PHY121 Physics I

Chapter 3 Vectors

Assoc. Prof. Dr. Eyüp DUMAN
University of Ankara Faculty of Engineering
Department of Energy Engineering

Chapter 3 Vectors

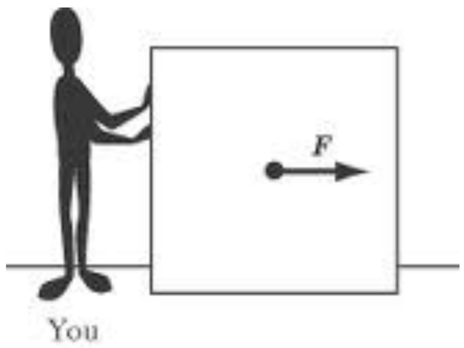
3.1. Vectors and Scalars
3.2. Adding Vectors Geometrically
3.3. Components of Vectors and Unit Vectors
3.4. Multiplying Vectors

3.1 Vectors and Scalars

Scalar quantities :

- A scalar quantity is completely specified by a single value with an appropriate unit and has no direction.

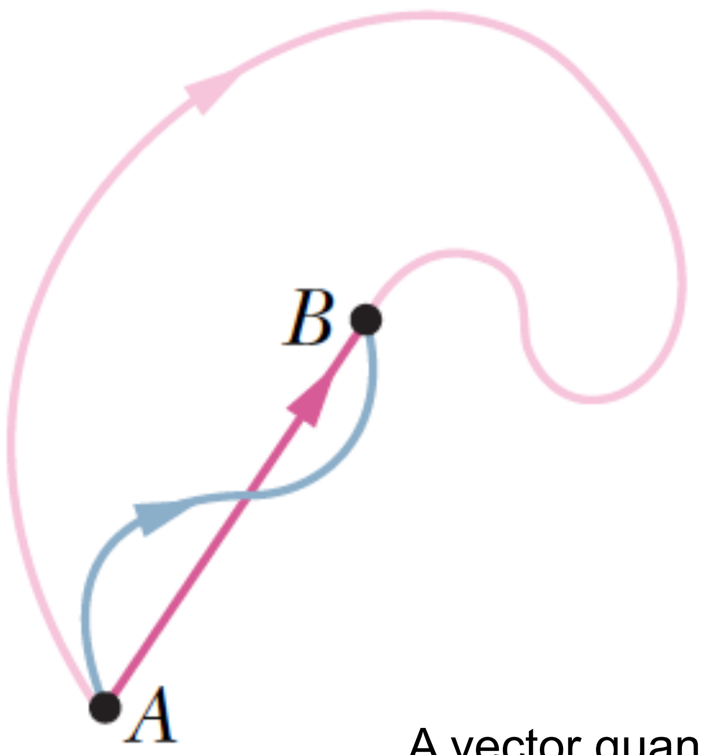
Vectorel quantities:



A vector quantity is completely specified by a number and appropriate units plus a direction.

3.1 Vectors and Scalars

Displecement as a vector quantity:



As a particle moves from A to B along an arbitrary paths represented by the blue or pink line, its displacement is a vector quantity shown by the arrow drawn from A to B.

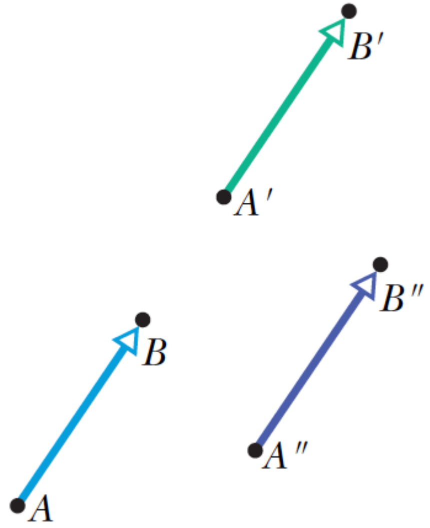
The displacement vector tells us nothing about the actual path that the particle takes.

A vector quantity can be shown as: $\vec{A}$

The magnitude of a vector: $|\vec{A}|$

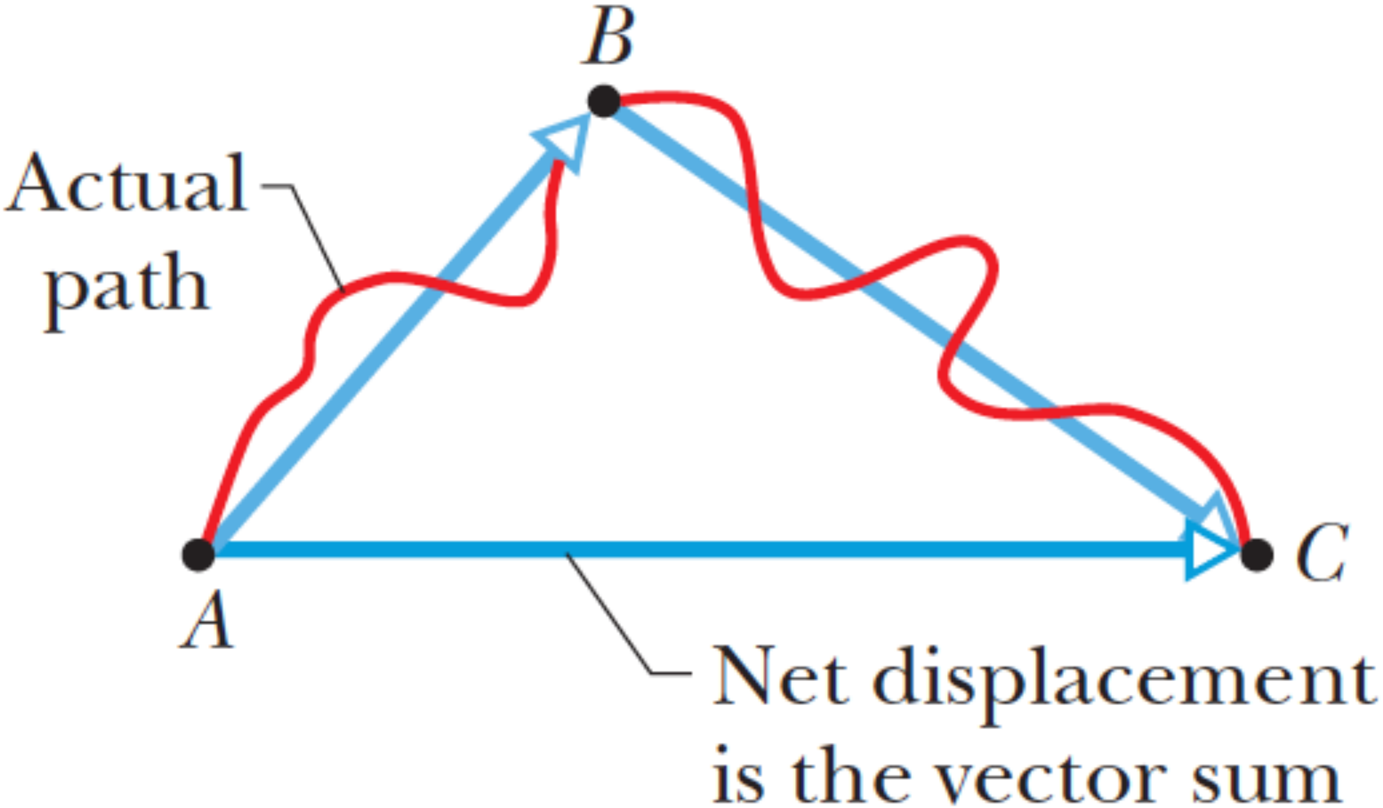
Equality of Two Vectors

Two vectors are equal only if they have the same magnitude and point in the same direction.

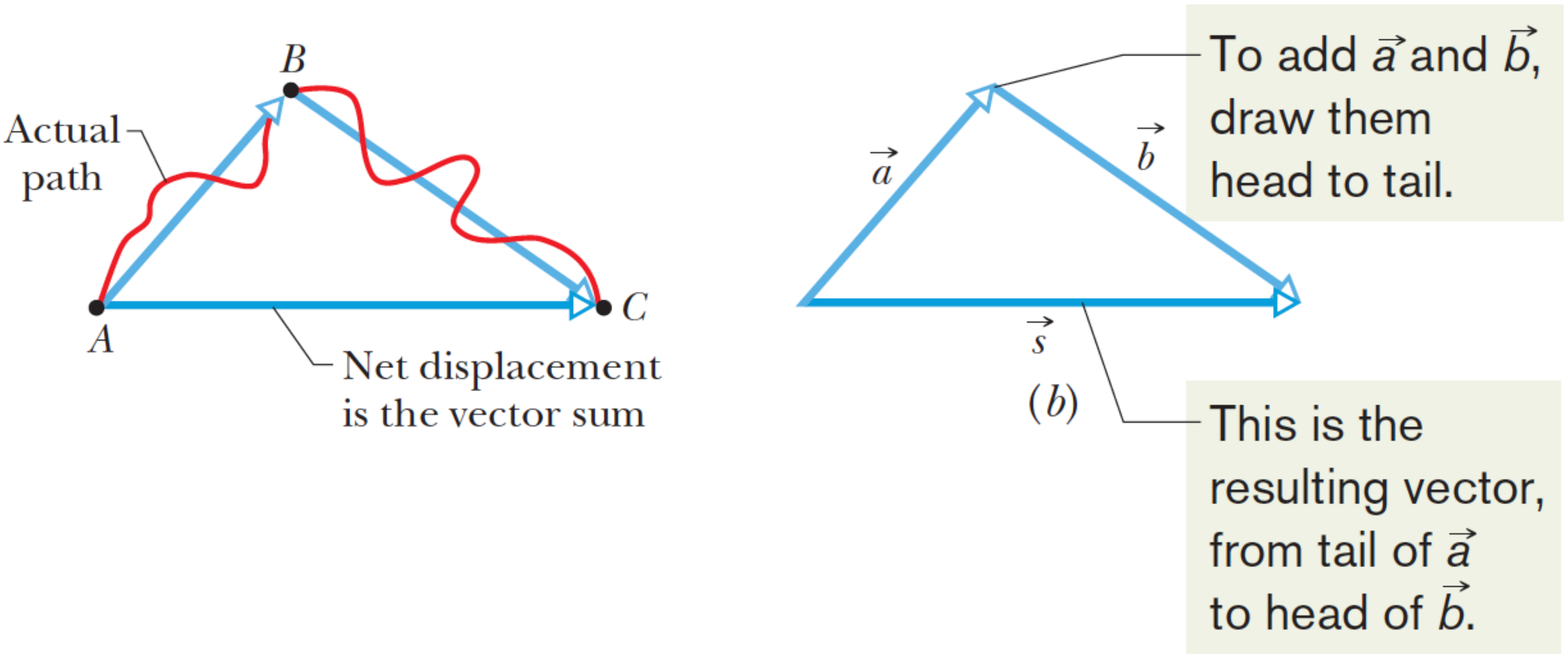


Negative of a Vector

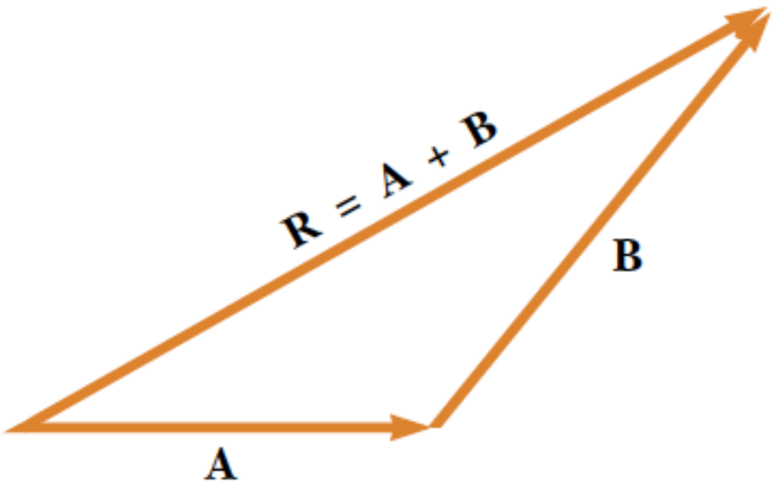
3.2. Adding Vectors Geometrically

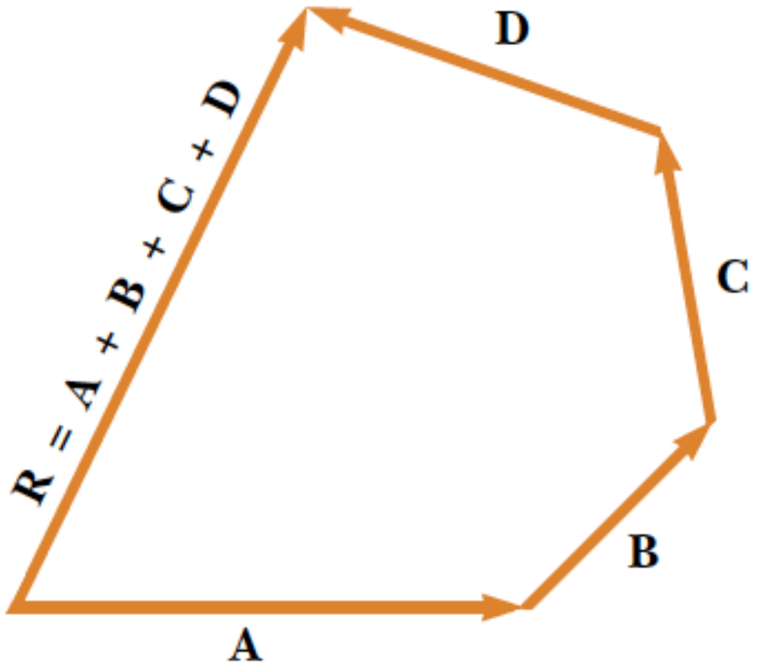


3.2. Adding Vectors Geometrically

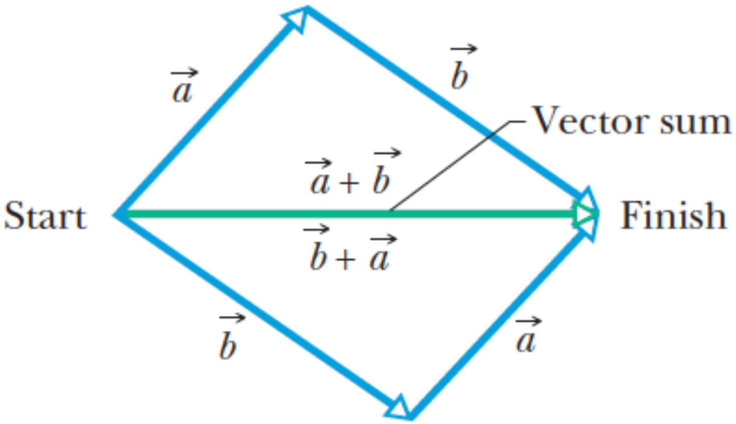


3.2. Adding Vectors Geometrically





3.2. Adding Vectors Geometrically



You get the same vector result for either order of adding vectors.

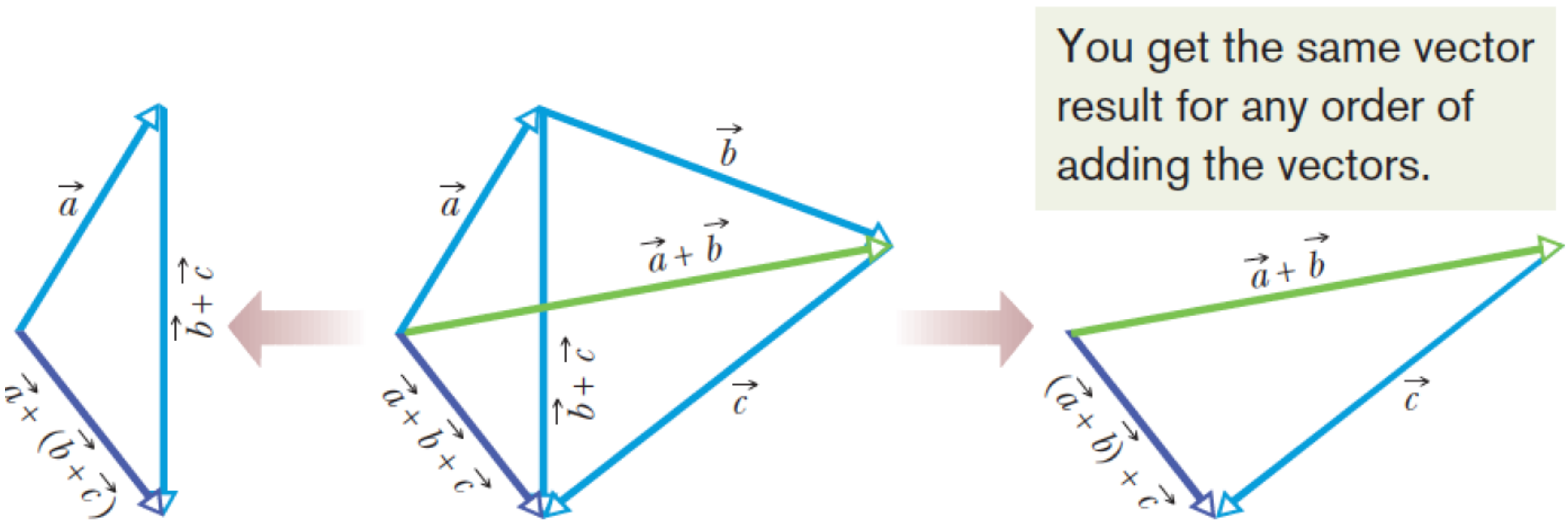
Commutative law of addition

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

3.2. Adding Vectors Geometrically

Associative law of addition:

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$



3.2. Adding Vectors Geometrically

Associative law of addition:

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$

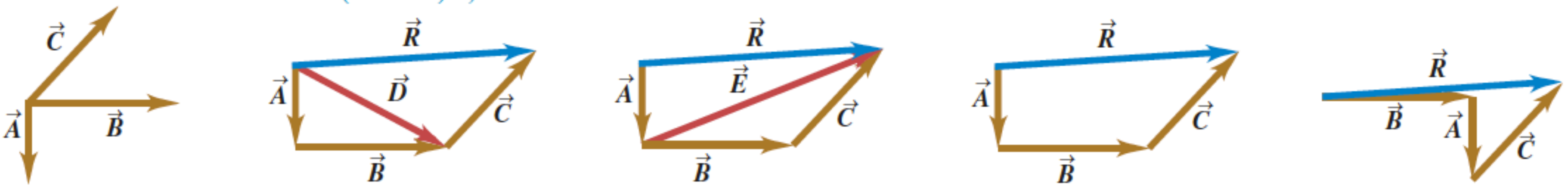
(a) To find the sum of these three vectors ...

(b) we could add $\vec{A}$ and $\vec{B}$ to get $\vec{D}$ and then add $\vec{C}$ to $\vec{D}$ to get the final sum (resultant) $\vec{R}$, ...

(c) or we could add $\vec{B}$ and $\vec{C}$ to get $\vec{E}$ and then add $\vec{A}$ to $\vec{E}$ to get $\vec{R}$, ...

(d) or we could add $\vec{A}$, $\vec{B}$, and $\vec{C}$ to get $\vec{R}$ directly, ...

(e) or we could add $\vec{A}$, $\vec{B}$, and $\vec{C}$ in any other order and still get $\vec{R}$.



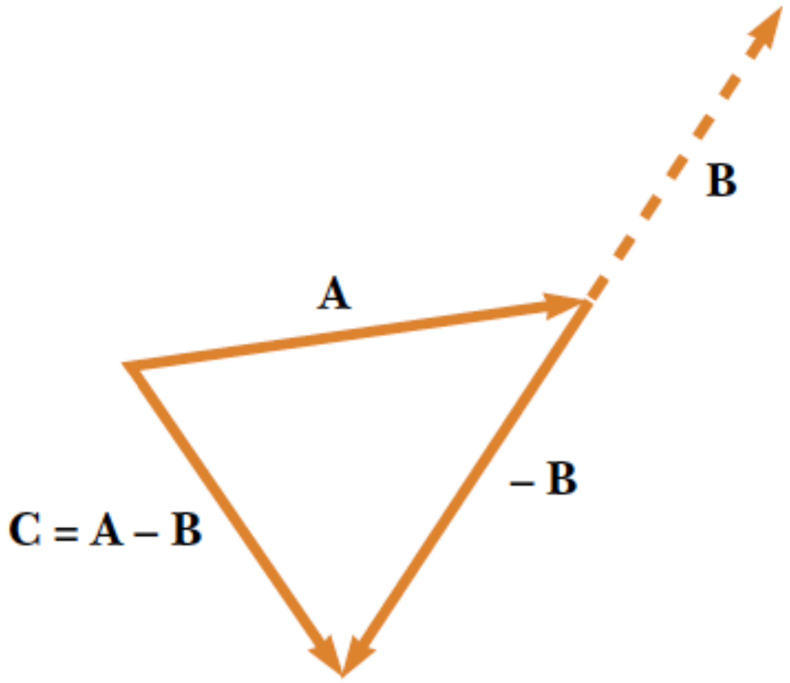
3.2. Adding Vectors Geometrically

Subtracting Vectors

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

$$\vec{A} = \vec{B}$$

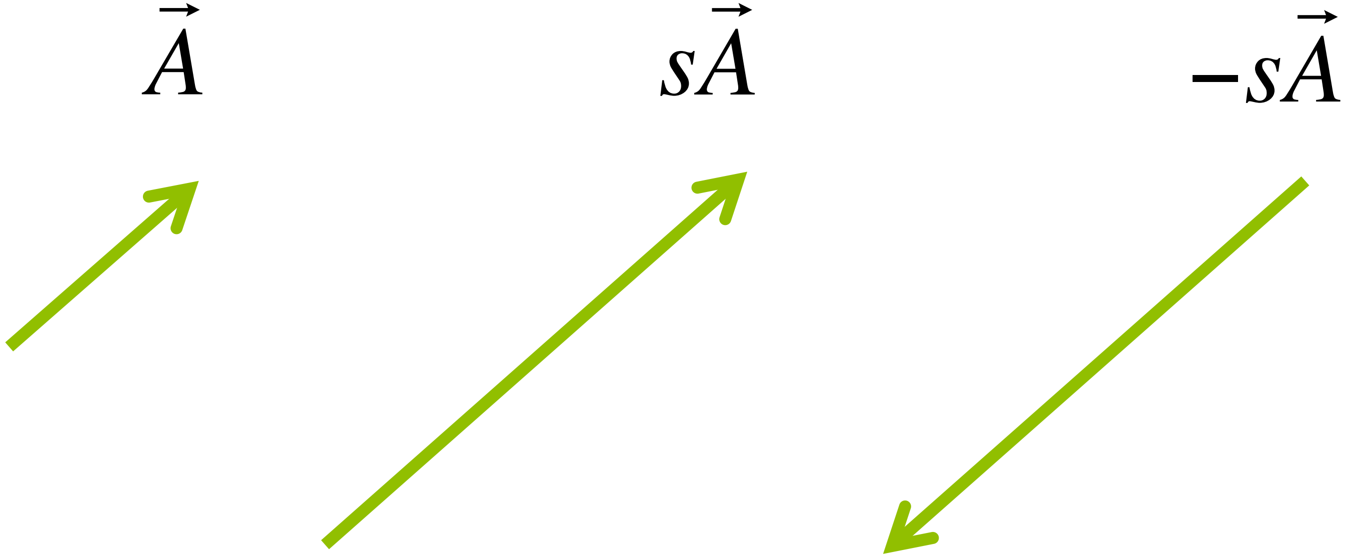
$$if\ \left|\vec{A}\right| = \left|\vec{B}\right|$$



(a)

3.2. Adding Vectors Geometrically

Multiplying a vector by a scalar



+ 3.2. Adding Vectors Geometrically

Equality

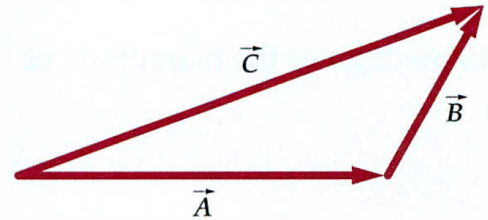
$\vec{A} = \vec{B}$

$if \ |\vec{A}| = |\vec{B}|$

and direction are same

Adding

$$\vec{C} = \vec{A} + \vec{B}$$



Negative of a vector

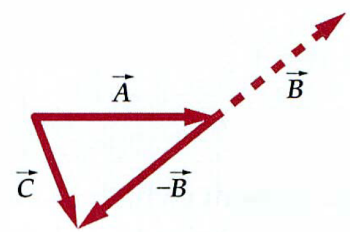
$\vec{A} = -\vec{B}$

$if \ |\vec{A}| = |\vec{B}|$ *and*

their direction are opposite

Substraction

$$\vec{C} = \vec{A} - \vec{B}$$



Multiplying a vector by a scalar

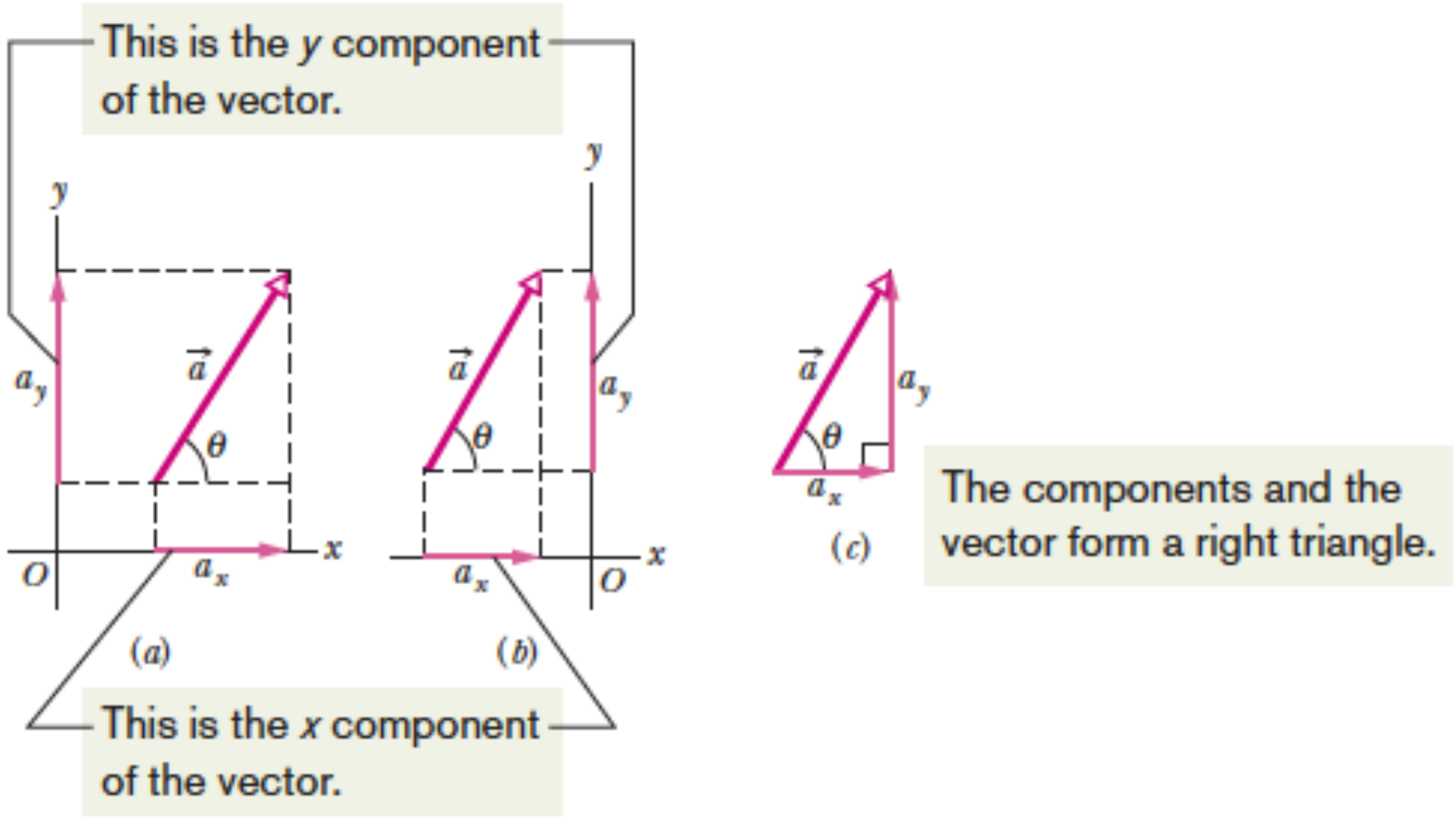
$\vec{B} = s\vec{A}$

$|\vec{B}| = s|\vec{A}|$ *and has same direction as* $\vec{A}$

if s *is positive*

or as $-\vec{A}$ *if* s *is negative*

3.3. Components of Vectors and Unit Vectors

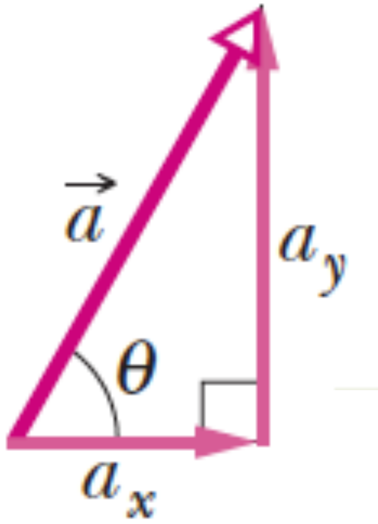


$$a_x = a\cos\theta \qquad a_y = a\sin\theta$$

3.3. Components of Vectors and Unit Vectors

$$a = \sqrt{a_x^2 + a_y^2}$$

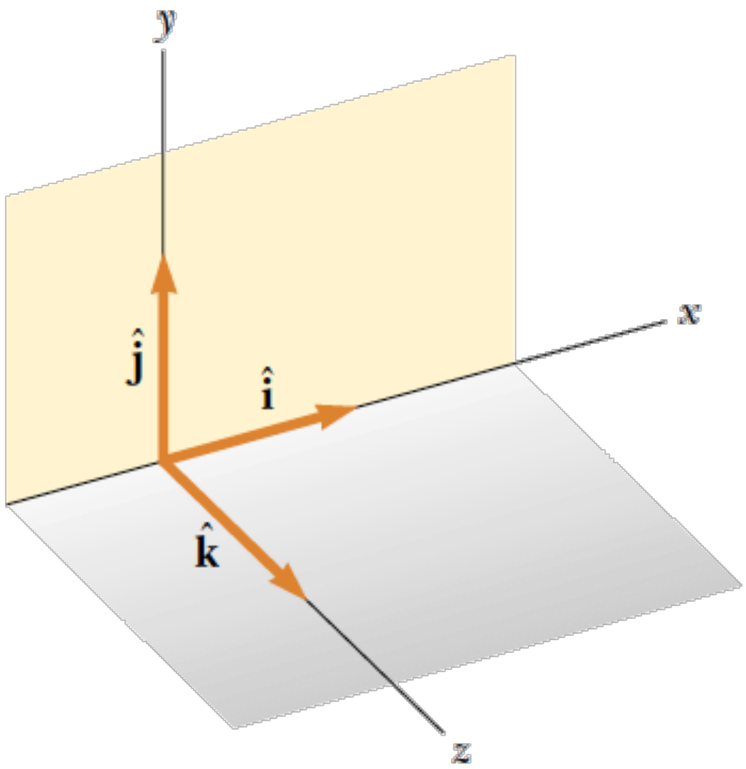
$$\tan\theta = \frac{a_x}{a_y} \qquad \theta = \tan^{-1}\left(\frac{a_x}{a_y}\right)$$



+ 3.3. Components of Vectors and Unit Vectors

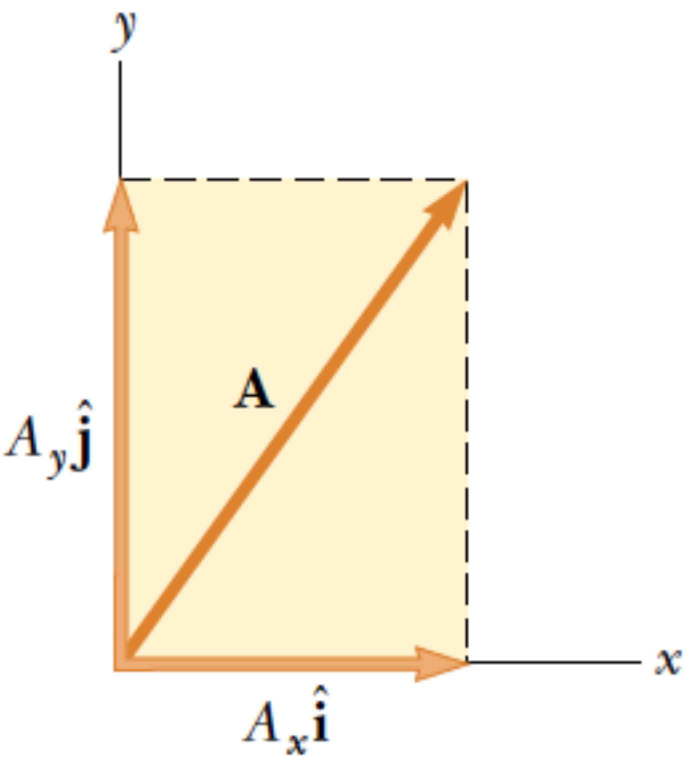
Unit vectors:

A unit vector is a dimensionless vector having a magnitude of exactly 1.



$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$

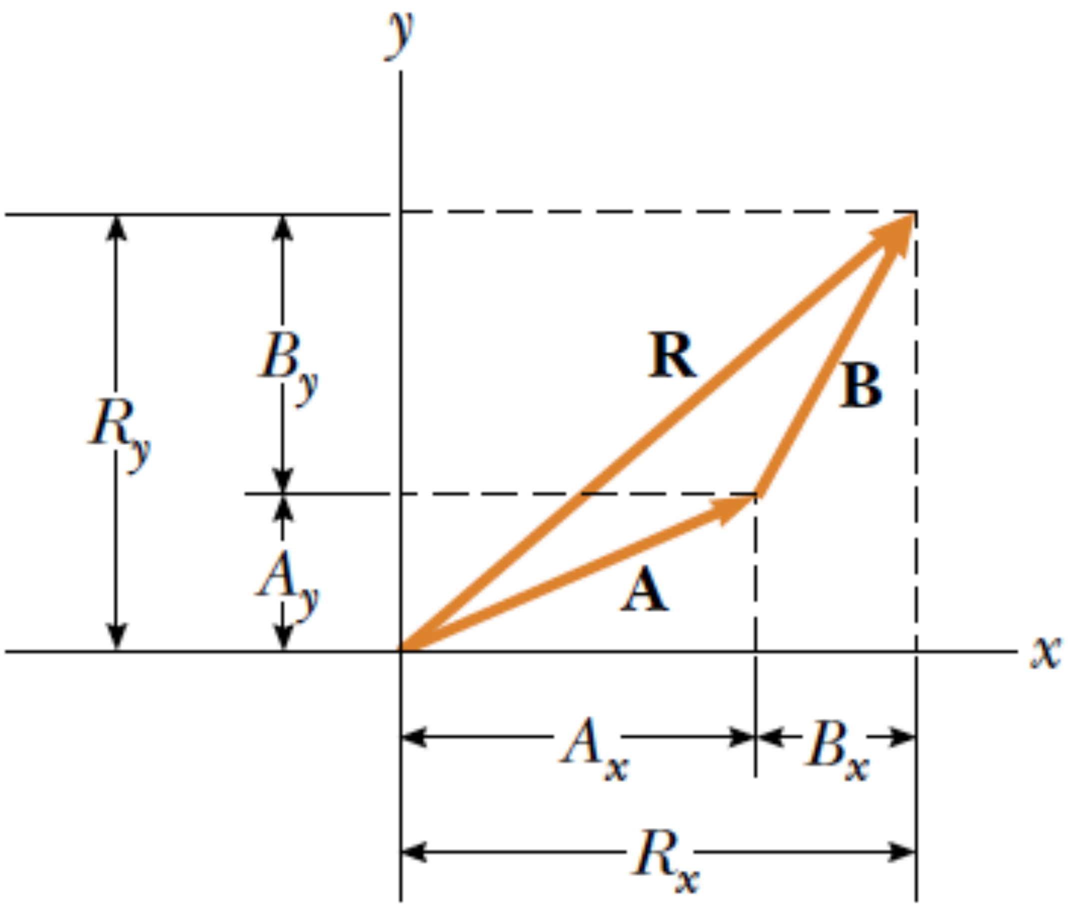
3.3. Components of Vectors and Unit Vectors



$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$

3.3. Components of Vectors and Unit Vectors

Adding Vectors by Components:



$$\vec{R} = \vec{A} + \vec{B}$$

$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

3.3. Components of Vectors and Unit Vectors

Equality

$\vec{A} = \vec{B}$

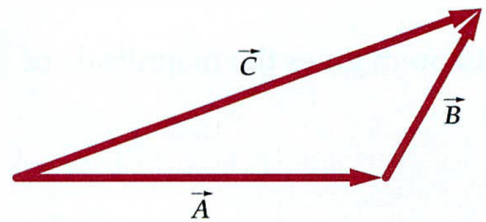
if $|\vec{A}| = |\vec{B}|$

and direction are same

$$A_x = B_x$$
$$A_y = B_y$$
$$A_z = B_z$$

Adding

$$\vec{C} = \vec{A} + \vec{B}$$



$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$
$$C_z = A_z + B_z$$

Negative of a vector

$\vec{A} = -\vec{B}$

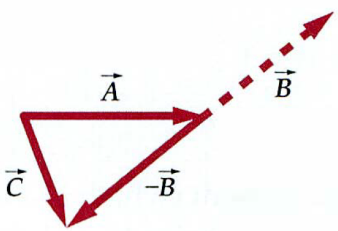
if $|\vec{A}| = |\vec{B}|$ *and*

their direction are opposite

$$A_x = -B_x$$
$$A_y = -B_y$$
$$A_z = -B_z$$

Substraction

$$\vec{C} = \vec{A} - \vec{B}$$



$$C_x = A_x - B_x$$
$$C_y = A_y - B_y$$
$$C_z = A_z - B_z$$

Multiplying a vector by a scalar

$\vec{B} = s\vec{A}$

$|\vec{B}| = s|\vec{A}|$ *and has same direction as* $\vec{A}$

if s is positive

or as $-\vec{A}$ *if s is negative*

$$B_x = sA_x$$
$$B_y = sA_y$$
$$B_z = sA_z$$

3.4. Multiplying Vectors

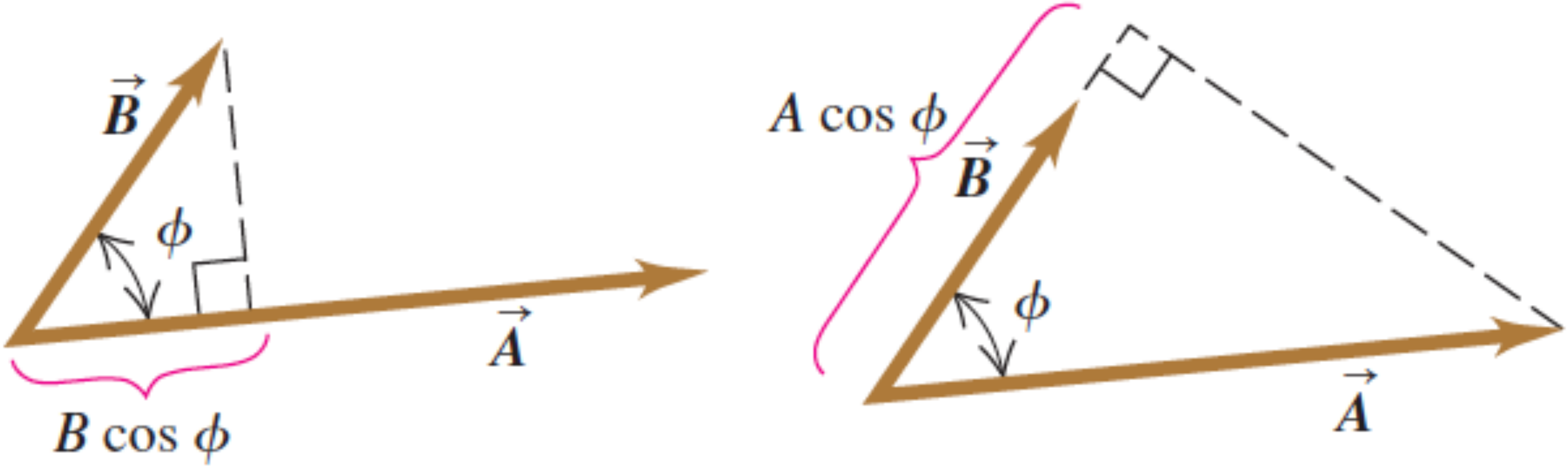
There are two ways to multiply a vector by a vector:

- Scalar product produces a scalar
- Vector product producs a new vector

3.4. Multiplying Vectors

Scalar product:

$$\vec{A} \cdot \vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi$$



$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

3.4. Multiplying Vectors

Scalar Product by Components

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = (1)(1)\cos 0° = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\imath} \cdot \hat{k} = \hat{\jmath} \cdot \hat{k} = (1)(1)\cos 90° = 0$$

$$\vec{A} \cdot \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \cdot (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

3.4. Multiplying Vectors

Vektörel Çarpım: Tanım

İki vektörün vektörel çarpımını

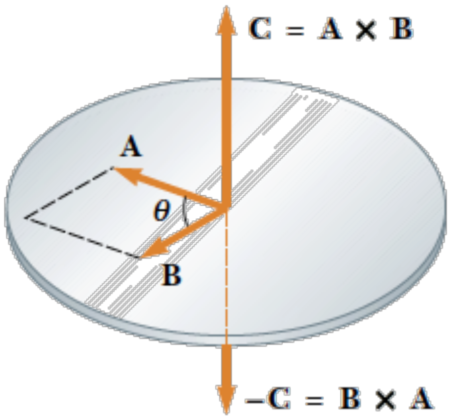
$$\vec{C} = \vec{A}x\vec{B}$$

şeklinde tanımlanır.

C vektörünün büyüklüğü

$$C \equiv AB \sin\ \theta$$

ile verilir. Oluşan **C** vektörünün yönü ise sağ el kuralı ile belirlenir.

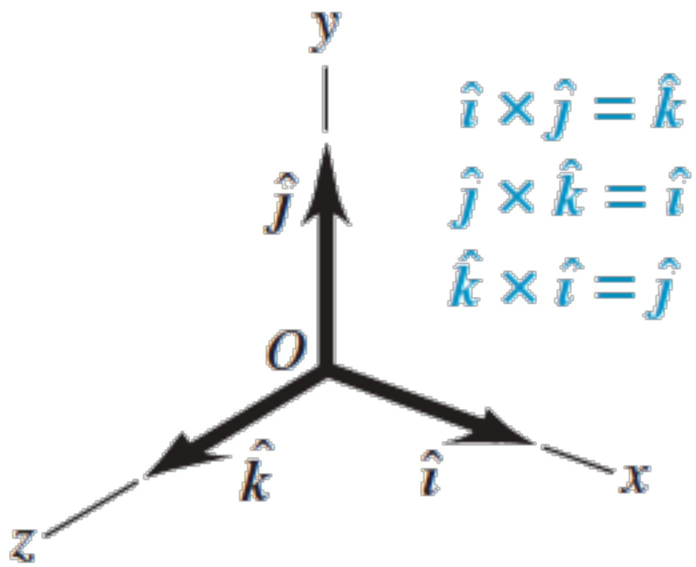


$$\vec{A}x\vec{B} = -\vec{B}x\vec{A}$$

3.4. Multiplying Vectors

Vektörel Çarpım: Bileşenlerin Kullanılması

İki vektörün vektörel çarpımını bileşenlerini kullanarak hesap edelim. Birim vektörlerin vektörel çarpımı vektörel çarpımın kullanılması tanımlanabilir.



$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

Vektörel Çarpım: Bileşenlerin Kullanılması

Bileşenlerini bildiğimiz ii vektörü vektörel çarpalım:

$$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

$$\vec{C} = \vec{A} \times \vec{B}$$

vektörünün bileşenleri böylece

$$C_x = A_yB_z - A_zB_y \qquad C_y = A_zB_x - A_xB_z \qquad C_z = A_xB_y - A_yB_x$$

olur.

3.4. Multiplying Vectors

Vektörel Çarpım: Bileşenlerin Kullanılması

İki vektörün vektörel çarpımı determinant kullanılarak hesaplanabilir:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{\mathbf{k}}$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\,\hat{\mathbf{i}} - (A_xB_z - A_zB_x)\,\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\,\hat{\mathbf{k}}$$

Örnek:1

Aşağıda verilen iki vektörün vektörel çarpımını hesap ediniz.

$$\vec{A} = 3\hat{i} - 4\hat{j} \; ; \; \vec{B} = -2\hat{i} + 3\hat{k}$$

Çözüm:1 (1.yöntem)

$$\vec{C} = \vec{A} x \vec{B}$$

$$\vec{C} = (3\hat{i} - 4\hat{j}) x (-2\hat{i} + 3\hat{k})$$

$$\vec{C} = 3\hat{i} x (-2\hat{i}) + 3\hat{i} x 3\hat{k} + (-4\hat{j}) x (-2\hat{j}) + (-4\hat{j}) x 3\hat{k}$$

$$\vec{C} = 6.0 + 9(-\hat{j}) + 8(-\hat{k}) - 12\hat{i}$$

$$\vec{C} = -12\hat{i} - 9\hat{j} - 8\hat{k}$$

3.4. Multiplying Vectors

Örnek:1

Aşağıda verilen iki vektörün vektörel çarpımını hesap ediniz.

$$\vec{A} = 3\hat{i} - 4\hat{j}\ ;\ \vec{B} = -2\hat{i} + 3\hat{k}$$

Çözüm:1 (2.yöntem)

$$\vec{C} = \vec{A}x\vec{B} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 0 \\ -2 & 0 & 3 \end{bmatrix} = (-4.3 - 0.0)\hat{i} - (0.0 - 3.3)\hat{j} + (3.0 - (-4).(-2))\hat{k}$$

$$\vec{C} = \vec{A}x\vec{B} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 0 \\ -2 & 0 & 3 \end{bmatrix} = -12\hat{i} - 9\hat{j} - 8\hat{k}$$

3.4. Multiplying Vectors

Örnek:2

Bir önceki örnekte vektörel çarpım sonucunda elde edilen **C** vektörünün hem **A** hem de **B** vektörüne dik olduğunu gösteriniz.

Çözüm:2

$$\vec{A} = 3\hat{i} - 4\hat{j} \; ; \; \vec{B} = -2\hat{i} + 3\hat{k} \qquad \vec{C} = -12\hat{i} - 9\hat{j} - 8\hat{k}$$

C vektörü **A**' ya dik ise **C.A** çarpımının sıfır olması gerekir:

$$\vec{C}.\vec{A} = (-12\hat{i} - 9\hat{j} - 8\hat{k}).(3\hat{i} - 4\hat{j})$$

$$\vec{C}.\vec{A} = (-12.3)\hat{i}.\hat{i} + (-9)(-4)\hat{j}.\hat{j} = 0$$

Benzer hesap **B** vektörü içinde yapılabilir.