PHY121 Physics I

Chapter 2 Motion Along a Straight Line

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+ Chapter 2 Motion Along a Straight Line

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2.2. Position and Displacement
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2.5. Acceleration
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2.7. Free Fall Acceleration
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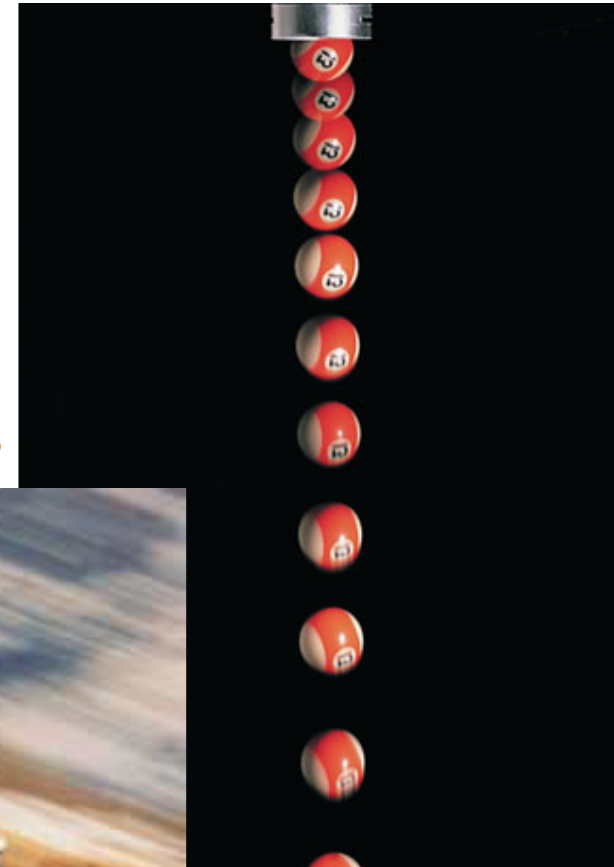


2.1 Motion

In physics motion is categorized into three types:

- **translational motion:** A car moving down a highway
- **rotational motion:** Earth's spin on its axis
- **vibrational motion:** back-and-forth movement of a pendulum

Some general properties of translation motion in this chapter:

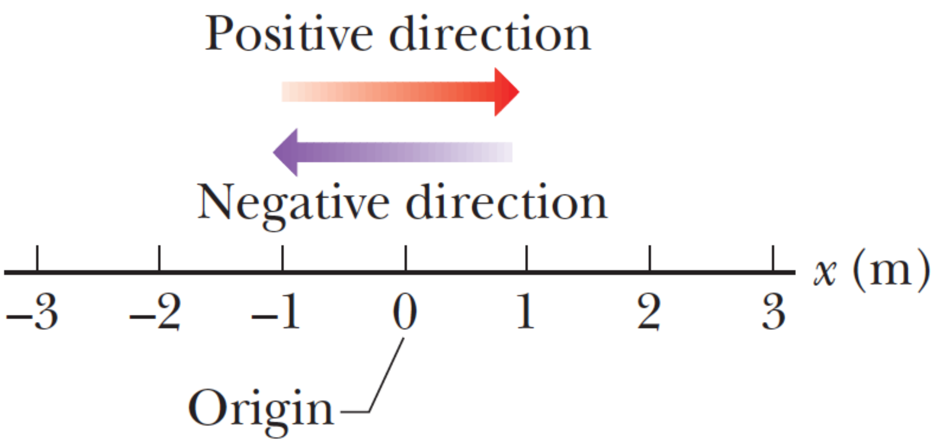
1. The motion is along a straight line only. The line may be vertical, horizontal, or slanted, but it must be straight.

2. We discuss only the motion itself and changes in the motion. What causes to this motion will be discussed later.

3. We will describe the moving object as **a particle** regardless of its size.

2.2.Position and Displacement

A particle's position is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.



You can freely choose the positive and negative direction.

2.2.Position and Displacement

The displacement of an object is defined as it’s change in position in some time interval:

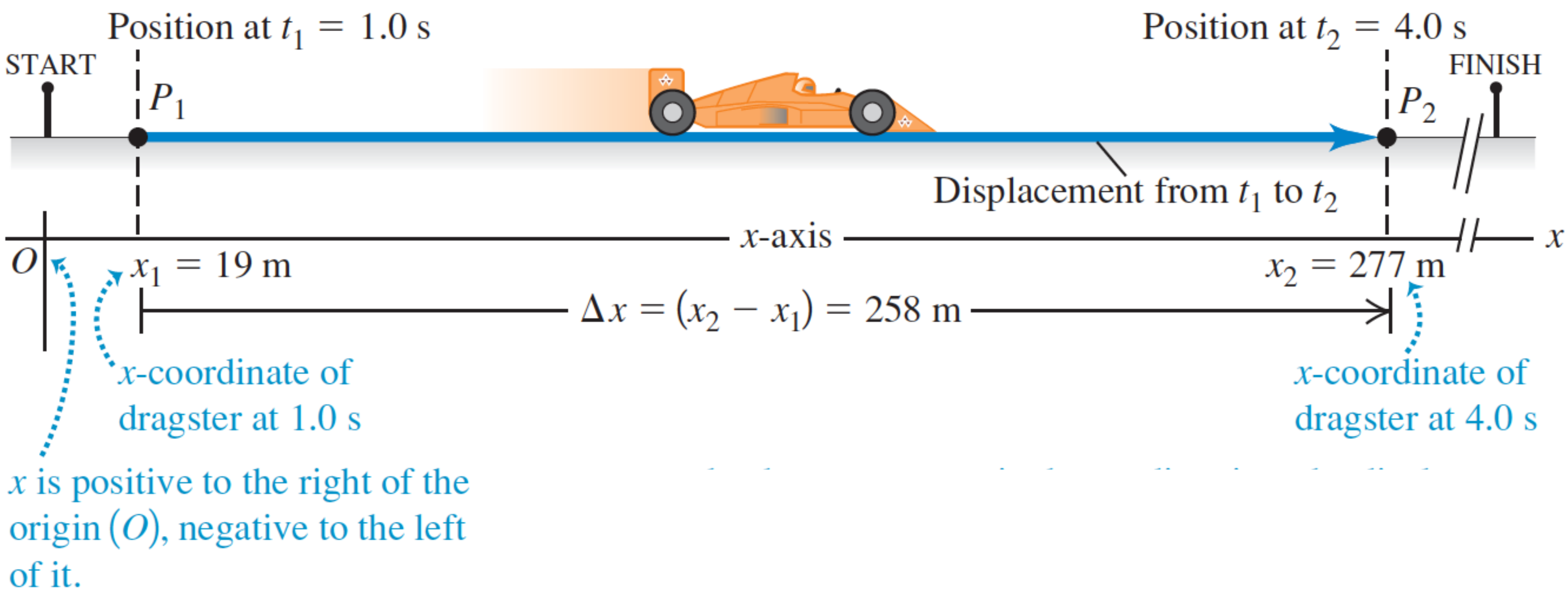
$$\boxed{\Delta x = x_f - x_i}$$

Displacement can be either positive or negative

Displacement is an example of a **vector** quantity, which is a quantity that has both a direction and a magnitude.

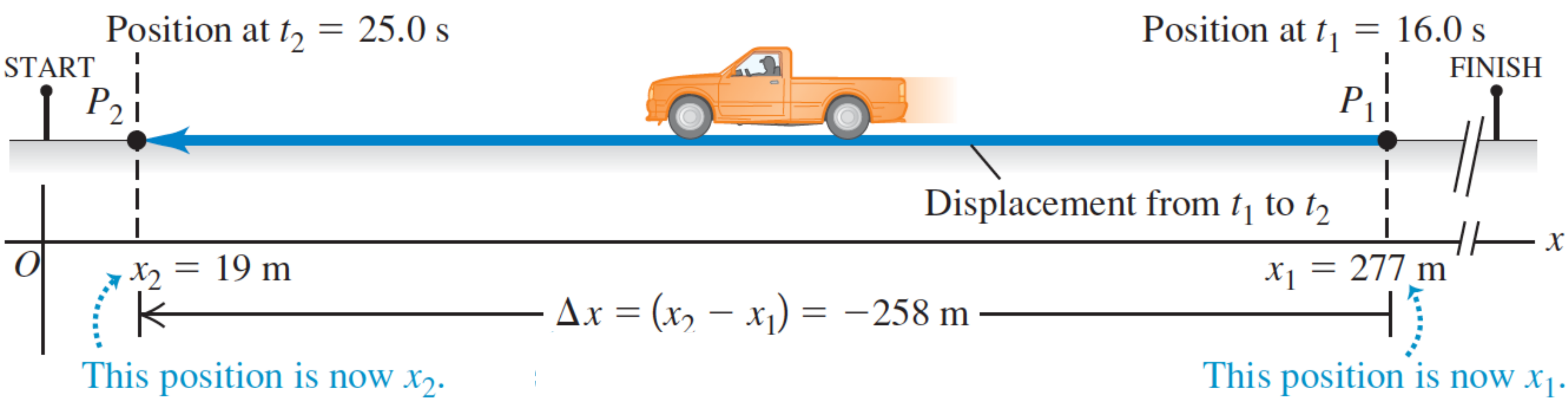
2.2.Position and Displacement

$$\boxed{\Delta x = x_s - x_i}$$



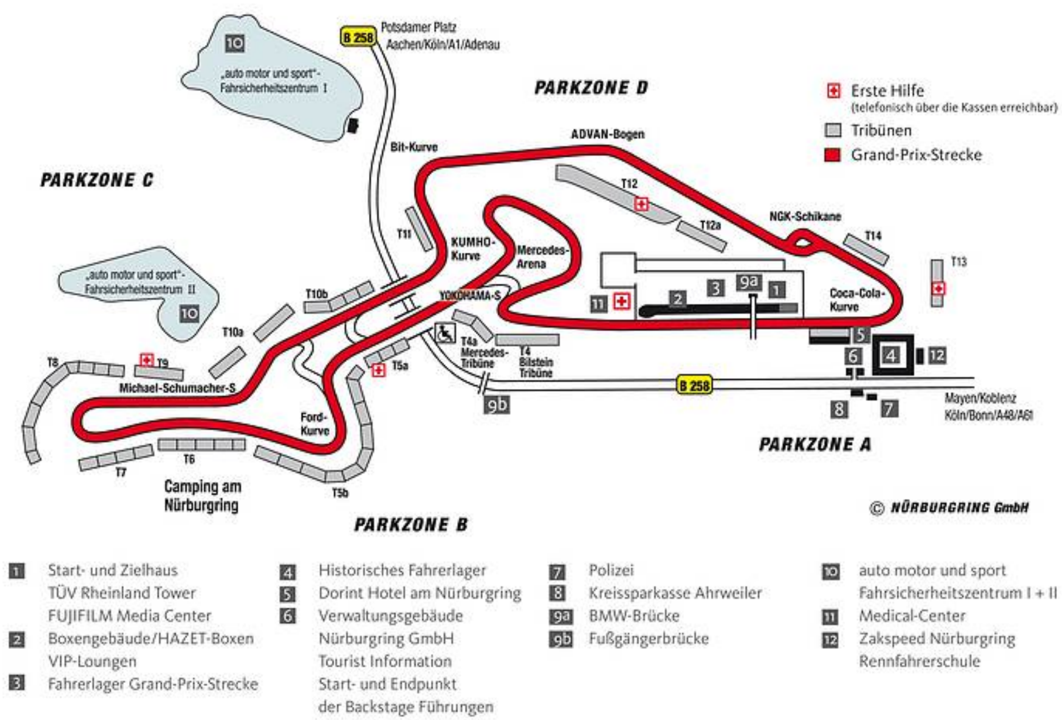
2.2.Position and Displacement

$$\boxed{\Delta x = x_f - x_i}$$



2.2.Position and Displacement

The total distance is different. Total distance is a traveled distance and is always a positive number.



What is the displacement of a Formula 1 car after a Grand Prix with 40 laps ?
What is the total distance a Formula 1 car after a Grand Prix with 40 laps?

2.3 Average Velocity and Speed

If an object has a displacement Δx in time interval Δt, then the average velocity is given by:

$$\boxed{v_{avg.} = \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}}$$

The unit of average velocity is m/s in SI unit system

The average velocity of a particle moving in one dimension can be positive or negative, depending on the sign of the displacement. (The time interval Δt is always positive.)

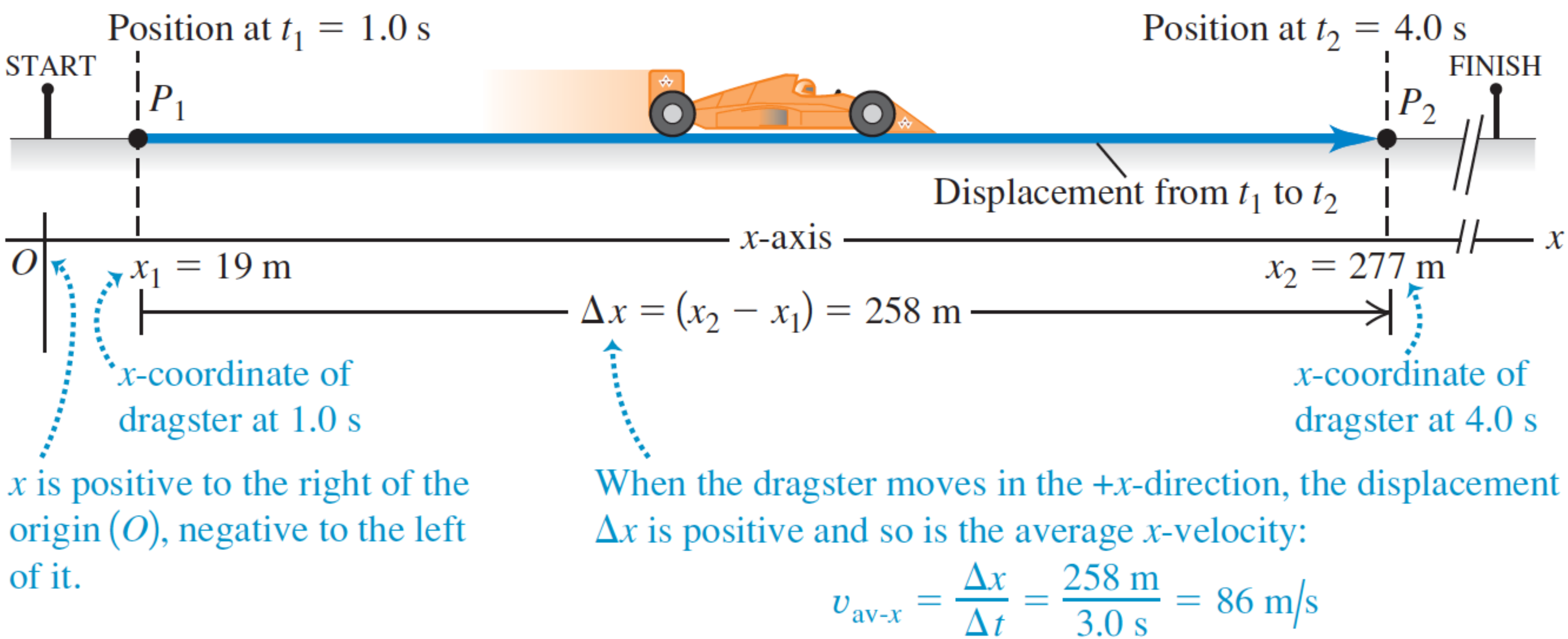
2.3 Average Velocity and Speed

Rules for the sign of the velocity

İf the x coordinate is	…average velocity
Positive and increasing (more positive)	Positive: Particle is moving in +x direction
Positive and decreasing (less positive)	Negative: Particle is moving in -x direction
Negative and increasing (less negative)	Positive: Particle is moving in +x direction
Negative and decreasing (more negative)	Negative: Particle is moving in -x direction

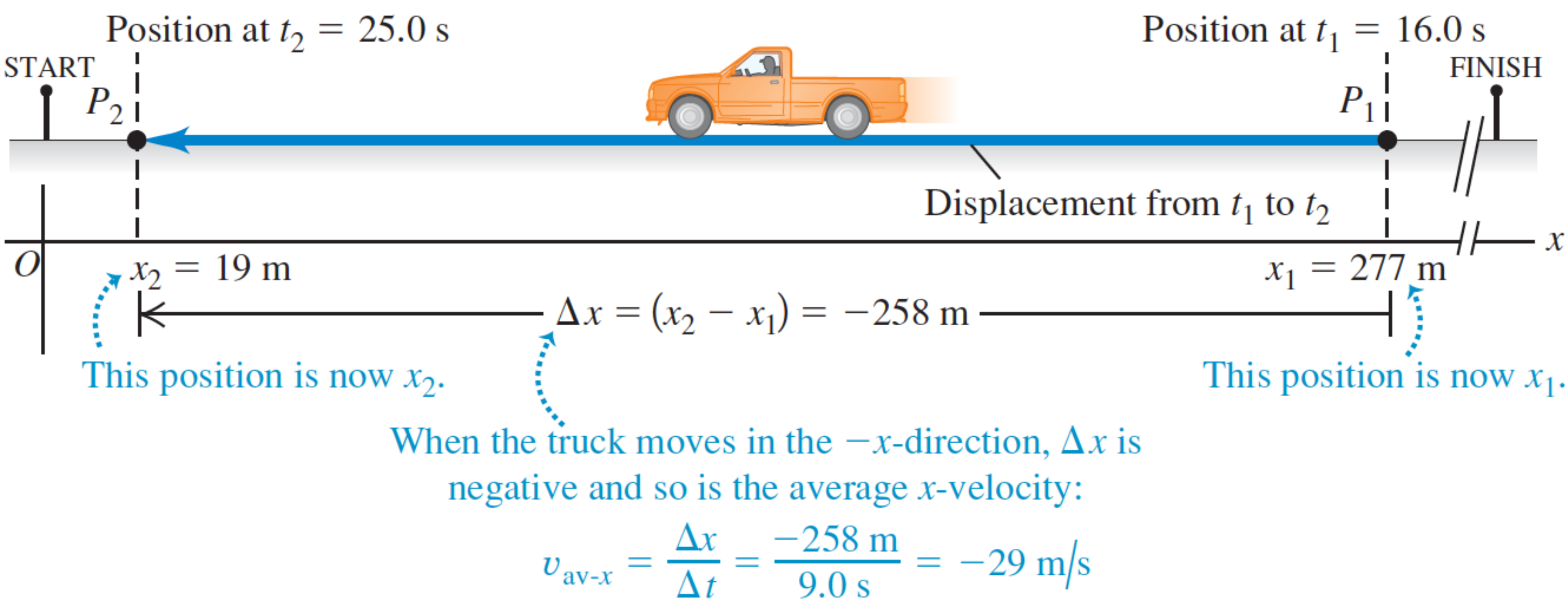
2.3 Average Velocity and Speed

$$\boxed{v_{avg.} = \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_s - x_i}{t_s - t_i}}$$

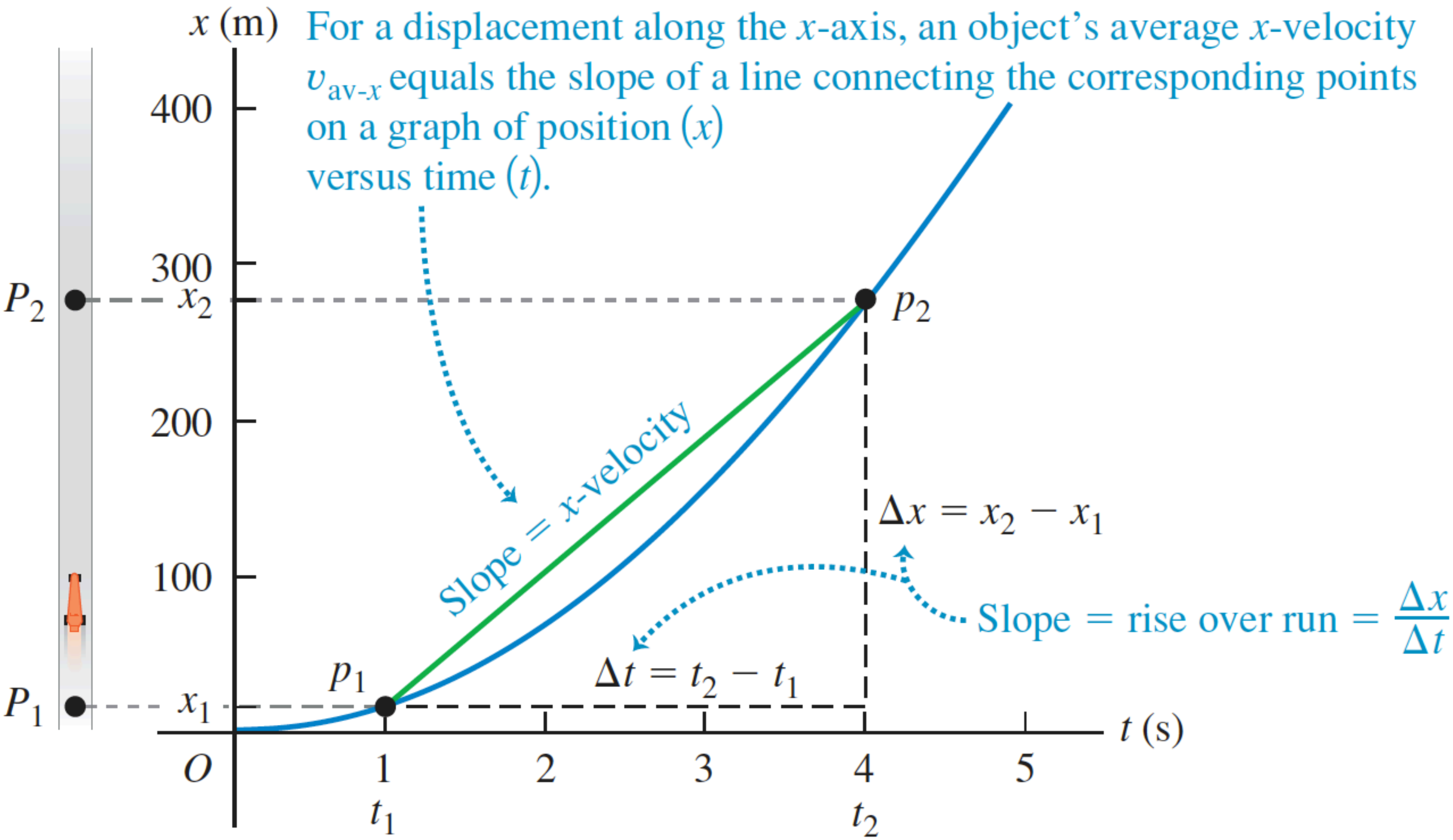


2.3 Average Velocity and Speed

$$v_{avg.} = \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_s - x_i}{t_s - t_i}$$



2.3 Average Velocity and Speed



2.3 Average Velocity and Speed

Velocity and speed is not same !

$$Average\ speed = \frac{Total\ distance}{Total\ time}$$

Speed is a scaler quantity.

2.3 Average Velocity and Speed

Example 1:

A car moves back and forth along a straight line. Find the displacement, average velocity, and average speed of the car between positions A and F.

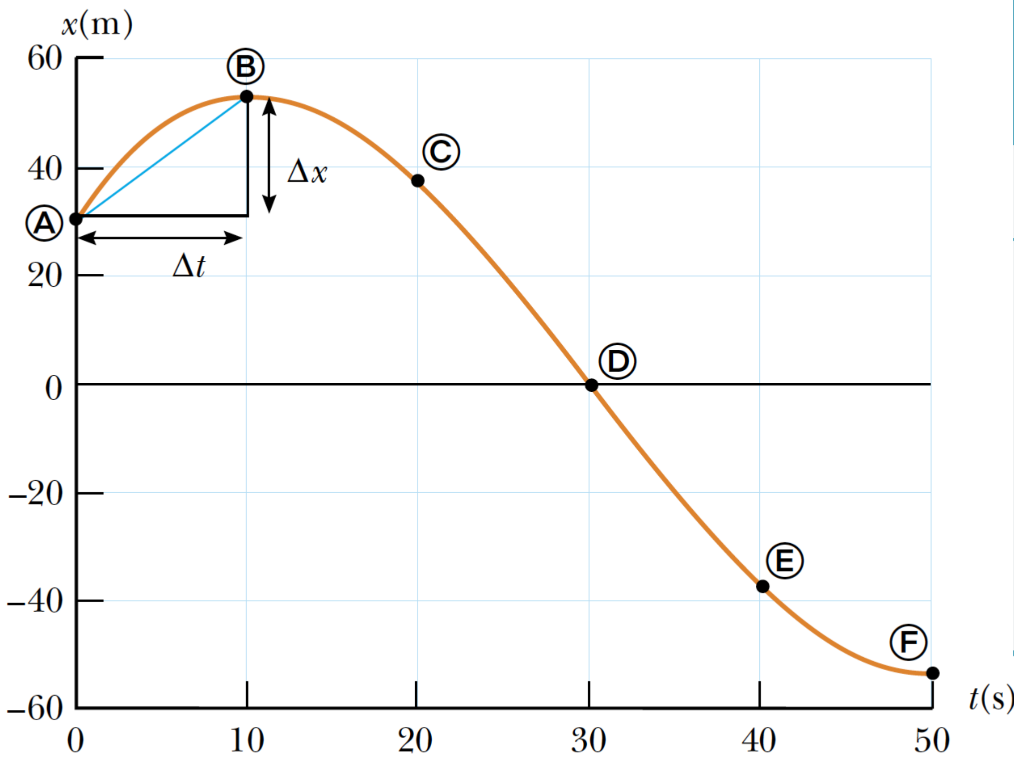


Table 2.1

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

2.3 Average Velocity and Speed

Answer 1:

Displacement between A and F

$$\Delta x = x_s - x_i = -53 - 30 = -83$$

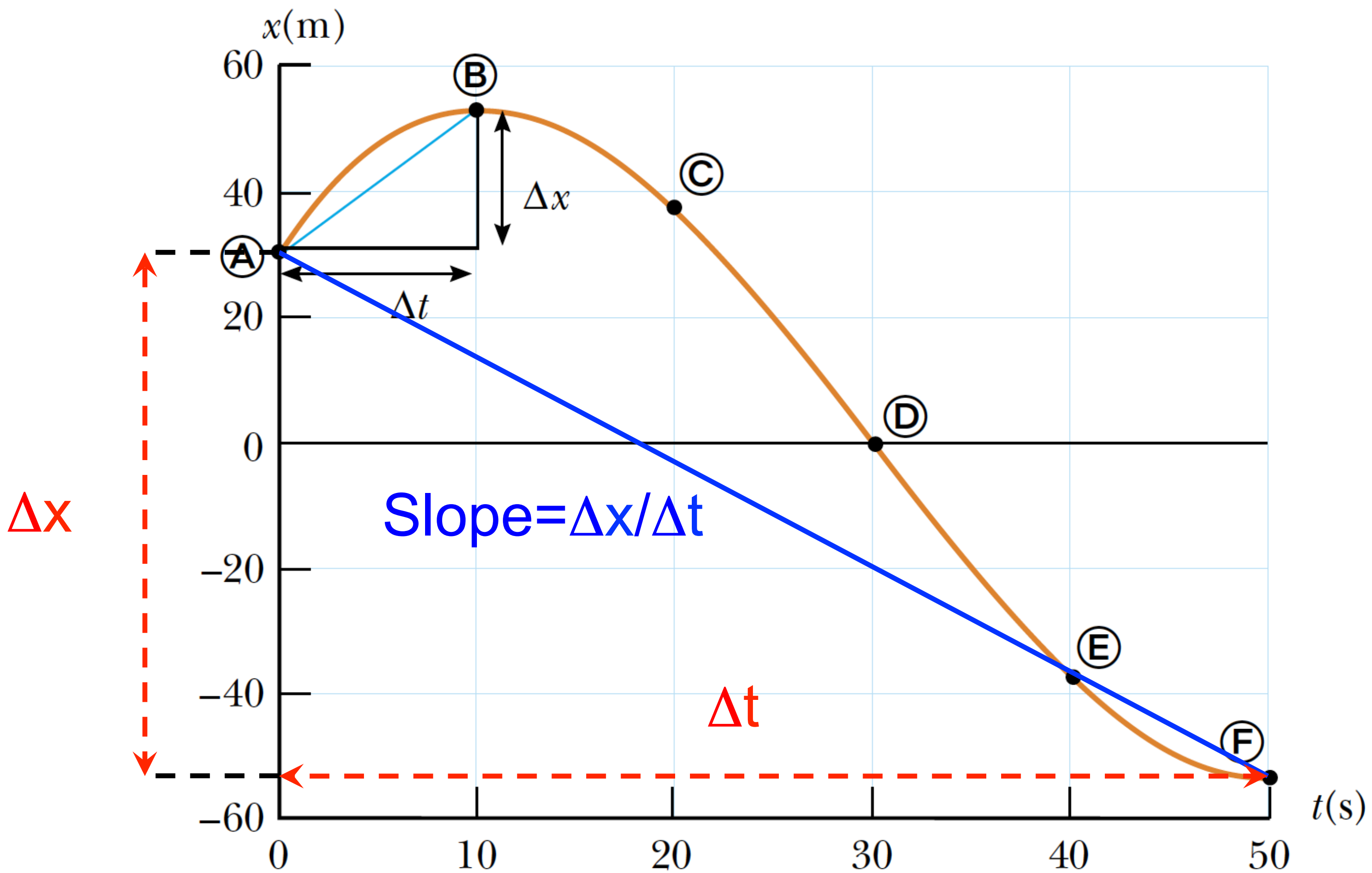
Average velocity between A and F

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_s - x_i}{t_2 - t_1} = \frac{-53 - 30}{50 - 0} = -1.7 m / s$$

2.3 Average Velocity and Speed

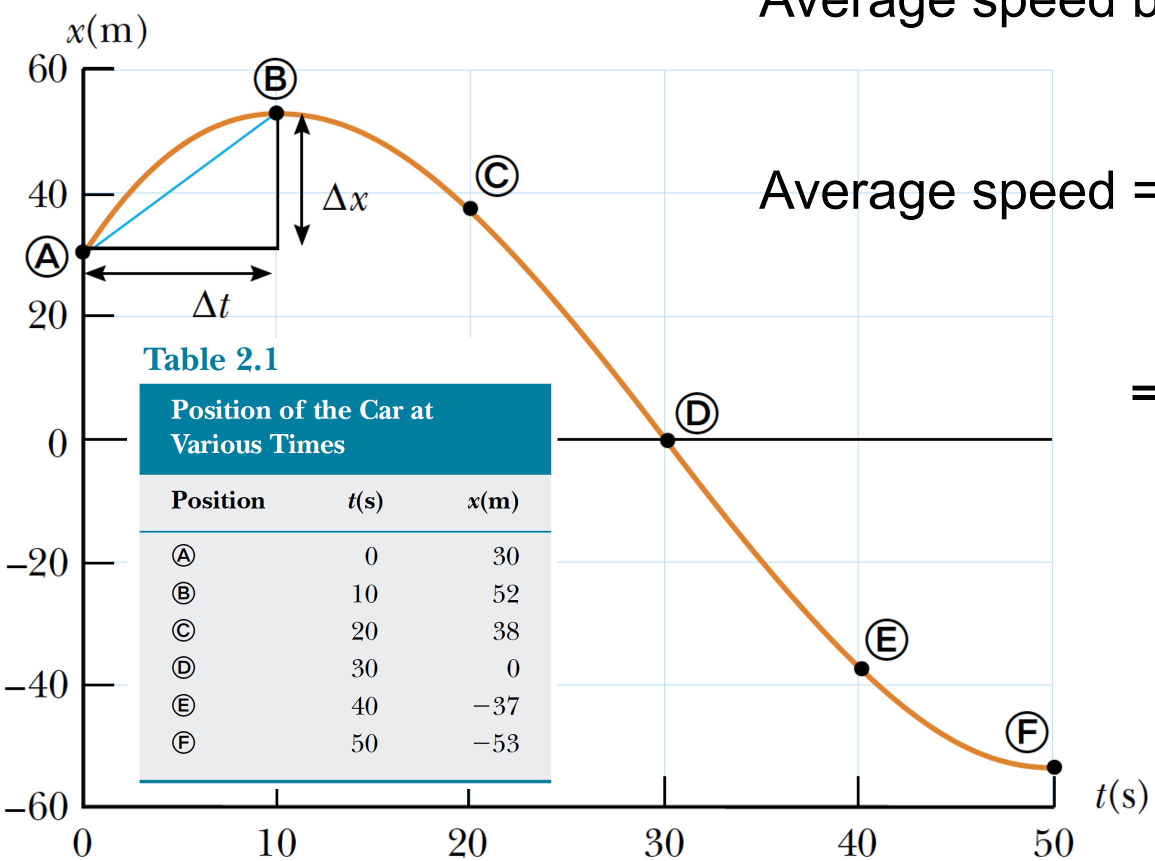
Answer 1:

Graphical method to find the average velocity



2.3 Average Velocity and Speed

Answer 1:



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Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Average speed between A and F:

Average speed = Total distance/ Total time

$$= \frac{22+52+53}{50} = 2.5 m/s$$

Example 2:

Each of the following automobile trips takes one hour.
The positive x-direction is to the east.

- Automobile A travels 50 km due east.
- Automobile B travels 50 km due west.
- Automobile C travels 60 km due east, then turns around and travels 10 km due west.
- Automobile D travels 70 km due east.
- Automobile E travels 20 km due west, then turns around and travels 20 km due east.

a) Rank the five trips in order of average x-velocity from most positive to most negative.
b) Which trips, if any, have the same average x-velocity?
c) For which trip, if any, is the average x-velocity equal to zero?

Discuss it!

2.3 Average Velocity and Speed

Example 3:

It normally takes you 10 min to travel 5 km to school along a straight road. You leave home 15 min before class begins. Delays caused by a broken traffic light slow down traffic to 20 km/h for the first 2 km of the trip. Will you be late for class?

2.3 Average Velocity and Speed

Answer 3:

$$\Delta t_{total} = \Delta t_{2km} + \Delta t_{3km}$$

$$\Delta v_{normal} = \frac{\Delta x_{normal}}{\Delta t_{normal}} = \frac{5\ km}{10\ \text{min.}} = 0.5\ km/\text{min.}$$

$$\Delta t_{2km} = \frac{\Delta x}{\Delta v_{avg.}} = \frac{2\ km}{20\ km/h} = 0.1\ h = 6\ \text{min.}$$

$$\Delta t_{3km} = \frac{\Delta x}{\Delta v_{normal}} = \frac{3\ km}{0.5\ km/\text{min}} = 6\ \text{min.}$$

$$\Delta t_{total} = \Delta t_{2km} + \Delta t_{3km}$$

$$\Delta t_{total} = 6 + 6 = 12\ \text{min.}$$

2.4.Instantaneous Velocity and Speed

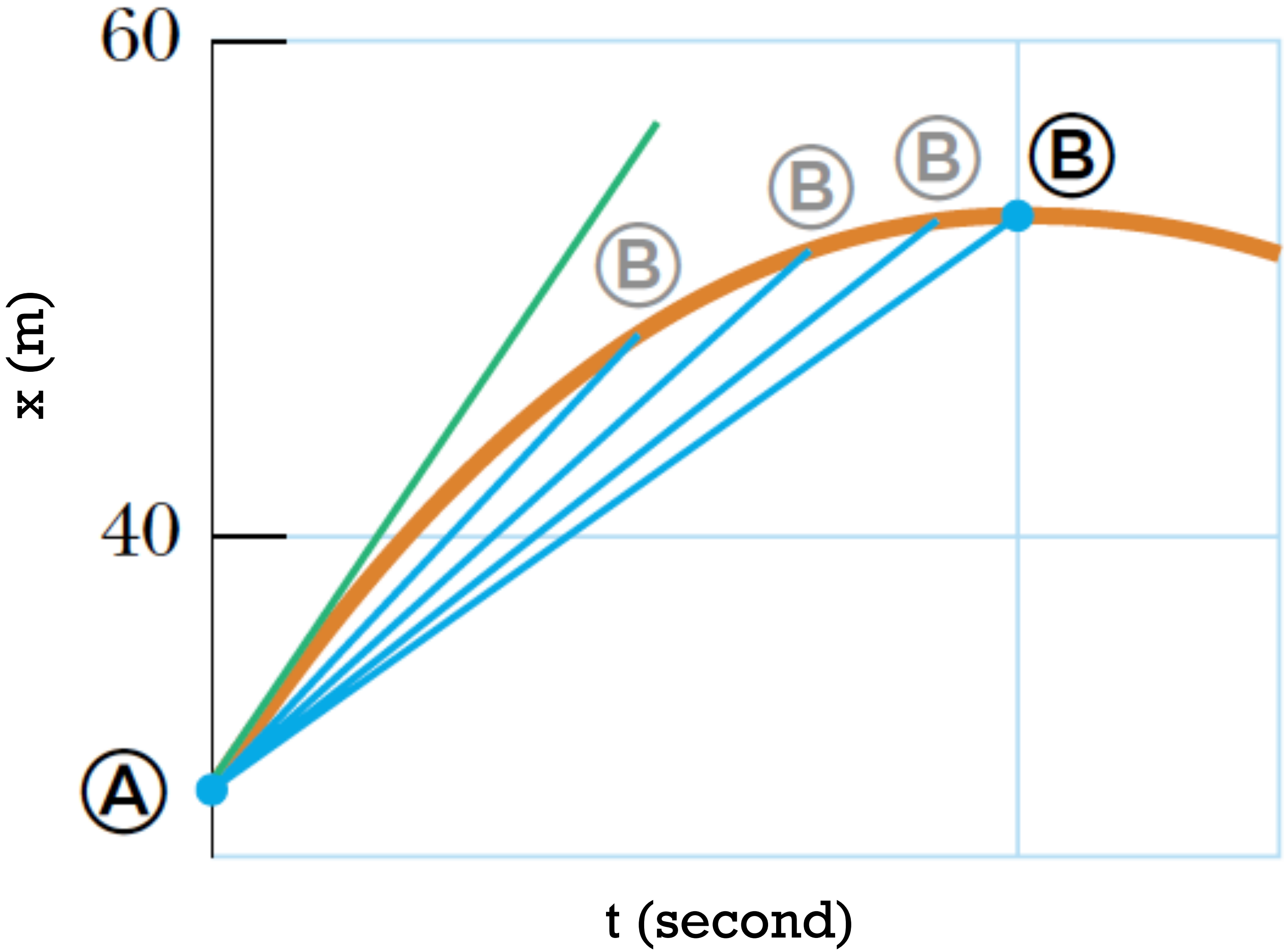
The winner of a 50-m swimming race is the swimmer whose average velocity has the greatest magnitude-that is, the swimmer who traverses a displacement of 50 m in the shortest elapsed time Δt.

2.4.Instantaneous Velocity and Speed

Often we need to know the velocity of a particle at a particular instant in time..

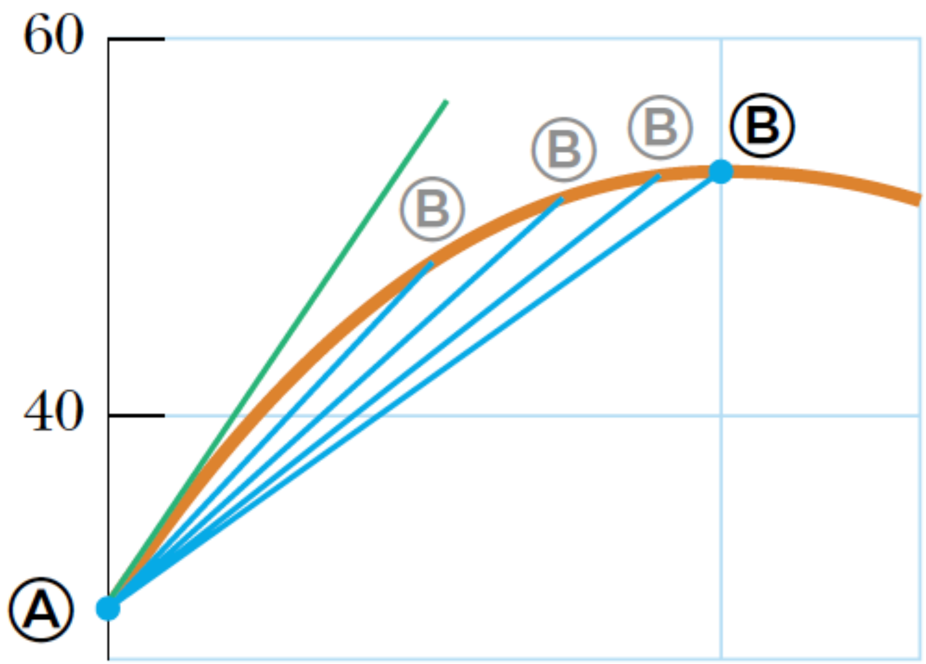
How can we find the instantaneous velocity?

2.4.Instantaneous Velocity and Speed



2.4.Instantaneous Velocity and Speed

The instantaneous velocity equals the limiting value of average velocity as Δt approaches zero.



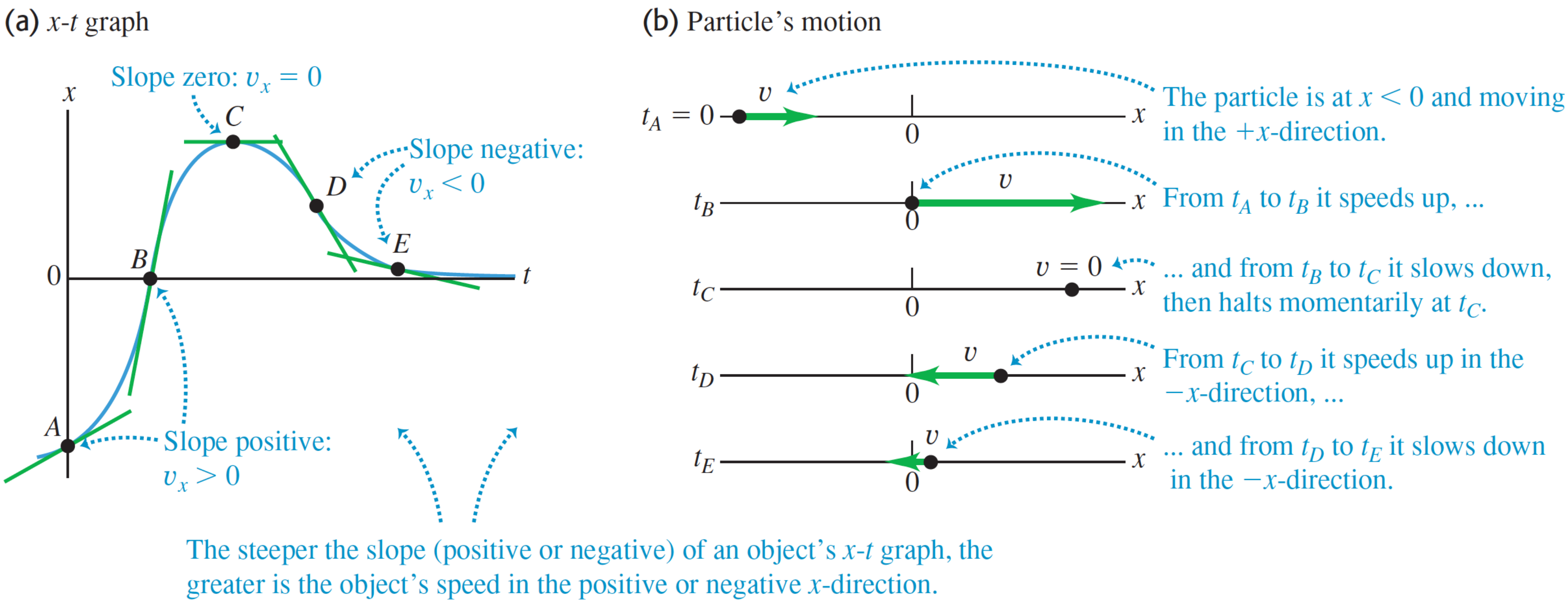
Anlık hız:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous velocity is a vector quantity !

2.4.Instantaneous Velocity and Speed

The instantaneous velocity can be positive, negative, or zero.



2.4.Instantaneous Velocity and Speed

Example 4:

A cheetah is crouched 20 m to the east of an observer. At time t=0 the cheetah begins to run due east toward an antelope that is 50 m to the east of the observer. During the first 2.0 s of the attack, the cheetah's coordinate x varies with time according to the equation

$$x = 20 + 5t^2$$

a) Find the cheetah's displacement between t=1 s and t= 2 s
b) Find its average velocity during that interval.
c) Find its instantaneous velocity at t=1 s by taking Δt= 0.1, Δt=0.01 ve Δt=0.001 s
d) Derive an expression for the cheetah's instantaneous velocity as a function of time.

2.4.Instantaneous Velocity and Speed

Answer 4:

a) The cheetah's positions at t=1 and t=2 s

$$\left.\begin{aligned} x_1 &= 20 + 5(1)^2 = 25\ \ m \\ x_2 &= 20 + 5(2)^2 = 40\ \ m \end{aligned}\right\} \quad \Delta x = x_2 - x_1 = 40 - 25 = 15\ \ m$$

b) The average x-velocity during this interval is

$$\Delta v_{ortalamaa} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{40 - 25}{1} = 15\ \ m/s$$

Answer 4:

c) With Δt=0.1 s the time interval is from t_1=1 s to a new t_2=1.1. At the position is

$$x_2 = 20 + (5.0m/s^2)(1.1s)^2 = 26.05m$$

The average x-velocity during this 0.1-s interval is

$$\Delta v_{avg.} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{26.05 - 25}{1.1 - 1.0} = 10.5\ m/s$$

Following this pattern, you can calculate the average x-velocities for 0.01-s and 0.001-s intervals: The results are 10.05 m/s ve 10.005 m/s. As Δt gets smaller, the average x-velocity gets closer to 10 m/s so we conclude that the instantaneous x-velocity at t=1.0 s is 10 m/s.

2.4.Instantaneous Velocity and Speed

Example 5:

A particle moves along the x axis. Its position varies with time according to the expression

$$x = -4t + 2t^2$$

where x is in meters and t is in seconds.

a) Determine the displacement of the particle in the time intervals t = 0 to t = 1 s and t =1 s to t = 3 s.
b) Calculate the average velocity during these two time intervals.
c) Find the instantaneous velocity of the particle at t =2.5 s.
d) Draw a position-time graph on graphing paper for the time interval t=0 to t=4 s.
e) Repeat a) b) and c using this position-time graph.

2.4.Instantaneous Velocity and Speed

Answer 5

a) for t=0, x_i=0 m
for t=1'de x_f=-2 m

$$\Delta x = x_s - x_i = -2 - 0 = -2m$$

for t=1, x_i = -2 m
for t=3 x_f = 6 m

$$\Delta x = x_s - x_i = 6 - (-2) = 8m$$

b) Average velocity in the time intervals t=0 to t= 1 s

$$\overline{v}_x = \frac{\Delta x}{\Delta t} = \frac{-2m}{1s} = -2m/s$$

Average velocity in the time intervals t=1 to t= 3 s

$$\overline{v}_x = \frac{\Delta x}{\Delta t} = \frac{8m}{2s} = 4m/s$$

2.4.Instantaneous Velocity and Speed

Answer 5

c) The instantaneous velocity at t=2.5 s would be the time derivative of position

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \left.\frac{dx}{dt}\right|_{t=2.5}$$

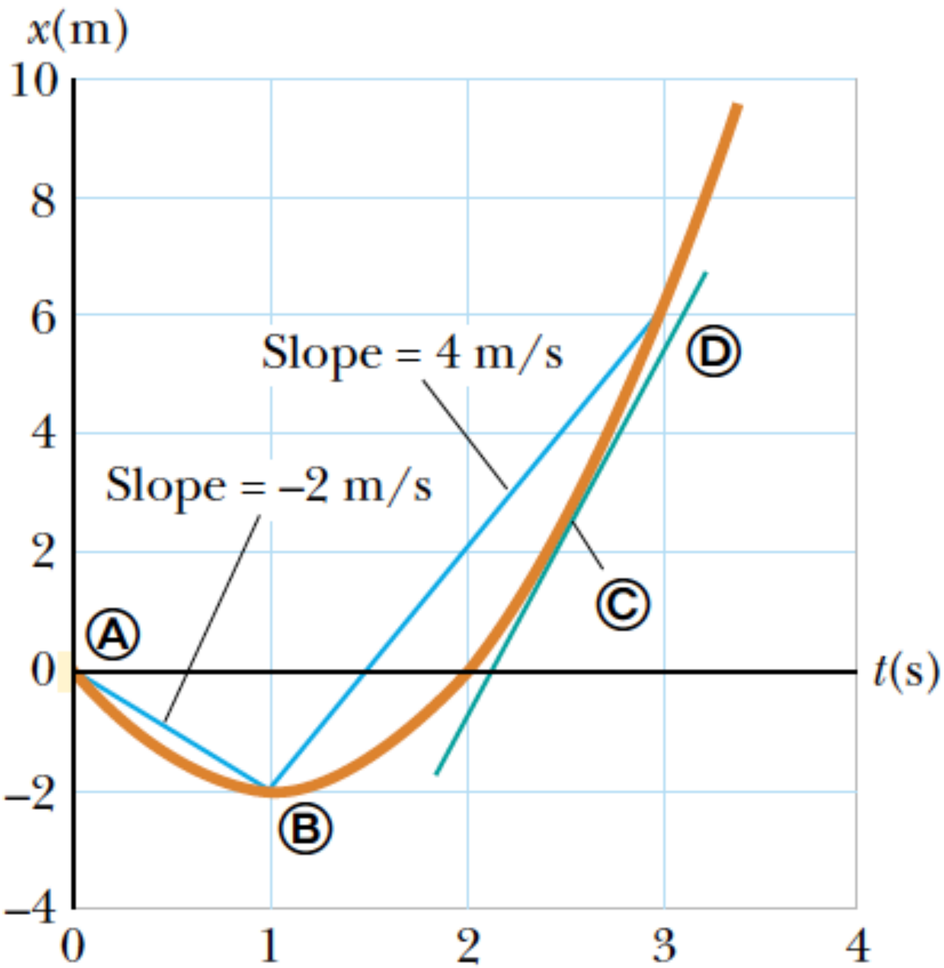
$$v_x = \frac{d}{dt}(-4t + 2t^2)\Big|_{t=2.5}$$

$$v_x = (-4 + 4t)\Big|_{t=2.5}$$

$$v_x = 6m/s$$

2.4.Instantaneous Velocity and Speed

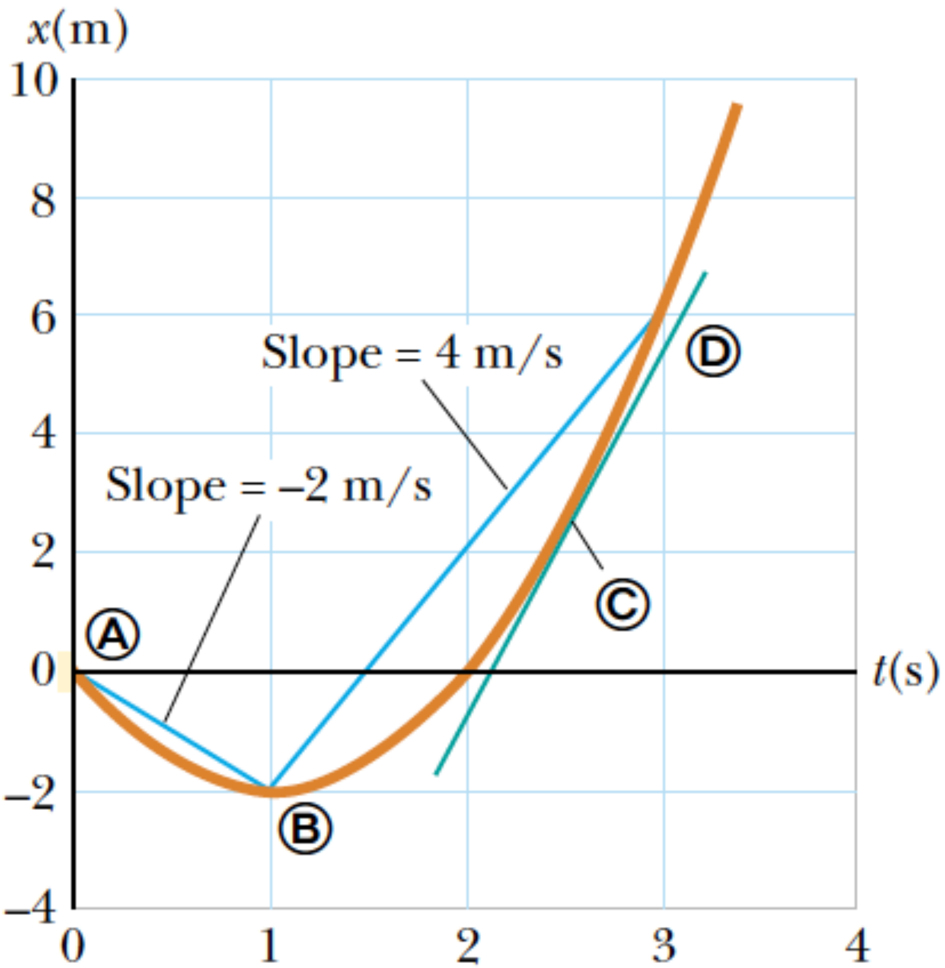
Answer 5

d) 

2.4.Instantaneous Velocity and Speed

Answer 5

d)



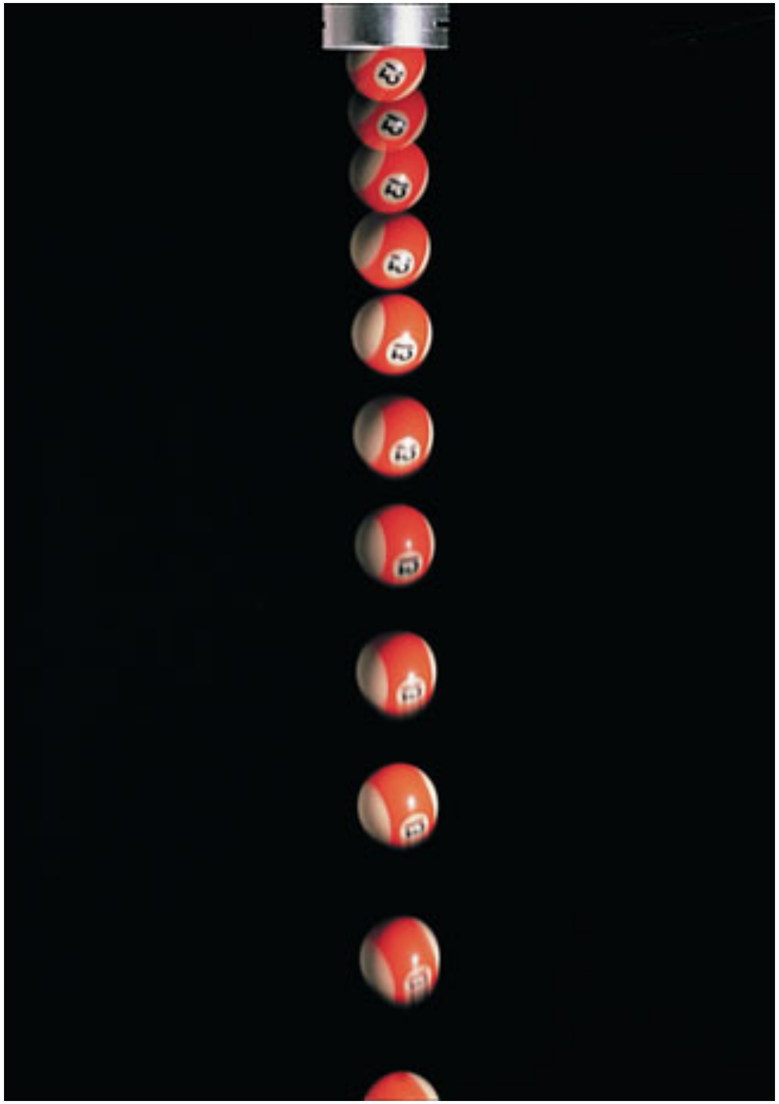
a) Position at any time can be easily read from the graph.

b) The average velocity of between any two points on curve would be the the slopes of the lines joining these points

c) The slope of tangent line at t=2.5 s give the instantaneous velocity at this point.

2.5. Acceleration

When a particle's velocity changes, the particle is said to undergo acceleration



2.5. Acceleration

The average acceleration of the particle is defined as the change in velocity Δv divided by the time interval Δt during which that change occurs:

$$\boxed{a_{avg} = \bar{a}_x = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}}$$

Acceleration is vectorel quantity too !

The unit of acceleration is m/s^2 in SI unit system.

+ 2.5. Acceleration

Rules for the sign of the acceleration

İf velocity is	…acceleration is
Positive and increasing (getting more positive)	Positive: Particle is moving in +x direction and speeding up
Positive and decreasing (getting less positive)	Negative: Particle is moving in +x direction and slowing down
Negative and increasing (getting less negative)	Positive: Particle is moving in -x direction and slowing down
Negative and decreasing (getting more negative)	Negative: Particle is moving in -x direction and speeding up

2.5. Acceleration

The instantaneous acceleration can be defined as the limit of the average acceleration as Δt approaches zero.

$$\boxed{a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}}$$

The instantaneous acceleration equals the derivative of the velocity with respect to time.

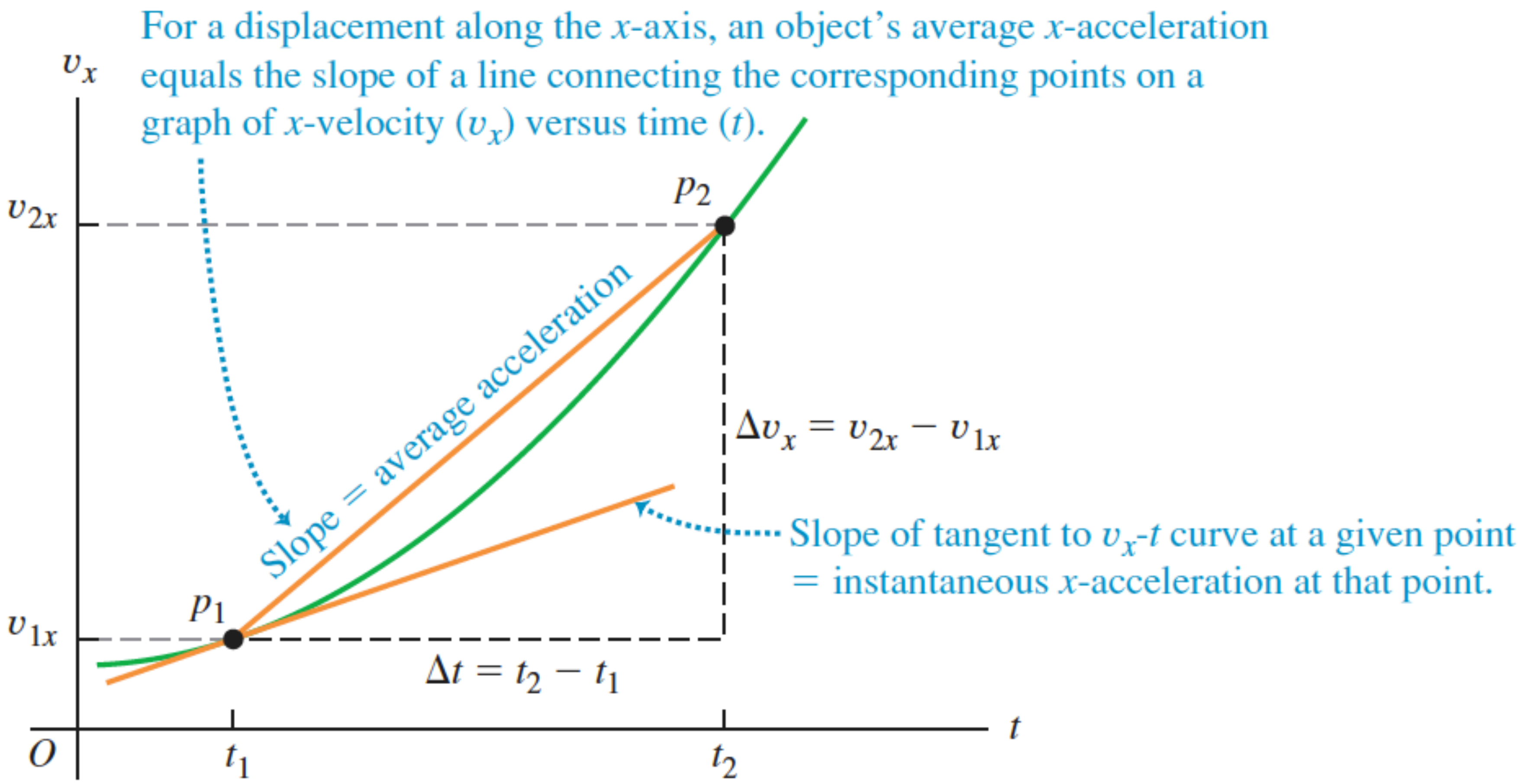
The acceleration can also be written

$$\boxed{a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}}$$

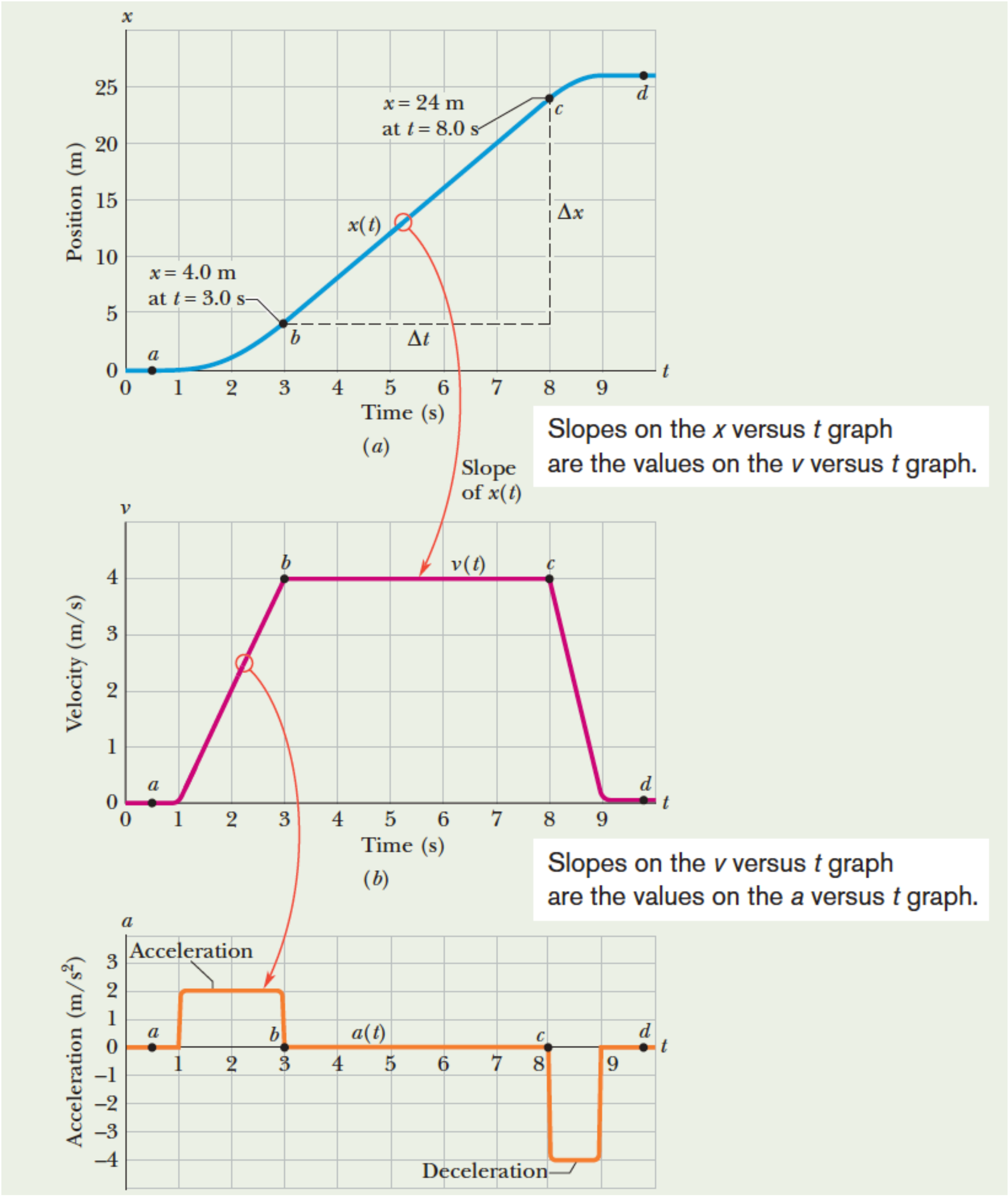
That is, in one-dimensional motion, the acceleration equals the second derivative of x with respect to time.

2.5. Acceleration

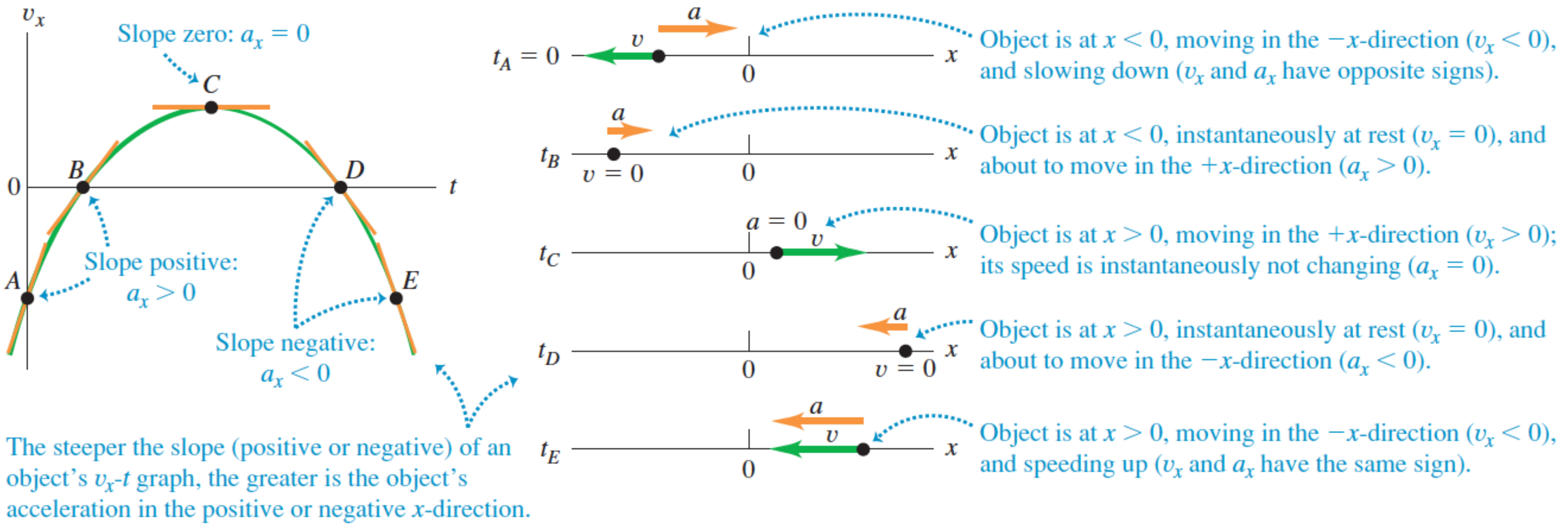
Finding Acceleration on a velocity-time graph



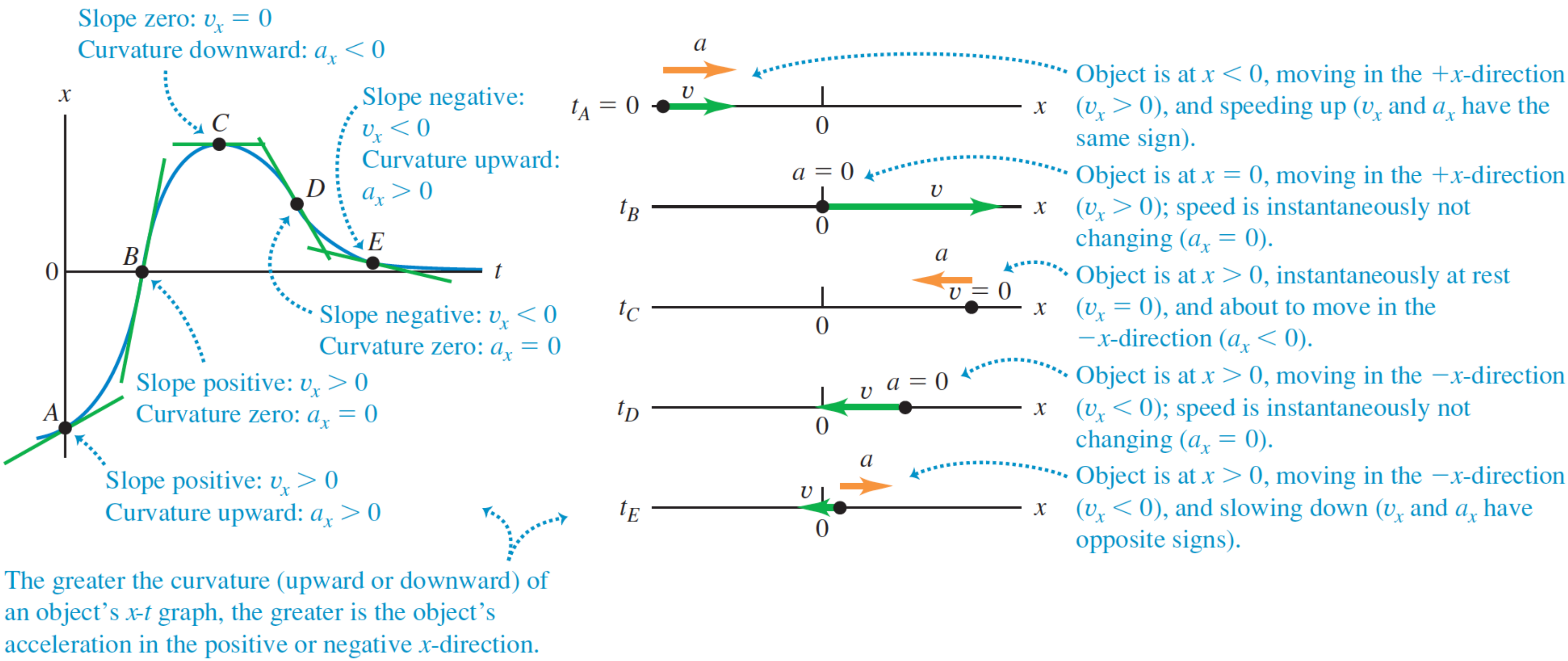
+ 2.5. Acceleration



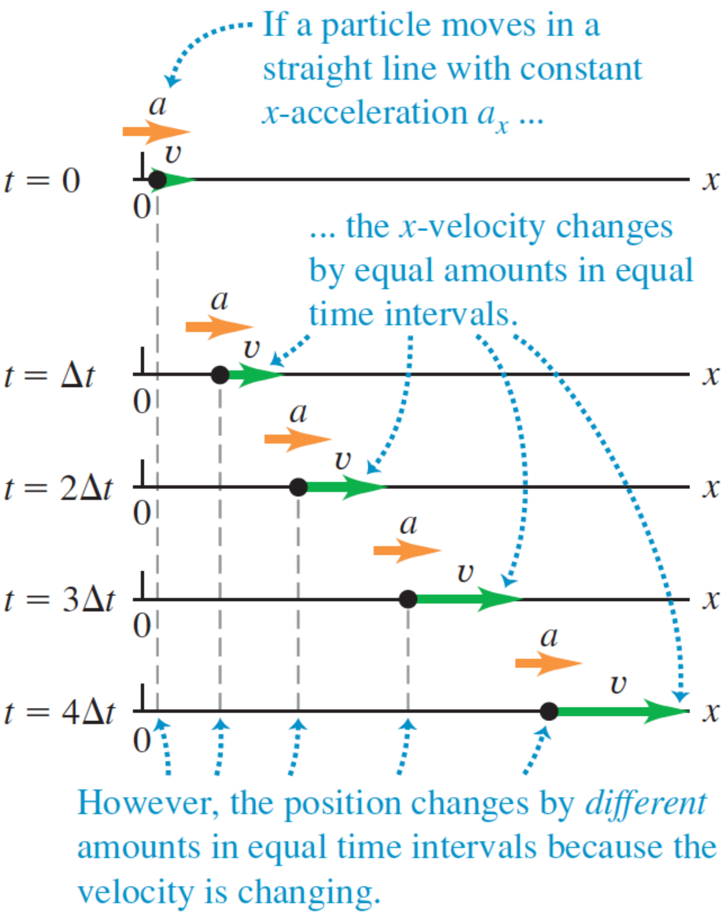
2.5. Acceleration



2.5. Acceleration



+ 2.6. Constant Acceleration

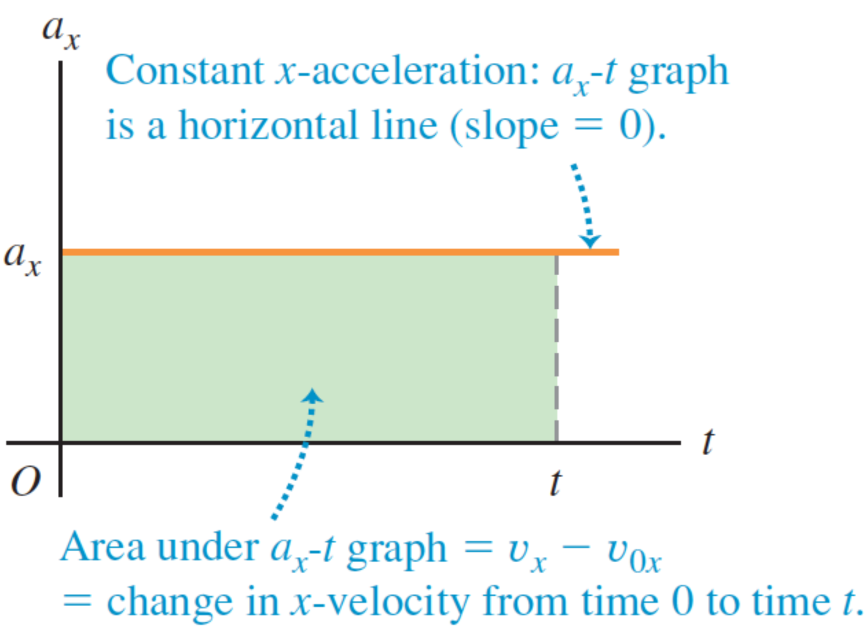


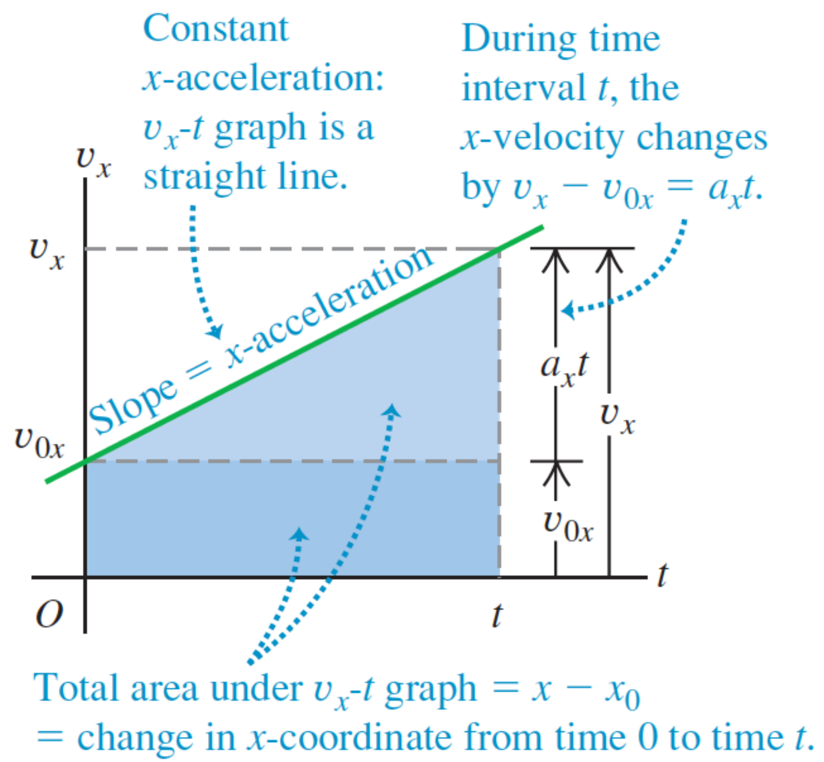
$$a_x = \frac{\Delta v}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

2.6. Constant Acceleration

An object's velocity at any time t for constant acceleration

$$\boxed{v_{xf} = v_{xi} + a_x t}$$





2.6. Constant Acceleration

Because velocity at constant acceleration varies linearly in time we can express the average velocity in any time interval as the arithmetic mean of the initial velocity v_{xi} and the final velocity v_{xf}

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{v_{xi} + v_{xf}}{2}$$

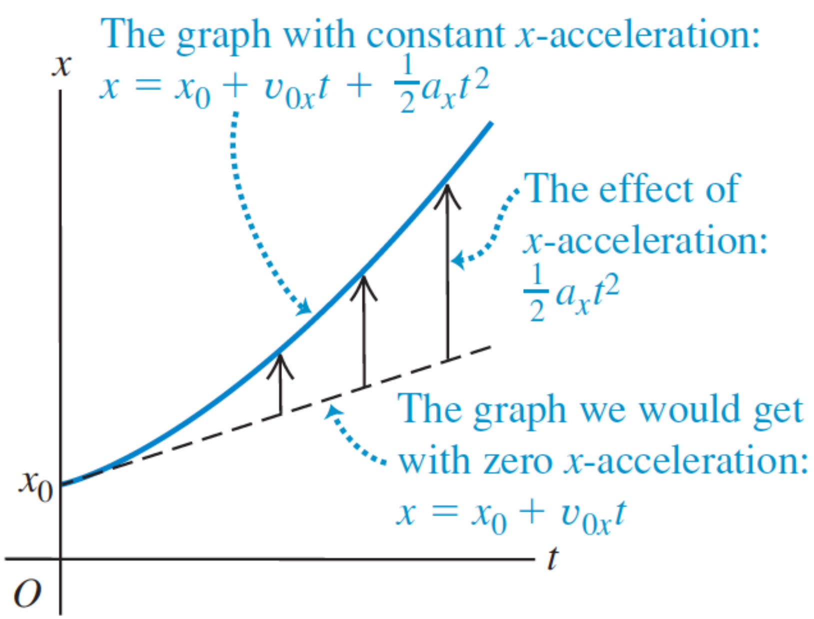
To obtain the position of an object as a function of time we can use ti=0 the definition of V_{xf} for constant acceleration:

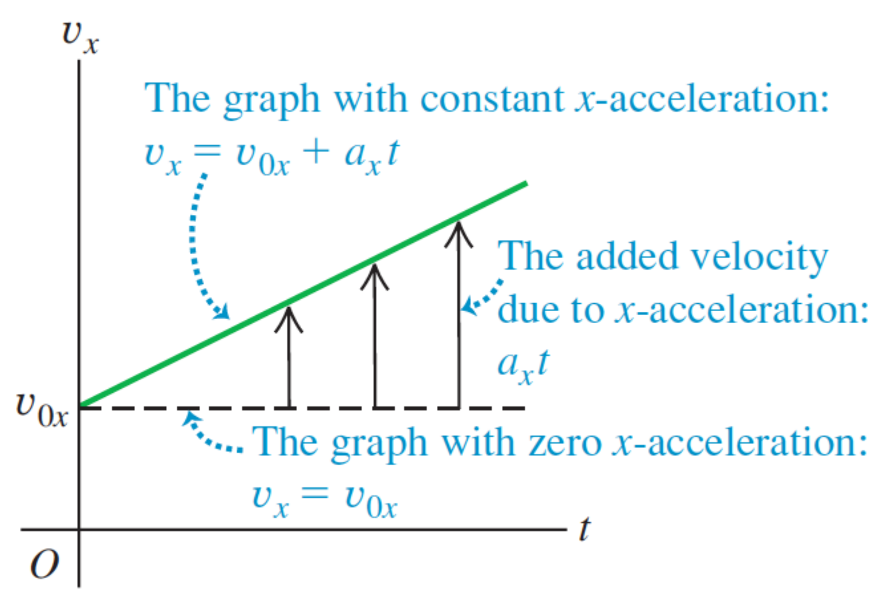
$$\boxed{x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t}$$

2.6. Constant Acceleration

The position of an object as a function of time

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$





2.6. Constant Acceleration

we can obtain an expression for the final velocity that does not contain time as a variable by substituting the value of t from

$$v_{xf} = v_{xi} + a_x t$$

this equation into this equation

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$$

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})(\frac{v_{xf} - v_{xi}}{a_x})$$

$$2a_x(x_f - x_i) = v_{xf}^2 - v_{xi}^2$$

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)}$$

2.6. Constant Acceleration

Kinematic equations for one dimensional motion with constant acceleration

$$\boxed{v_{xs} = v_{xi} + a_x t}$$

$$\boxed{x_s = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$

$$\boxed{v_{xs}^2 = v_{xi}^2 + 2a_x(x_s - x_i)}$$

$$\boxed{x_s = x_i + \frac{1}{2}(v_{xi} + v_{xs})t}$$

The choice of which equation you use in a given situation depends on what you know beforehand.

2.6. Constant Acceleration

Example 10:

On a highway at night you see a stalled vehicle and brake your car to a stop with an acceleration of magnitude 5 m/s^2. What is the car's stopping distance if its initial speed is (a) 15 m/s or (b) 30 m/s?

2.6. Constant Acceleration

Answer 10: a)

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

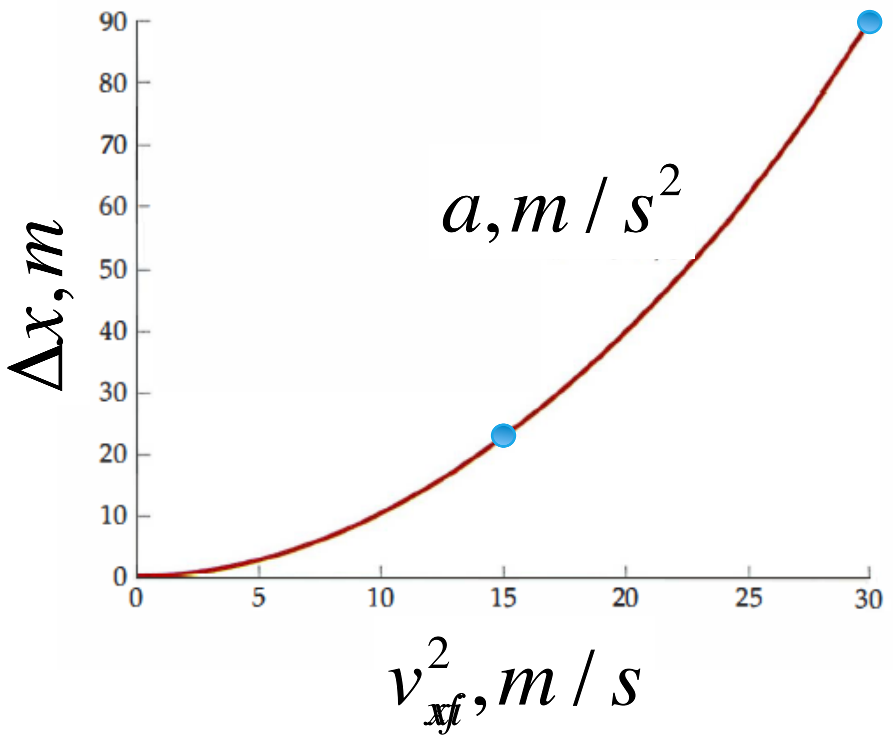
$$0 = v_{xi}^2 + 2a_x\Delta x$$

$$\Delta x = -\frac{v_{xi}^2}{2a_x} = -\frac{(15m/s)^2}{2(-5m/s^2)} = 22.5m$$

2.6. Constant Acceleration

Answer 10:b)

$$\Delta x = -\frac{v_{xi}^2}{2a_x} = 4(22.5m) = 90m$$



2.6. Constant Acceleration

Example 11:

In last Example 10, (a) how much time does it take for the car to stop if its initial velocity is 30 m/s? (b) How far does the car travel in the last second?

2.6. Constant Acceleration

Example 11:

In last Example 10, (a) how much time does it take for the car to stop if its initial velocity is 30 m/s? (b) How far does the car travel in the last second?

Answer 11:a)

$$v_{xf} = v_{xi} + a_x t$$

$$0 = 30m/s - (5m/s^2)t$$

$$t = 6s$$

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$$

$$90m = 0 + \frac{1}{2}(30m/s + 0).t$$

$$t = 6s$$

2.6. Constant Acceleration

Example 11:

In last Example 10, (a) how much time does it take for the car to stop if its initial velocity is 30 m/s? (b) How far does the car travel in the last second?

Answer 11:b)

$$\Delta x = -\frac{v_{xf}^2}{2a_x} = -\frac{5m/s^2}{2(-5m/s^2)} = 2.5m$$

2.6. Constant Acceleration

Example 12:

In a crash test, a car traveling 100 km/h hits an immovable concrete wall. What is its acceleration? Assume that the acceleration of ca is constant and stopping distance for the car is about 0.75 m.

2.6. Constant Acceleration

Example 12:

In a crash test, a car traveling 100 km/h hits an immovable concrete wall. What is its acceleration? Assume that the acceleration of ca is constant and stopping distance for the car is about 0.75 m.

Answer 12:

$$100\frac{km}{h}x\frac{1000m}{1km}x\frac{1h}{3.6ks}=27.8m/s$$

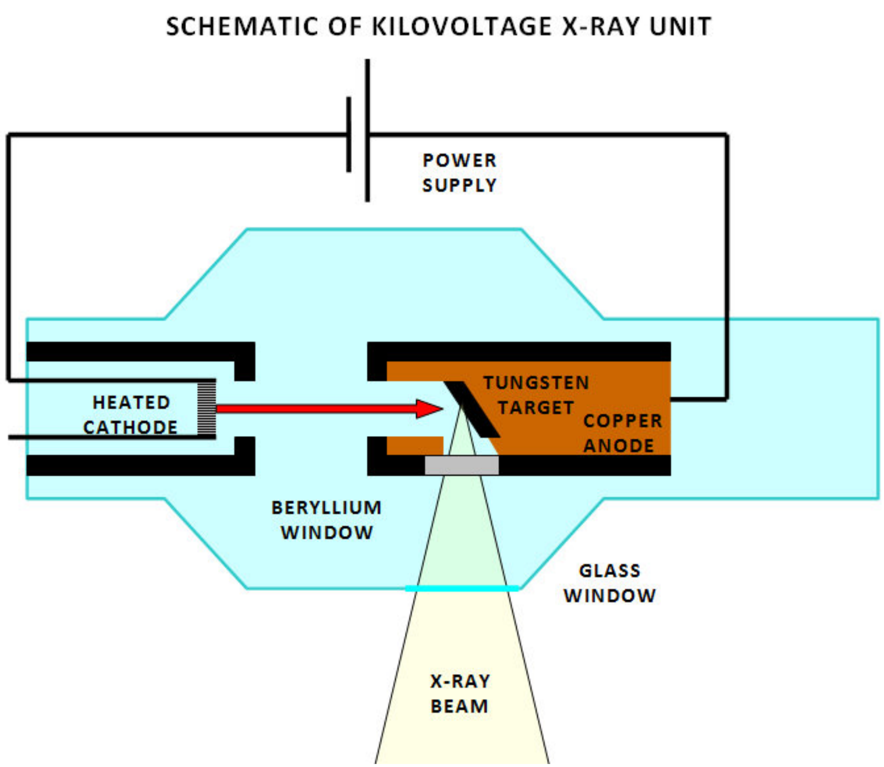
$$v_{xf}^2=v_{xi}^2+2a_x(x_f-x_i)$$

$$a_x=\frac{v_{xf}^2-v_{xi}^2}{2(x_f-x_i)}=\frac{(27.8m/s)^2}{20.75m}=-514m/s^2$$

+ 2.6. Constant Acceleration

Example 13:

An electron in a cathode-ray tube accelerates from rest with a constant acceleration of 5.33×10^{12} m/s^2 for 0.15 μs. The electron then drifts with constant velocity for 0.2 μs. Finally, it comes to rest with an acceleration of -2.67×10^{13} m/s^2. How far does the electron travel?



2.6. Constant Acceleration

Answer 13:

Find the displacement and final velocity for the first 0.15 μs interval

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(5.33x10^{12}m/s^2)(0.15x10^{-6}s)^2 \Rightarrow \Delta x_1 = 0.06m = 6.0cm$$

$$v_{xf} = v_{xi} + a_x t = (5.33x10^{12}m/s^2)(0.15x10^{-6}s) = 8x10^5 m/s$$

Use this final velocity as the constant velocity to find the displacement while it drifts at constant velocity.

$$x_s = x_i + v_{xi}t = (8x10^5 m/s)(0.2x10^{-6}s) \Rightarrow \Delta x_2 = 0.16m = 16.0cm$$

Answer 13:

Use this same velocity as the initial velocity to find the displacement for the third interval, in which the electron slows down.

$$v_{xs}^2 = v_{xi}^2 + 2a_x(x_s - x_i)$$

$$0 = (8x10^5 m/s)^2 + 2(-2.67x10^{-13} m/s^2)\Delta x$$

$$\Delta x_3 = 1.2cm$$

Add the displacements found in steps 1, 2, and 3 to find the total displacement

$$\Delta x_{total} = x_1 + x_2 + x_3 = 23.2cm$$

Example 14:

A car is speeding at 90 km/h in a school zone where the speed limit were 10 m/s. A police car starts from rest just as the speeder passes and accelerates at a constant rate of 5 m/s^2.

(a) When does the police car catch the speeding car?

(b) How fast is the police car traveling when it catches up with the speeder?

(c) How far have the cars traveled when the police car catches the speeder?

(d) How fast is the police car travelling when it is 25 m behind the speeding car?

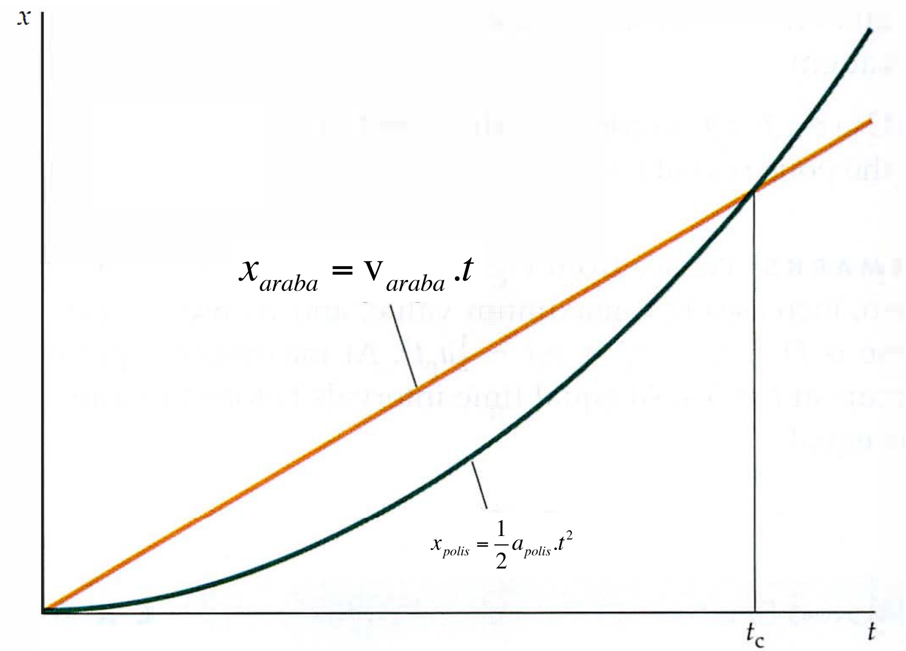
(e) When is the distance between police car and speeder maximum ? What is the value of maximum distance between police car and speeder in m?

2.6. Constant Acceleration

Answer 14 a)

$$x_{speeder} = v_{speeder}.t$$

$$x_{police} = \frac{1}{2}a_{police}.t^2$$



$$V_{speeder} = 90\frac{km}{h}x\frac{1000m}{1km}x\frac{1h}{3.6ks} = 15m/s$$

$$v_{speeder}.t_c = \frac{1}{2}a_{police}.t_c^2$$

$$t_c = \frac{2v_{speeder}}{a_{police}} = \frac{2.(25m/s)}{5m/s^2} = 10s$$

2.6. Constant Acceleration

Answer :b)

$$v_{xspolice} = v_{xipolice} + a_{police} t_c$$

$$v_{xspolice} = 0 + (5m/s^2).10s = 50m/s$$

Answer :c)

$$x_{police} = \frac{1}{2} a_{police} .t_c^2 = \frac{1}{2}(5m/s^2).(10s)^2 = 250m$$

$$x_{speeder} = \mathrm{v}_{speeder} .t = (25m/s).(10s) = 250m$$

2.6. Constant Acceleration

Answer : d)

$$\left.\begin{aligned} x_{speeder} &= \mathrm{v}_{speeder}.t \\ x_{police} &= \frac{1}{2}a_{police}.t^2 \end{aligned}\right\} \quad x_{speeder} - x_{police} = \mathrm{v}_{speeder}.t - \frac{1}{2}a_{speeder}.t^2$$

$$25 = 25t - \frac{1}{2}5t^2$$

$$\boxed{t_1 = 8.873s} \qquad \boxed{t_2 = 1.127s}$$

$$v_{xspolice} = v_{xipolice} + a_{police}t_1$$

$$v_{xspolice} = 0 + (5m/s^2).(8.873s) = 44.4m/s$$

$$v_{xspolice} = v_{xipolice} + a_{police}t_2$$

$$v_{xspolice} = 0 + (5m/s^2).(1.127s) = 5.64m/s$$

2.6. Constant Acceleration

Answer 14:f)

The separation between the police and speeding car at any time is given by

$$D = x_{speeding} - x_{police} = \mathrm{v}_{speeding}.t - \frac{1}{2}a_{police}.t^2$$

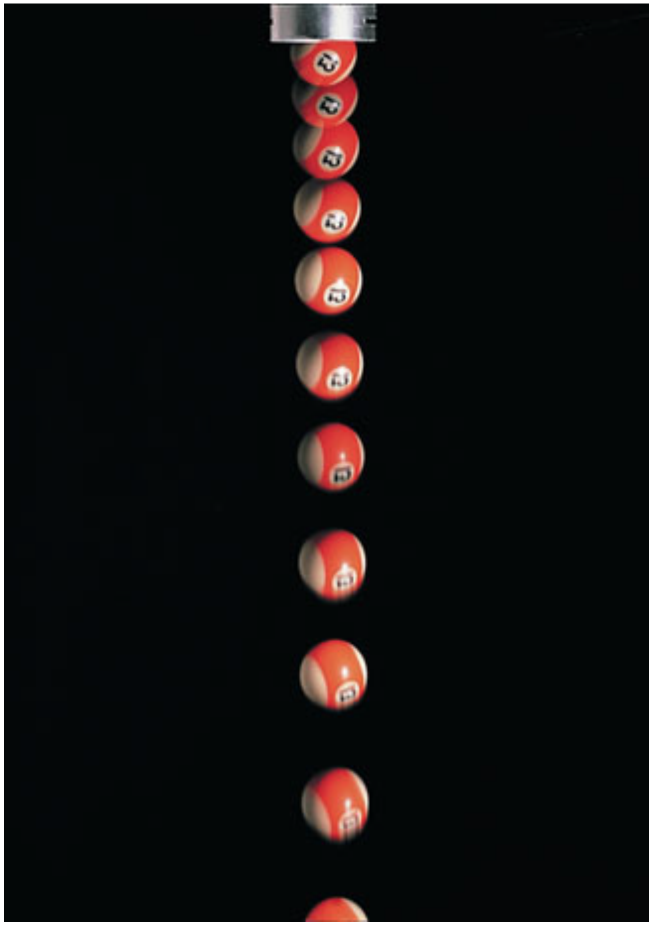
At maximum separation is dD/dt=0

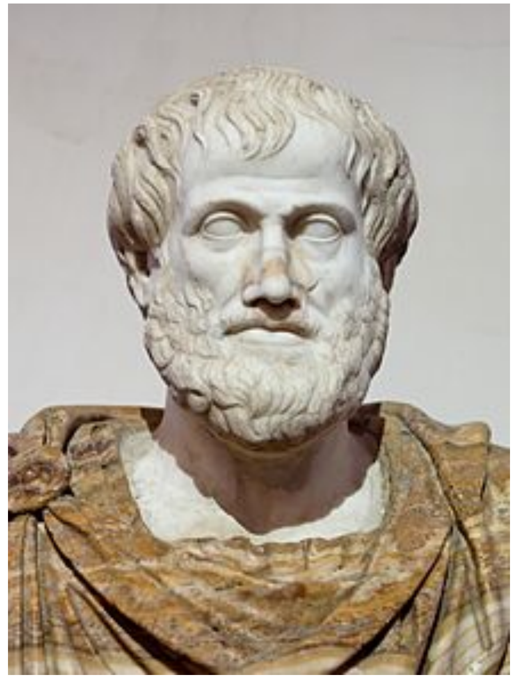
$$\frac{d}{dt}(x_{speeder} - x_{police}) = \mathrm{v}_{speeder}. - a_{police}.t = 0 \quad \Rightarrow 25 - 5.t = 0 \Rightarrow t = 5s$$

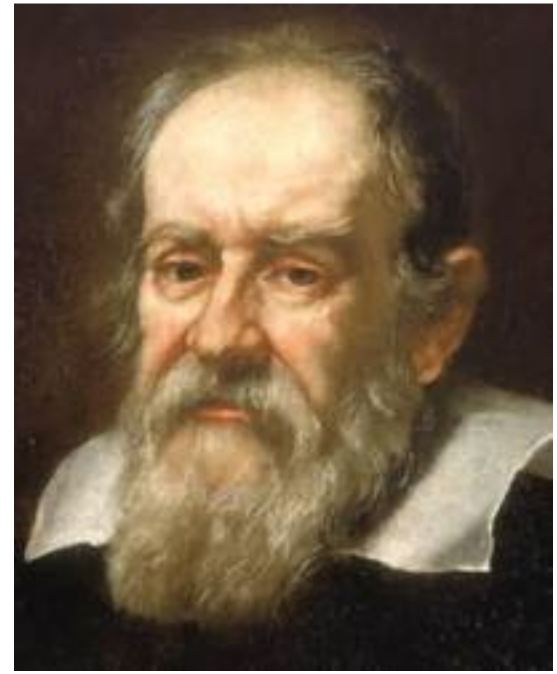
The separation between the police and speeding car at t=5 s:

$$x_{speeder} - x_{police} = \mathrm{v}_{speeder}.t - \frac{1}{2}a_{police}.t^2 = 25.5 - -\frac{1}{2}5.5^2 = 62.5m$$

2.7. Free Fall Acceleration

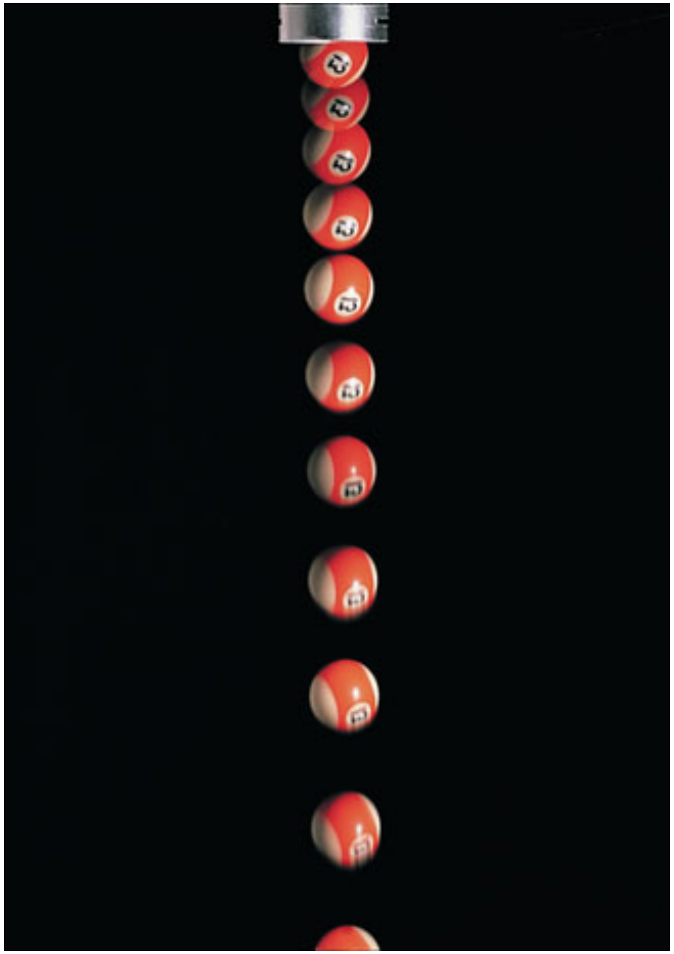






A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion

2.7. Free Fall Acceleration



A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion

Objects thrown upward or downward and those released from rest are all falling freely once they are released.

Any freely falling object experiences an acceleration directed downward, regardless of its initial motion.

We shall denote the magnitude of the free-fall acceleration by the symbol g.

$$g = 9.8m/s^2 \approx 10m/s^2$$

2.7. Free Fall Acceleration

If we neglect air resistance and assume that the free-fall acceleration does not vary with altitude over short vertical distances, then the motion of a freely falling object moving vertically is equivalent to motion in one dimension under constant acceleration.

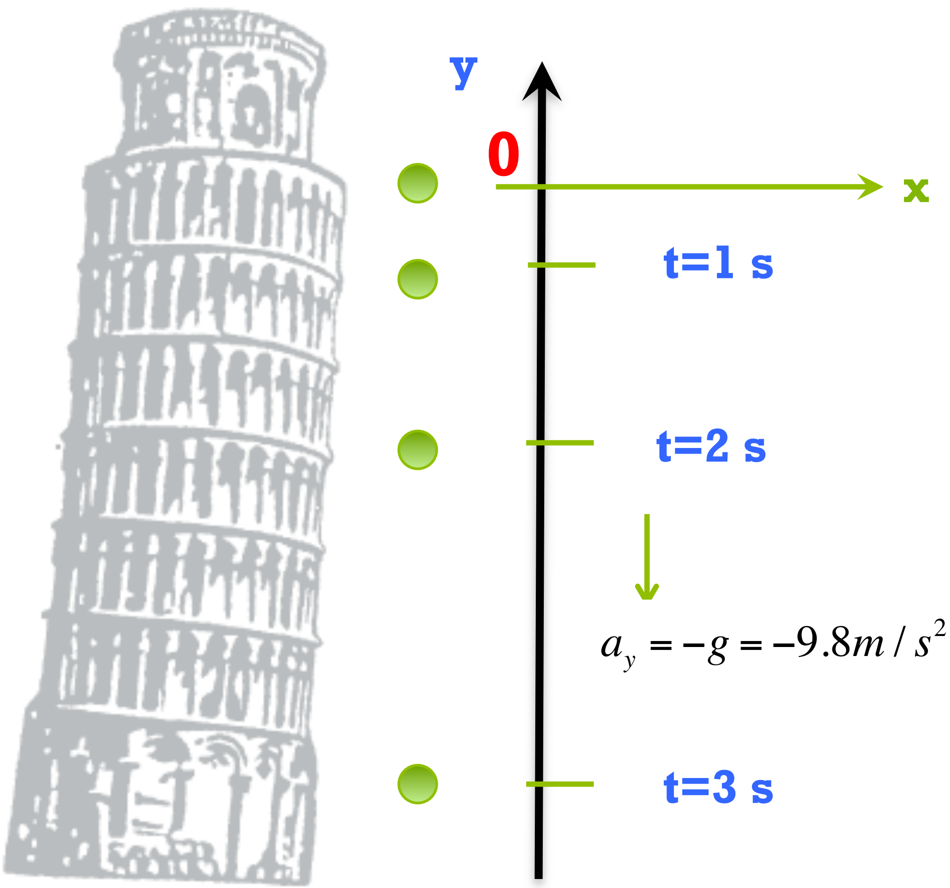
Therefore, the equations developed in Section 2.6 for objects moving with constant acceleration can be applied.

$$a_y = -g = -9.8 m/s^2 \approx -10 m/s^2$$

2.7. Free Fall Acceleration

Example 15:

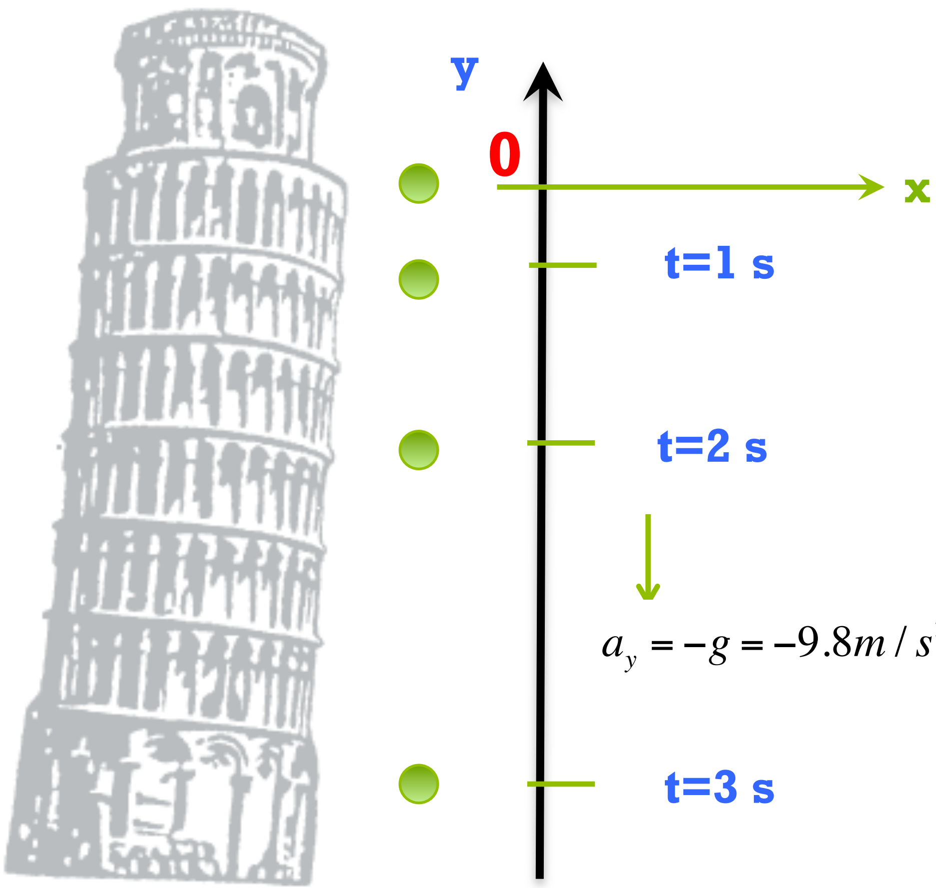
A one-euro coin is dropped from the Leaning Tower of Pisa and falls freely from rest. What are its position and velocity after 1.0 s, 2.0 s, and 3.0 s?



$$a_y = -g = -9.8 m/s^2 \approx -10 m/s^2$$

2.7. Free Fall Acceleration

Answer 15:



$$a_y = -g = -9.8 m/s^2 \approx -10 m/s^2$$

$$\boxed{y_s = y_i + v_{yi}t + \frac{1}{2}a_y t^2}$$

$$\boxed{v_{ys} = v_{yi} + a_y t}$$

2.7. Free Fall Acceleration

Answer 15

for t=1.0 s

$$y_s = 0 + 0t + \frac{1}{2}(-10m/s^2)x(1s)^2 = -5m$$

$$v_{ys} = 0 + (-10m/s^2)x(1s) = -10m/s$$

for t=2.0 s

$$y_s = 0 + 0t + \frac{1}{2}(-10m/s^2)x(2s)^2 = -20m$$

$$v_{ys} = 0 + (-10m/s^2)x(2s) = -20m/s$$

for=3.0 s

$$y_s = 0 + 0t + \frac{1}{2}(-10m/s^2)x(3s)^2 = -45m$$

$$v_{ys} = 0 + (-10m/s^2)x(3s) = -30m/s$$

2.7. Free Fall Acceleration

Example 16:

You throw a ball vertically upward from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of the ball is 15m/s then in free fall. On its way back down, it just misses the railing. Find

(a) the ball's position and velocity 1.00 s and 4.00 s after leaving your hand;

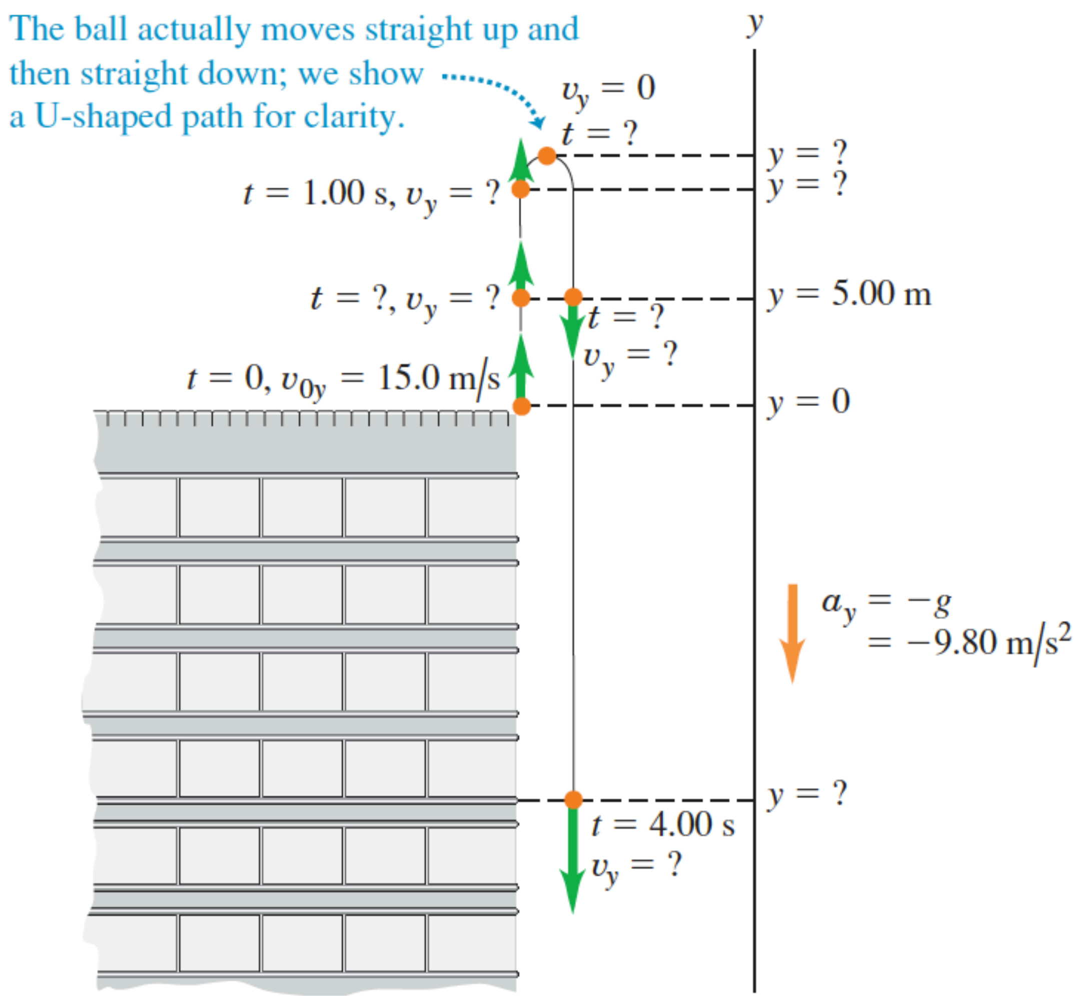
(b) the ball's velocity when it is 5.00 m above the railing;

(c) the maximum height reached;

(d) the ball's acceleration when it is at its maximum height.

2.7. Free Fall Acceleration

Answer 16:



2.7. Free Fall Acceleration

Answer 16:a)

The position and y-velocity at time t are given by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad v_f = v_i + a_y t$$

for t=1 s

$$y_f = 0 + (15m/s).(1s) + \frac{1}{2}(-9.8m/s^2).(1s)^2 = 10.1m$$

$$v_f = 15m/s + (-9.8m/s^2).(1s) = 5.2m/s$$

2.7. Free Fall Acceleration

Answer 16:a)

for t=4 s

$$y_f = 0 + (15m/s).(4s) + \frac{1}{2}(-9.8m/s^2).(4s)^2 = -18.4m$$

$$v_f = 15m/s + (-9.8m/s^2).(4s) = -24.2m/s$$

Answer 16:b)

When the ball is 5.00 m above the origin we have y_f=5.0 m

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$v_{yf}^2 = (15m/s)^2 + 2(-9.8m/s^2).(5-0)$$

$$v_{yf}^2 = 127.69 \Rightarrow v_{yf} = \pm 11.3m/s$$

2.7. Free Fall Acceleration

Answer 16:c)

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$0 = (15m/s)^2 + 2(-9.8m/s^2).(y_f - 0)$$

$$y_f = y_{\max} = h = 11.5m$$

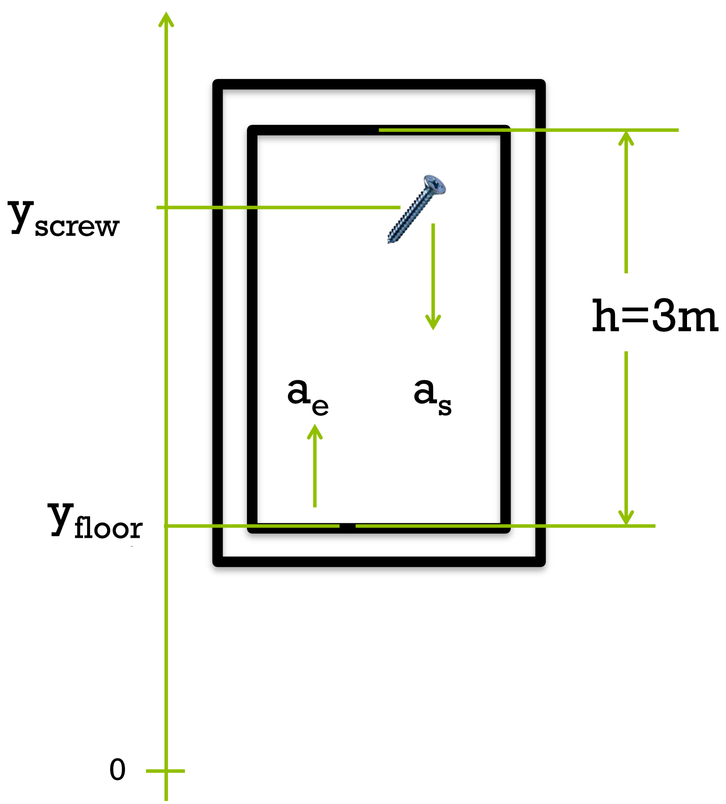
Answer 16:d)

Discuss !

2.7. Free Fall Acceleration

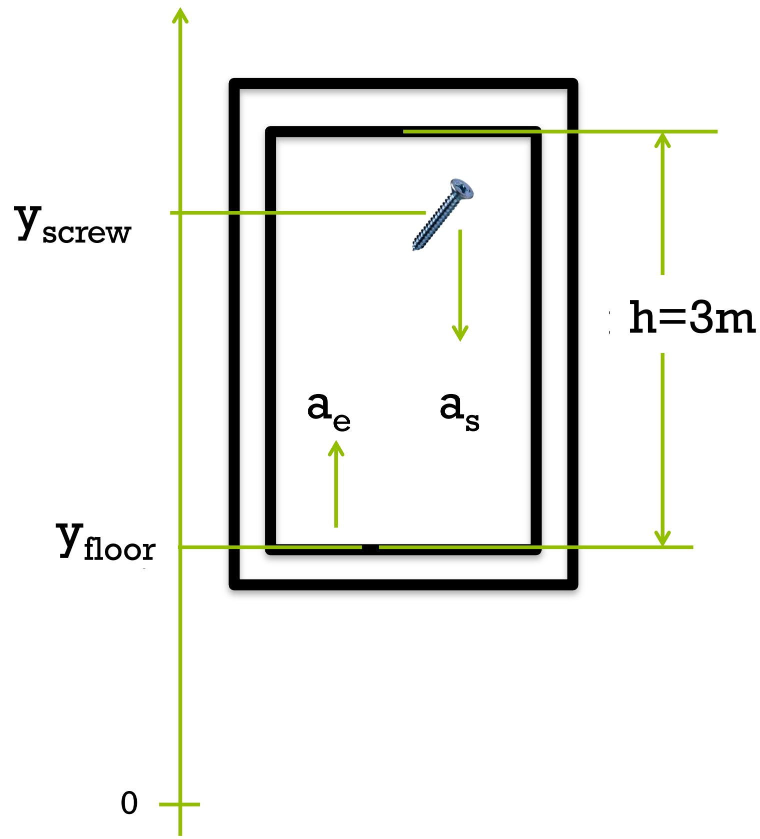
Example 17:

While standing in an elevator, you see a screw fall from the ceiling. The ceiling is 3 m above the floor. How long does it take for the screw to hit the floor if the elevator is moving upward and gaining speed at a constant rate of a = 4.0 m/s²?



+ 2.7. Free Fall Acceleration

Answer 17



Screw: $$y_{screw,f} = y_{screw,i} + v_{screw,i}t + \frac{1}{2}a_{s,y}t^2$$

Floor: $$y_{floor,f} = y_{floor,i} + v_{floor,i}t + \frac{1}{2}a_e t^2$$

At time when the screw hits the floor, these positions are equal:

$$y_v = y_t$$

$$y_{si,f} + \cancel{v_{syi}t} + \frac{1}{2}a_s t^2 = y_{fi} + \cancel{v_{fyi}t} + \frac{1}{2}a_e t^2$$

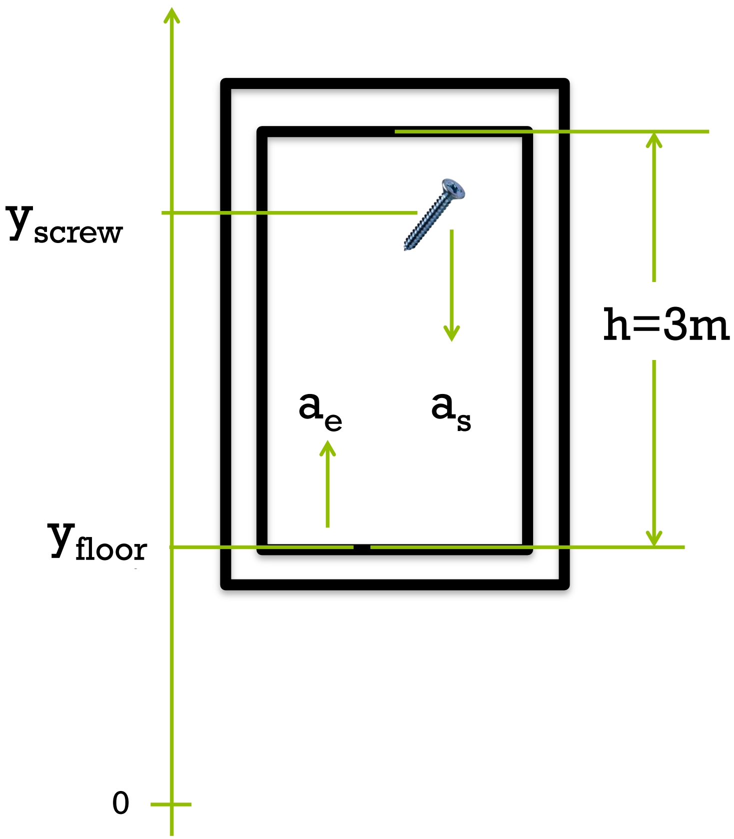
$$y_{si,f} + \frac{1}{2}a_s t^2 = y_{si} + \frac{1}{2}a_e t^2$$

$$y_{si} = h = 3m, a_s = -g, y_{fi} = 0, a_e = 4m/s^2$$

$$h - \frac{1}{2}gt^2 = 0 + \frac{1}{2}a_e t^2$$

2.7. Free Fall Acceleration

Answer 17:



$$h - \frac{1}{2}gt^2 = 0 + \frac{1}{2}a_e t^2$$

$$h = \frac{1}{2}(g + a_e)t^2$$

$$t = \sqrt{\frac{2h}{g + a_e}}$$

$$t = \sqrt{\frac{2.(3m)}{9.8m/s^2 + 4m/s^2}} = 0.659s$$

2.7. Free Fall Acceleration

Homework 1:

Consider the elevator and screw in Example 17. Assume the velocity of the elevator is 16 m/s upward when the screw separates from the ceiling.

(a) How far does the elevator rise while the screw is falling? How far does the screw fall?
(b) What is the velocity of the screw and the velocity of the elevator at impact?
(c) What is the velocity of the screw relative to the floor at impact?

2.7. Free Fall Acceleration

Answer HM1: a)

The distance the floor rises in time t_1=0.659 s.

$$y_{floor,f} = y_{floor,i} + v_{floor,i}t + \frac{1}{2}a_e t^2$$

$$y_{floor,f} = 0 + (16m/s).(0659s) + \frac{1}{2}(4m/s^2).(0.659)^2$$

$$y_{floor,f} = 11.42m$$

2.7. Free Fall Acceleration

Answer HM1: b)

The position of screw in this time should be same:

$$y_{screw,f} = y_{screw,i} + v_{screw,i}t + \frac{1}{2}gt^2$$

$$y_{floor,f} = 3m + (16m/s).(0659s) + \frac{1}{2}(-9.8m/s^2).(0.659)^2$$

$$y_{floor,f} = 11.42m$$

Since the screw starts out 3 m above the floor the distance, the screw falls in time is 8.42 m.

2.7. Free Fall Acceleration

Answer HM1: c)

The impact velocity of the screw and of the floor at impact

$$v_{screw,f} = v_{screw,i} - gt$$

$$v_{screw,f} = 16m/s - (9.8m/s^2).(0.659) = 9.5m/s$$

$$v_{floor,f} = v_{floor,i} + at$$

$$v_{screw,f} = 16m/s + (4m/s^2).(0.659) = 18.6m/s$$

2.7. Free Fall Acceleration

Homework 2:

Your friend climbs a tree to get a better view of the musician at an outdoor music concert. Unfortunately, he leaves his binoculars behind. You throws them up to your friend, but your strength is greater than your accuracy. The binoculars pass your friend's outstretched hand after 0.69 s and again 1.68 s later. How high is your friend? Find the initial velocity of the binoculars and the velocity of the binoculars as they pass your friend on the way down.

2.7. Free Fall Acceleration

Answer HM2

$$h = v_{yi}t_1 + \frac{1}{2}a_y t_1^2 \qquad h = v_{yi}t_2 + \frac{1}{2}a_y t_2^2$$

$$h = v_{yi}t_1 - \frac{1}{2}g t_1^2 \qquad h = v_{yi}t_2 - \frac{1}{2}g t_2^2$$

$$h = v_{yi}t_2 - \frac{1}{2}g t_2^2$$

$$h = \left(\frac{h + \frac{1}{2}g t_1^2}{t_1}\right)t_2 - \frac{1}{2}g t_2^2 = 8.02m$$

2.7. Free Fall Acceleration

Answer HM 2

The initial velocity of the binocular:

$$h = v_{yi}t_1 - \frac{1}{2}gt_1^2$$

$$8.02m = v_{yi}(0.69s) - \frac{1}{2}(9.8m/s^2).(0.69s)^2$$

$$v_{yi} = 15m/s$$

2.7. Free Fall Acceleration

Answer HM 2

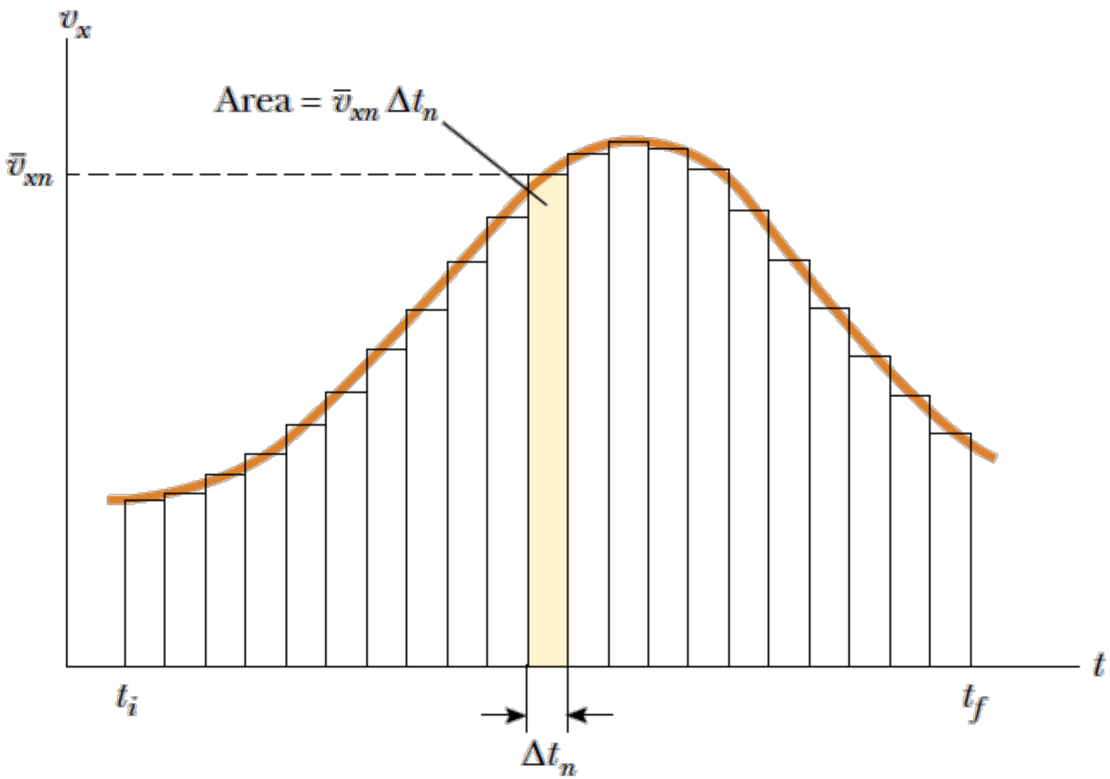
The velocity of the binocular as they pass your friend on the way down

$$v_{yf} = v_{yi} + at_2$$

$$v_{yf} = 15m/s - (9.8m/s^2).(2.37s)$$

$$v_{yi} = -8.23m/s$$

2.8. Graphical Integration in Motion Analysis

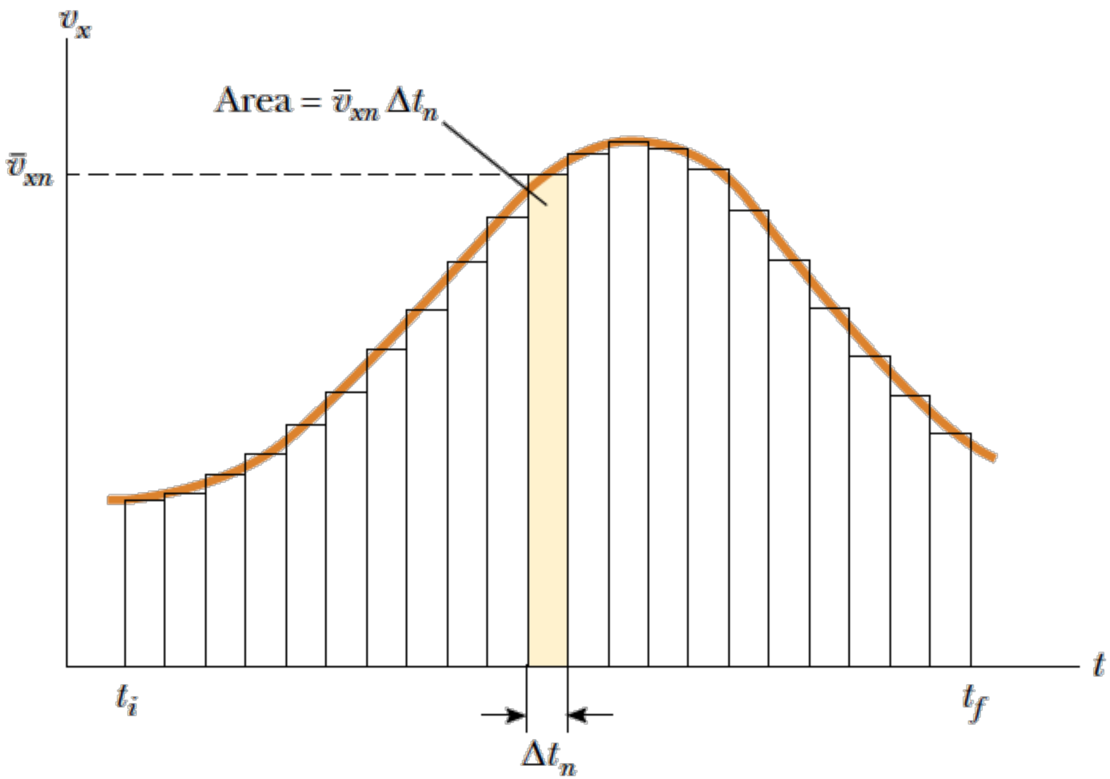


$$\Delta x = \sum_n \bar{v}_{xn} \Delta t_n$$

$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n$$

Displacement = area under the v_x-t graph

2.8. Graphical Integration in Motion Analysis



$$\Delta x = \sum_n \bar{v}_{xn} \Delta t_n$$

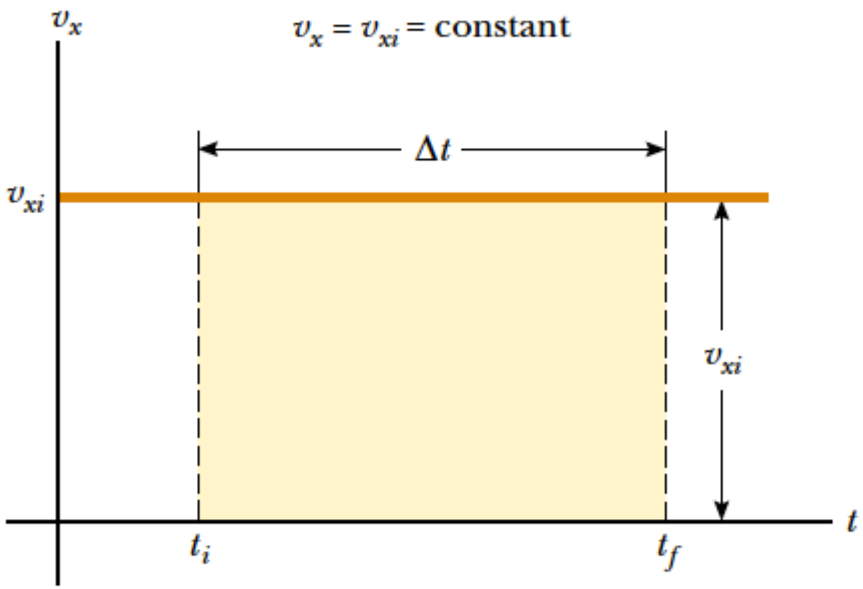
$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n$$

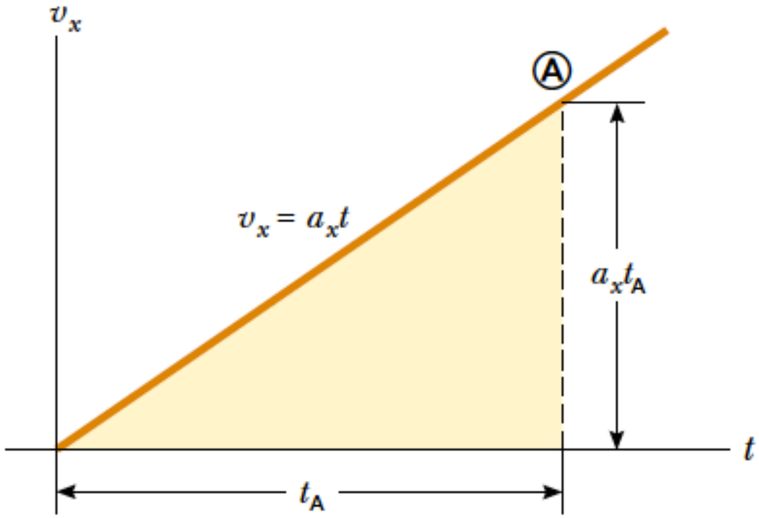
Displacement = area under the v_x-t graph

2.8. Graphical Integration in Motion Analysis

Definite integral

$$\lim_{\Delta t_n} \sum_n v_{xn} \Delta t_n = \int_{ti}^{ts} v_x(t)\,dt$$





2.8. Graphical Integration in Motion Analysis

Definite integral

$$\lim_{\Delta t_n} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$

So for time interval $\Delta t = t_2 - t_1$, the displacement is given by:

$$\boxed{x_2 - x_1 = \int_{t1}^{t2} v(t)\,dt}$$

Same procedure can be applied to displacement-time curve

$$\boxed{v_2 - v_1 = \int_{t1}^{t2} a(t)\,dt}$$

2.8. Graphical Integration in Motion Analysis

Homework 3:

Sally is driving along a straight highway in her 1965 Mustang. At t=0 when she is moving at 10 m/s in the positive x-direction, she passes a signpost at x= 50 m. Her x-acceleration as a function of time is

$$a = 2.0 m/s^2 - (0.10 m/s^3)t$$

(a) Find her x-velocity and position x as functions of time.
(b) When is her x-velocity greatest?
(c) What is that maximum x-velocity?
(d) Where is the car when it reaches that maximum x-velocity?
(e) Draw the a-t, v-t and x-t graph in graphing paper.

2.8. Graphical Integration in Motion Analysis

Answer HM 3:a)

The integral of acceleration give us the velocity of car as a function of time

$$v_2 - v_1 = \int_0^t a(t)\,dt$$

$$v(t) = v_2 = v_1 + 2.0t - \frac{1}{2}(0.10)t^2$$

2.8. Graphical Integration in Motion Analysis

Answer HM3:a)

The integral of velocity give us the position as a function of time:

$$x_2 - x_1 = \int_0^t v(t)\,dt$$

$$x_2 = x_1 + \int_0^t \left[v_1 + 2.0t - \frac{1}{2}(0.10)t^2 \right] dt$$

$$x_2 = x_1 + v_1 t + 2.0\frac{t^2}{2} - -\frac{1}{2}(0.10)\frac{t^3}{3}$$

$$x_2 = x_1 + v_1 t + t^2 - \frac{0.10}{6}t^3$$

2.8. Graphical Integration in Motion Analysis

Answer HM3:b)

We have to find the first derivative of velocity and then equal to zero and solve for t:

$$\frac{dv}{dt} = 2.0m/s^2 - (0.10m/s^3)t = 0$$

$$t = 20s$$

Answer HM3:c)

We find the maximum x-velocity by substituting, the time from part (b) when velocity is maximum, into the equation for from part (a):

$$v_{\max} = (2.0).(20s) - \frac{1}{2}(0.10).(20)^2 = 30m/s$$

2.8. Graphical Integration in Motion Analysis

Answer HM3:d)

To find the car's position at the time that we found in part (b), we substitute into the expression for x from part (a):

$$x_2 = x_1 + v_1 t + t^2 - \frac{0.10}{6} t^3$$

$$x_{\max} = 50m + (10m/s).(20s) + (20s)^2 - \frac{0.10}{6}(20s)^3 = 517m$$

2.8. Graphical Integration in Motion Analysis

Answer HM3:e)