rightarrow\quad\overline{\nabla}^2\overline{\mathscr{H}}-\mu\varepsilon\left(\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2}\right)=0$$

Daha açık bir şekilde yazarsak:

$$\overline{\nabla}^2 = \overline{\nabla}\cdot\overline{\nabla} = \left(\hat{a}_x\frac{\partial}{\partial x} + \hat{a}_y\frac{\partial}{\partial y} + \hat{a}_z\frac{\partial}{\partial z}\right)\cdot\left(\hat{a}_x\frac{\partial}{\partial x} + \hat{a}_y\frac{\partial}{\partial y} + \hat{a}_z\frac{\partial}{\partial z}\right) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right), \text{ve}$$

$$\overline{\mathscr{H}} = \overline{\mathscr{H}}(x,y,z;t) = \hat{a}_x\mathscr{H}_x(x,y,z;t) + \hat{a}_y\mathscr{H}_y(x,y,z;t) + \hat{a}_z\mathscr{H}_z(x,y,z;t)$$

$$\Rightarrow \quad \overline{\nabla}^2\overline{\mathscr{H}} = \overline{\nabla}^2\overline{\mathscr{H}}(x,y,z;t) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(\hat{a}_x\mathscr{H}_x(x,y,z;t) + \hat{a}_y\mathscr{H}_y(x,y,z;t) + \hat{a}_z\mathscr{H}_z(x,y,z;t)\right)$$

$$= \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial x^2} + \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial x^2} + \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial x^2}$$

$$+ \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial y^2} + \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial y^2} + \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial y^2}$$

$$+ \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial z^2} + \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial z^2} + \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial z^2}$$

veya

$$\overline{\nabla}^2\overline{\mathscr{H}} = \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial x^2} + \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial y^2} + \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial z^2}$$

$$+ \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial x^2} + \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial y^2} + \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial z^2}$$

$$+ \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial x^2} + \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial y^2} + \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial z^2}$$

Bu durumda:

$$\overline{\nabla}^2\overline{\mathscr{H}} - \mu\varepsilon\frac{\partial^2\overline{\mathscr{H}}}{\partial t^2} =$$

$$\hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial x^2} + \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial y^2} + \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial z^2}$$

$$+ \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial x^2} + \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial y^2} + \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial z^2}$$

$$+ \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial x^2} + \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial y^2} + \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial z^2}$$

$$- \hat{a}_x\mu\varepsilon\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial t^2} - \hat{a}_y\mu\varepsilon\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial t^2} - \hat{a}_z\mu\varepsilon\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial t^2}$$

$$= \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial x^2} + \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial y^2} + \hat{a}_x\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial z^2} - \hat{a}_x\mu\varepsilon\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial t^2}$$

$$+ \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial x^2} + \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial y^2} + \hat{a}_y\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial z^2} - \hat{a}_y\mu\varepsilon\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial t^2}$$

$$+ \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial x^2} + \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial y^2} + \hat{a}_z\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial z^2} - \hat{a}_z\mu\varepsilon\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial t^2} = 0$$

En son elde ettiğimiz ifadede, her bir satırdaki terimler farklı yönlerdedir; dolayısıyla her biri 0'a eşittir. Bir başka deyişle, yukarıdaki ifade 3 ayrı skaler denkleme ayrıştırılabilir:

$$\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial x^2}+\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial y^2}+\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial z^2}-\mu\varepsilon\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial t^2}=0$$

$$\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial x^2}+\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial y^2}+\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial z^2}-\mu\varepsilon\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial t^2}=0$$

$$\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial x^2}+\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial y^2}+\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial z^2}-\mu\varepsilon\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial t^2}=0$$

Ya da, kısaca

$$\overline{\nabla}^2\mathscr{H}_x(x,y,z;t)-\mu\varepsilon\frac{\partial^2(\mathscr{H}_x(x,y,z;t))}{\partial t^2}=0$$

$$\overline{\nabla}^2\mathscr{H}_y(x,y,z;t)-\mu\varepsilon\frac{\partial^2(\mathscr{H}_y(x,y,z;t))}{\partial t^2}=0$$

$$\overline{\nabla}^2\mathscr{H}_z(x,y,z;t)-\mu\varepsilon\frac{\partial^2(\mathscr{H}_z(x,y,z;t))}{\partial t^2}=0$$

Bu denklemler de, en başta tanımladığımız

$$\left(\frac{\partial^2 u}{\partial x^2}\right)+\kappa\left(\frac{\partial^2 u}{\partial t^2}\right)=0$$

dalga denklemine çok benzemektedir. Dolayısıyla, zamanla değişen manyetik alan da, dalga denklemini sağlamaktadır; ya da bir başka deyişle, zamanla değişen manyetik alan da bir dalgadır!..