PHY121 Physics I

Chapter 4 Motion in Two (Three) Dimensions

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Chapter 4 Motion in Two (Three) Dimensions

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4.2. Average and Instantaneous Velocity
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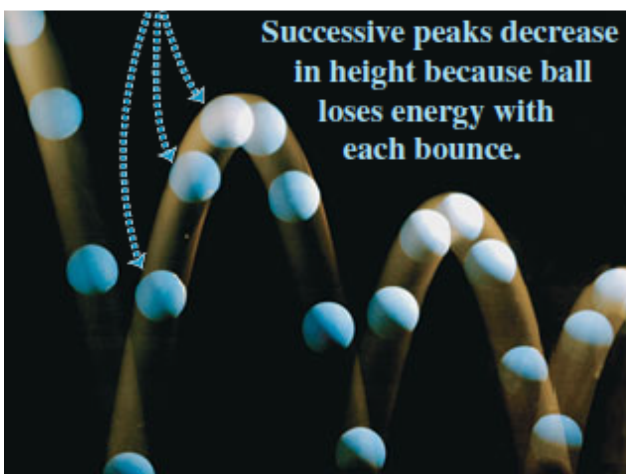
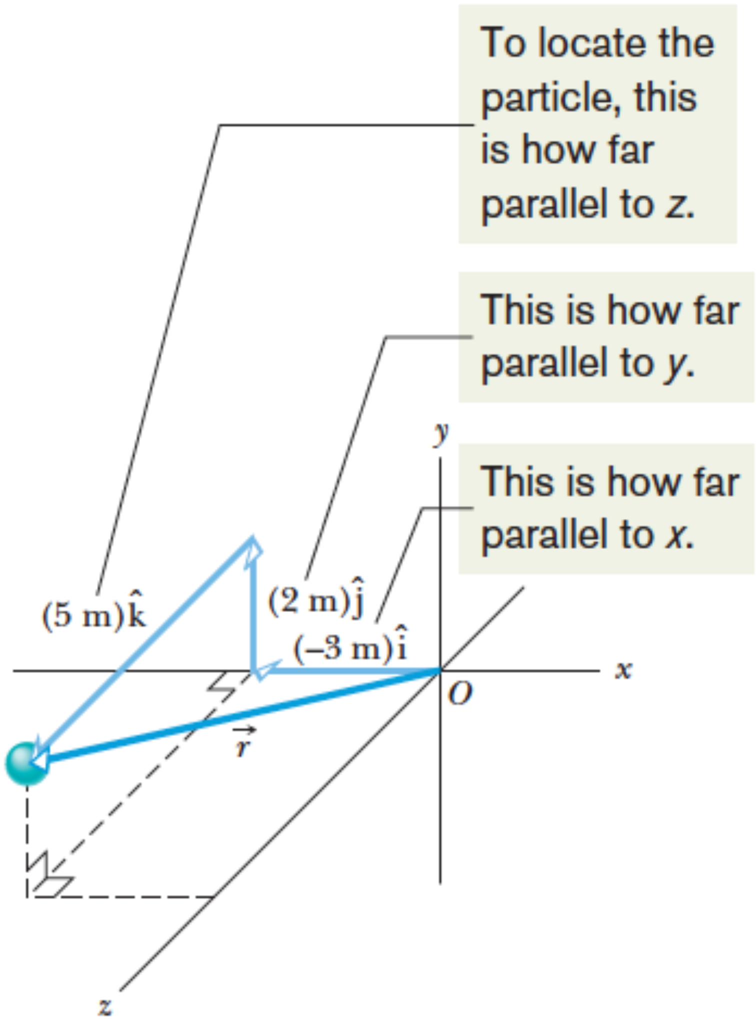


4.1.Position and Displacement



Position Vector:

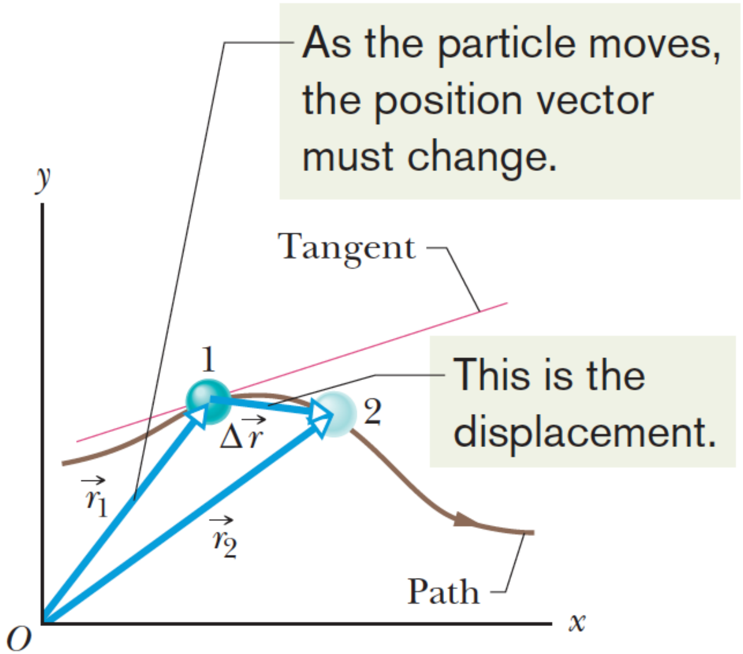
- The position of a particle is given by a position vector **r**:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

4.1.Position and Displacement

Yerdeğiştirme Vektörü

Eğer cismin konumu zaman içinde ve $\mathbf{r}_1$'den $\mathbf{r}_2$'ye değişiyorsa bu aralıkta cismin yerdeğiştirmesi son konum vektörü ile ilk konum vektörü arasındaki vektörel fark olacaktır:



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

4.1.Position and Displacement

Example 4.1

Sample problem in HR page 63.

4.1.Position and Displacement

Example 4.1

Bir tavşan park alanında koşturuyor. Tavşanın konumunun t zamanının fonksiyonu olarak koordinatları şöyle veriliyor:

$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30$$

Buna göre tavşanın t=15 sn'de konum vektörünü yazınız. Vektörün büyüklüğü ve x-ekseni ile yaptığı açıyı hesap ediniz.

Bölüm 4.1.Konum ve Yerdeğiştime

Çözüm:1

Tavşanın konum vektörü en genel haliyle:

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

t=15 sn için x ve y bileşenlerini hesap edersek:

$$x = -0.31.(15)^2 + (7.2).(15) + 28 = 66m$$

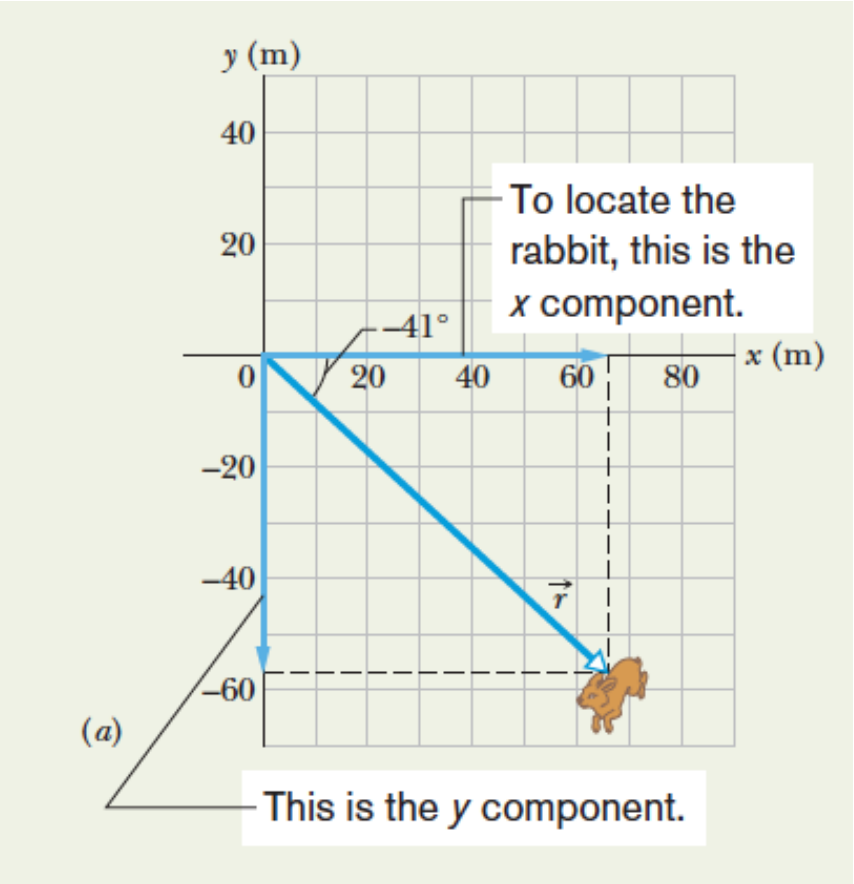
$$y = 0.22(15)^2 - (9.1).(15) + 30 = -57m$$

Buna göre konum vektörü:

$$\vec{r} = 66\hat{i} - 57\hat{j}$$

4.1.Position and Displacement

Çözüm:1



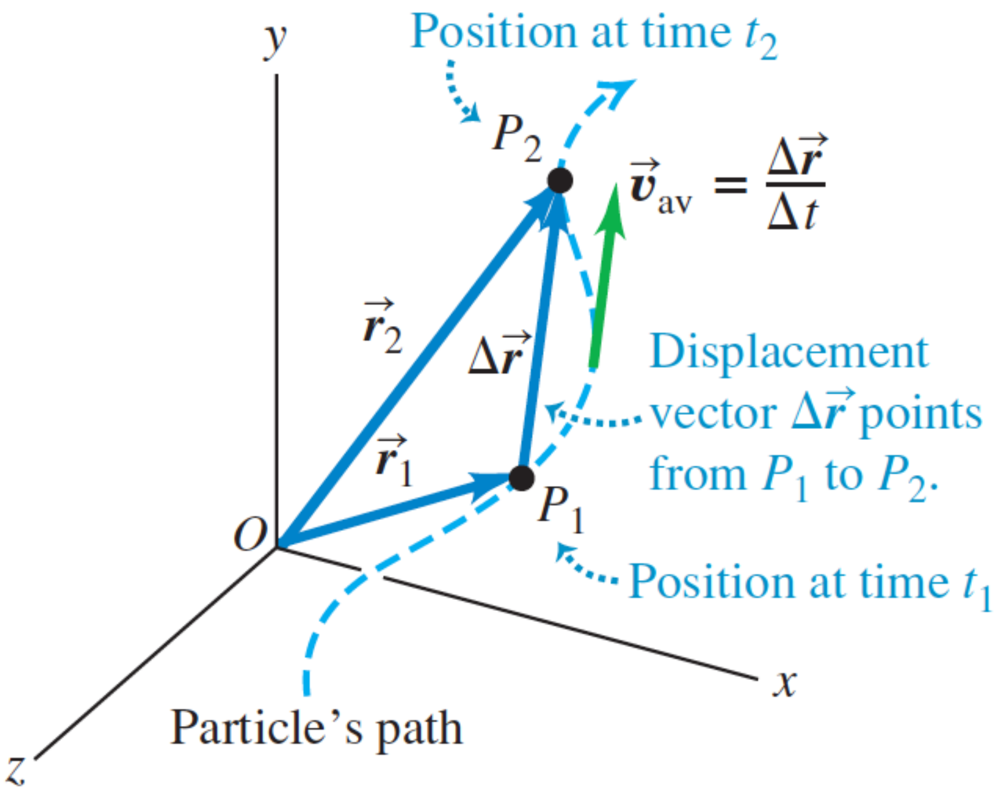
Konum vektörünün büyüklüğü ve açısını hesap edelim:

$$|\vec{r}| = \sqrt{x^2 + y^2} = \sqrt{66^2 + (-57)^2} = 87m$$

$$\tan\theta = \frac{y}{x} = (\frac{-57}{66}) \Rightarrow \theta = -41^0$$

4.2. Average and Instantaneous Velocity

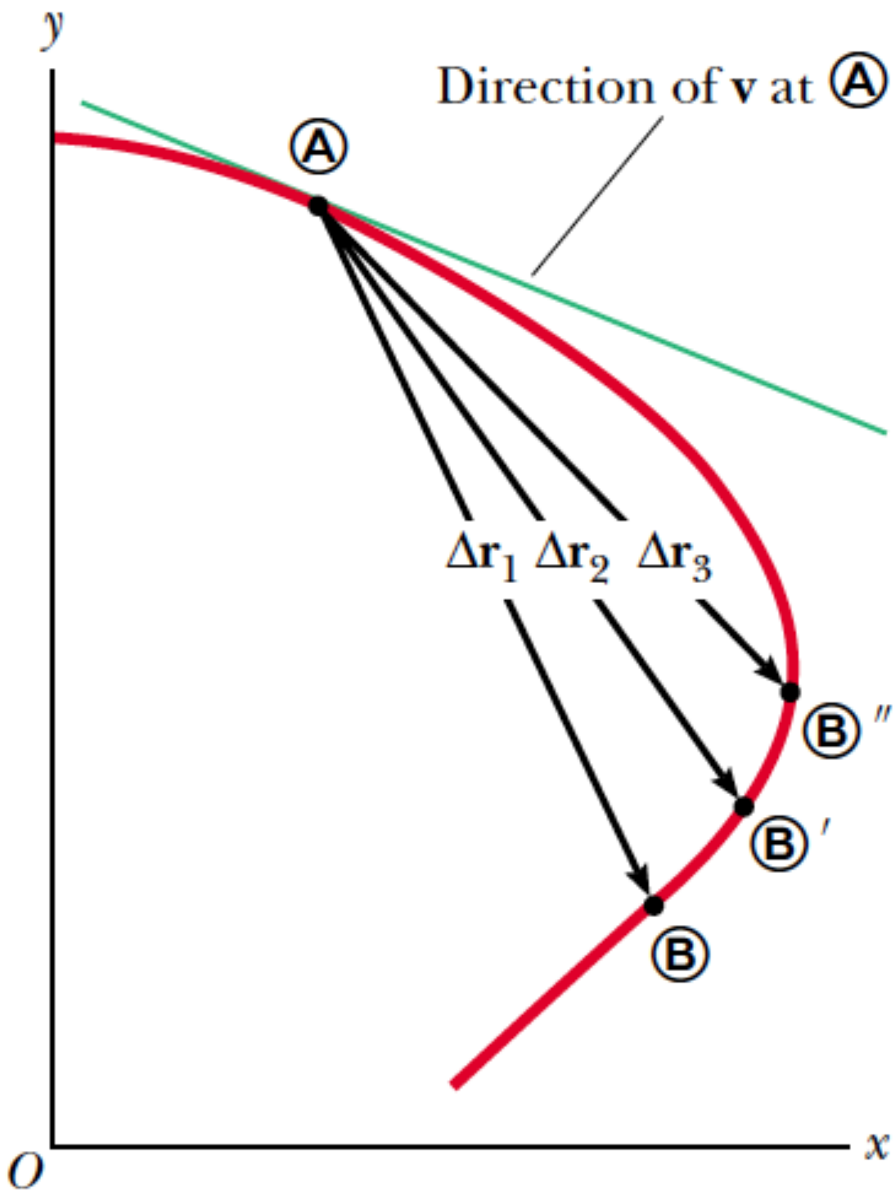
Average Velocity Vector:



If a particle moves through a displacement Δ**r** in a time interval Δt, the average velocity:

$$\vec{v}_{avg.} = \frac{\Delta\vec{r}}{\Delta t}$$

+ 4.2. Average and Instantaneous Velocity



Instantaneous Velocity Vector:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

The direction of the instantaneous velocity **V** of a particle is always tangent to the particle's path at the particle's position.

4.2. Average and Instantaneous Velocity

Instantaneous Velocity Vector:

the components of instantenous vector in three dimensions, :

$$\vec{v} = \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$v_x = \frac{dx}{dt}; v_y = \frac{dy}{dt}; v_z = \frac{dz}{dt}$$

Anlık hız vektörünün büyüklüğü de sürati verir:

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

4.2. Average and Instantaneous Velocity

The magnitute of instantaneous velocity vector: SPEED

The magnitude of the instantaneous velocity vector **v** is called the speed, which is a scalar quantity.

$$\left|\vec{v}\right| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

4.2. Average and Instantaneous Velocity

Example 4.2

Sample problem in HR page 63.

4.2. Average and Instantaneous Velocity

Example 4.2

Bir önceki örnekteki tavşanın t=15 sn'deki anlık hızını bulunuz

Çözüm:2

Bir önceki örnekte tavşanın konum vektörü

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

ve bileşenleri

$$x = -0.31.(15)^2 + (7.2).(15) + 28 = 66m$$

$$y = 0.22(15)^2 - (9.1).(15) + 30 = -57m$$

olarak verilmişti.

4.2. Average and Instantaneous Velocity

Çözüm:2

Anlık hız vektörünü konum vektörünün zamana göre birinci türevini alarak bulabiliriz:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28) = -0.62t + 7.2$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.11t + 30) = 0.44t - 9.1$$

Buradan t=15 sn için anlık hız vektörünün bileşenleri

$$v_x = -(0.62).15 + 7.2 = -2.1 m/s$$

$$v_y = (0.44).15 - 9.1 = -2.5 m/s$$

4.2. Average and Instantaneous Velocity

Çözüm:2

Buna göre anlık hız vektörü:

$$\vec{v} = -2.1\hat{i} - 2.5\hat{j}$$

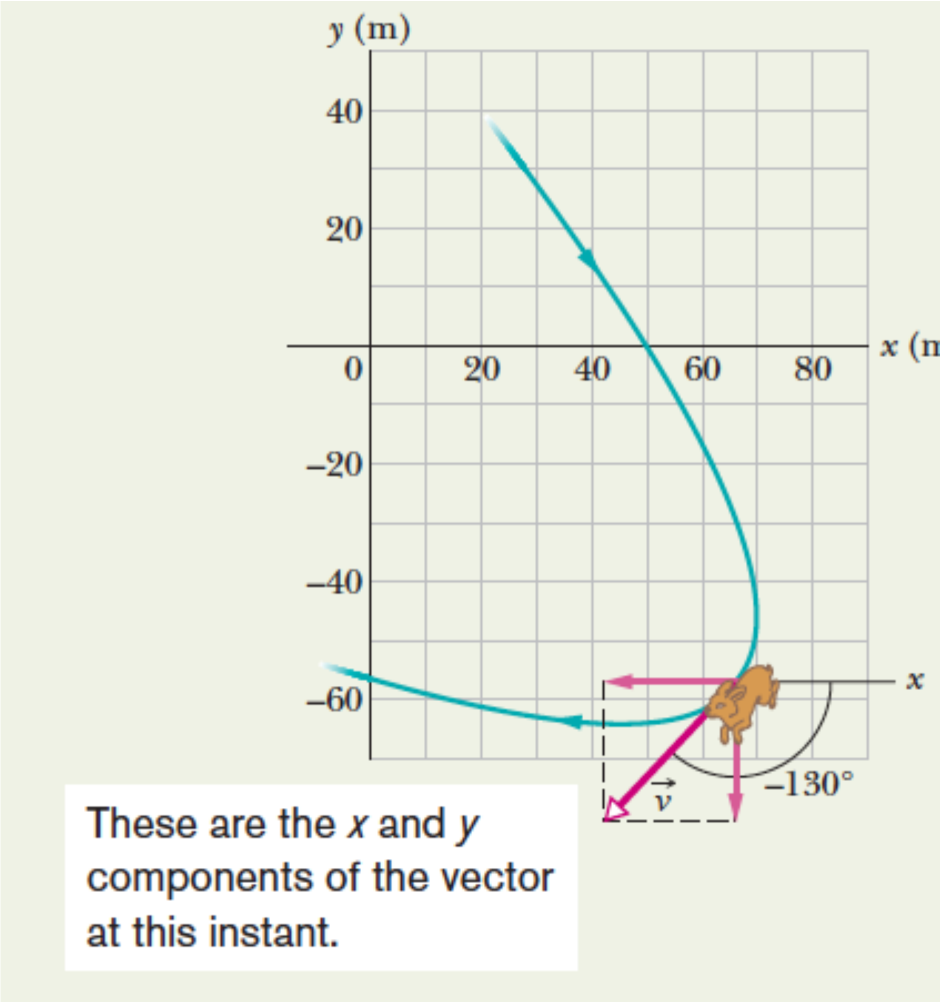
Anlık hız vektörünün büyüklüğü ve açısı

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2} = 3.3 m/s$$

$$\tan\theta = \frac{v_y}{v_x} = \left(\frac{-2.5}{-2.1}\right) \Rightarrow \theta = -130^0$$

4.2. Average and Instantaneous Velocity

Çözüm:2



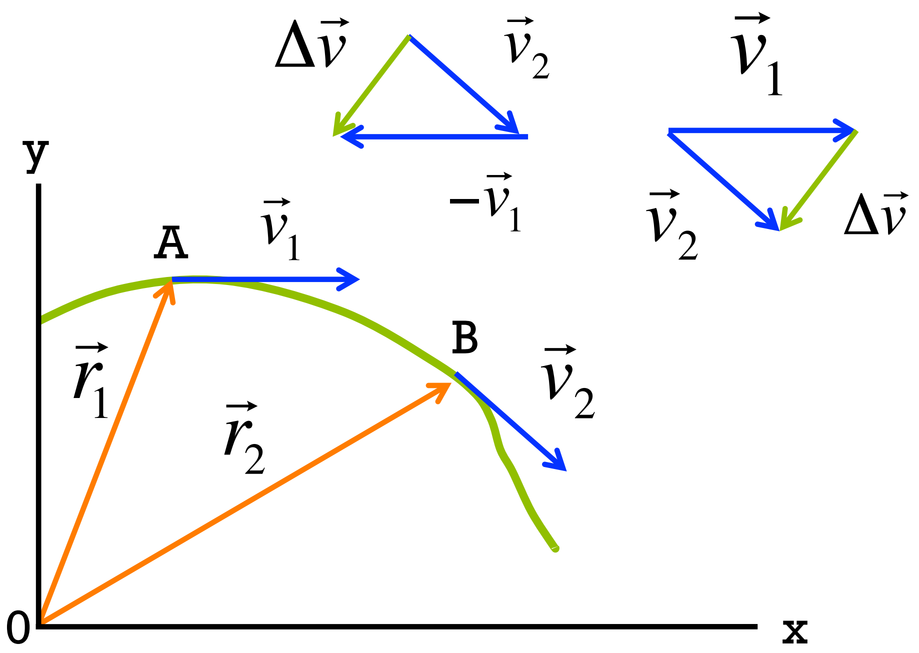
$$\vec{v} = -2.1\hat{i} - 2.5\hat{j}$$

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2} = 3.3 m/s$$

$$\tan\theta = \frac{v_y}{v_x} = \left(\frac{-2.5}{-2.1}\right) \Rightarrow \theta = -130^0$$

4.3. Average and Instantaneous Acceleration

Average accelaration vector:



$$\vec{a}_{avg.} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

4.3. Average and Instantaneous Acceleration

Instantaneous accelaration vector:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

The components of instantaneous acceleration are:

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$a_x = \frac{dv_x}{dt}; a_y = \frac{dv_y}{dt}; a_z = \frac{dv_z}{dt}$$

4.3. Average and Instantaneous Acceleration

Example 4.3.

A car is traveling east at 60 km/h. It rounds a curve, and 5 s later it is traveling north at 60 km/h. Find the average acceleration of the car

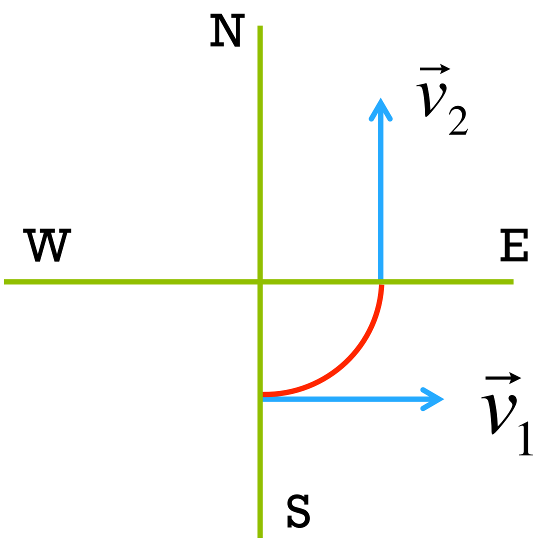
4.3. Average and Instantaneous Acceleration

Example 4.3.

A car is traveling east at 60 km/h. It rounds a curve, and 5 s later it is traveling north at 60 km/h. Find the average acceleration of the car

Answer 4.3

Average acelaration is not zero! WHY ?



$$\vec{v}_1 = 60\hat{i} \qquad \vec{v}_2 = 60\hat{j}$$

$$\vec{a}_{avg.} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{60\hat{j} - 60\hat{i}}{5}$$

$$\vec{a}_{avg.} = -12\hat{i} + 12\hat{j}$$

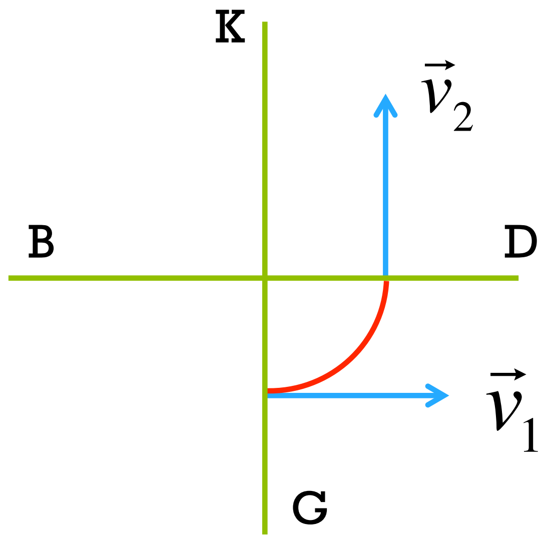
4.3. Average and Instantaneous Acceleration

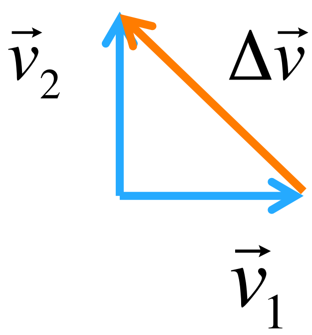
Example 4.3.

A car is traveling east at 60 km/h. It rounds a curve, and 5 s later it is traveling north at 60 km/h. Find the average acceleration of the car

Answer 4.3

The direction of acceleration vector is in the same direction as Δ**v**





4.2. Average and Instantaneous Velocity

Homework 4.1

A robotic vehicle is exploring the surface of Mars. The stationary Mars lander is the origin of coordinates, and the surrounding Martian surface lies in the xy-plane. The rover, which we represent as a point, has x- and y-coordinates that vary with time:

$$x = 2 - 0.25t^2; y = t + 0.025t^3$$

a) Find the rover's coordinates and distance from the lander at t=2.0 s.
b) Find the rover's displacement and average velocity vectors for the interval t=0.0 ve t=2.0 s.
c) Find a general expression for the rover's instantaneous velocity vector
d) Find the instantaneous velocity of the robot at t=2.0 s in component form and in terms of magnitude and direction
e) Find the components of the average acceleration for the interval to t=0 to t=2.0 s
f) Find the instantaneous acceleration at t=2.0 s.

4.2. Average and Instantaneous Velocity

Çözüm:3a

t=2.0 sn'deki koordinatlar

$$x = 2 - (0.25)(2)^2 = 1.0m$$

$$y = 2 + (0.025).(2)^3 = 2.2m$$

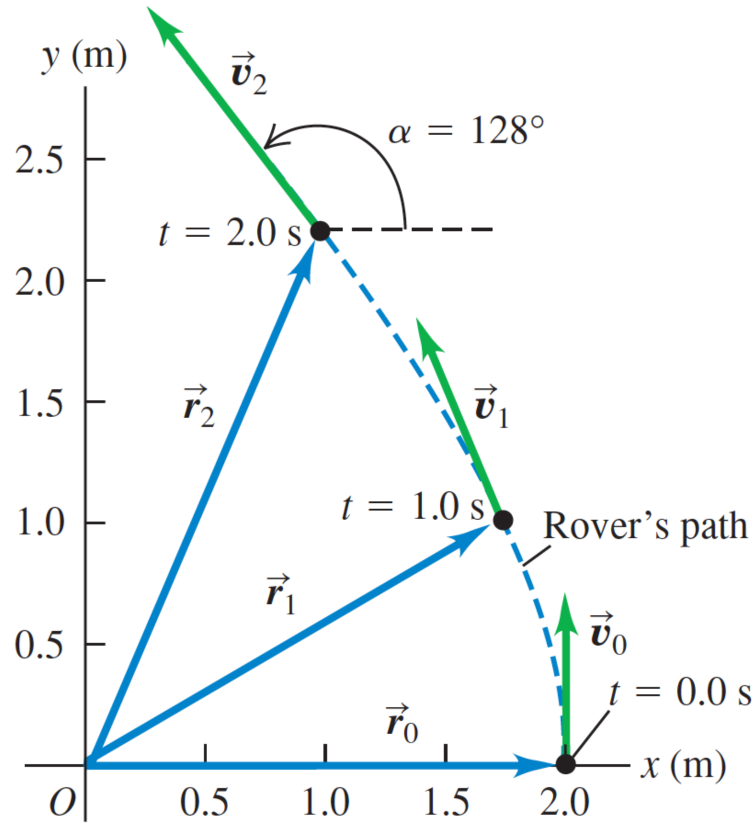
r₂ konum vektörü

$$\vec{r}_2 = 1.0\hat{i} + 2.2\hat{j}$$

Robot bu anda orijinden

$$|\vec{r}| = \sqrt{x^2 + y^2} = \sqrt{1^2 + (2.2)^2} = 2.4m$$

kadar uzakta bulunmakta.



4.2. Average and Instantaneous Velocity

Çözüm:3b

t=0.0 sn'deki koordinatlar

$$x = 2 - (0.25)(0)^2 = 2.0m$$

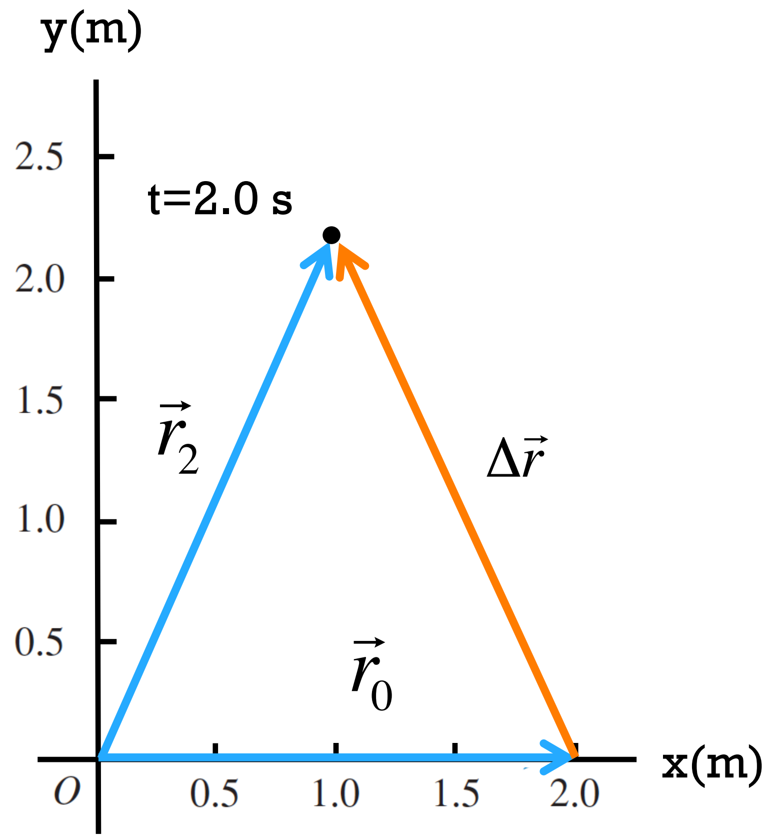
$$y = 0 + (0.0025).(0)^3 = 0m$$

Buna göre **r$_o$** konum vektörü:

$$\vec{r}_0 = 2.0\hat{i}$$

r$_2$ konum vektörü ise

$$\vec{r}_2 = 1.0\hat{i} + 2.2\hat{j}$$



Buna göre t=0.0 s ve t=2.0 sn aralığındaki yerdeğiştirme

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_0 = -1.0\hat{i} + 2.2\hat{j}$$

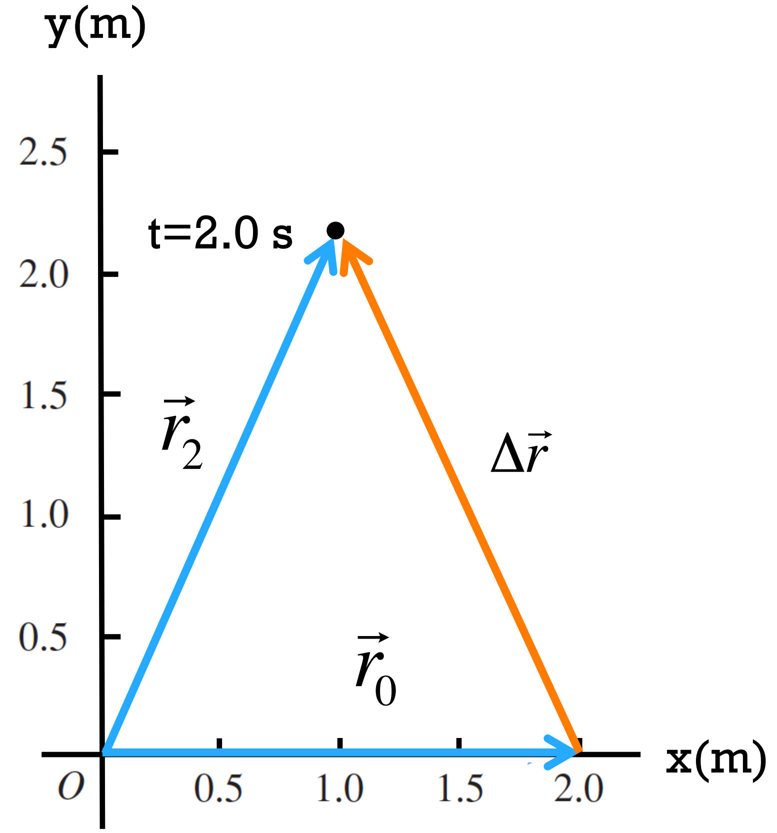
4.2. Average and Instantaneous Velocity

Çözüm:3b

Ortalama hız ise

$$\vec{v}_{ort} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v}_{ort} = \frac{-1.0\hat{i} + 2.2\hat{j}}{2.0 - 0.0} = -0.5\hat{i} + 1.1\hat{j}$$



4.2. Average and Instantaneous Velocity

Çözüm:3c

Anlık hızı konum vektörünün zamana göre türevini alarak bulabiliriz:

$$v_x = \frac{dx}{dt} = (-0.25).(2t)$$

$$v_x = \frac{dy}{dt} = 1.0 + (0.025)(3t^2)$$

Anlık hız vektörü:

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{v} = (-0.25).(2t)\hat{i} + \left[1.0 + (0.025)(3t^2)\right]\hat{j}$$

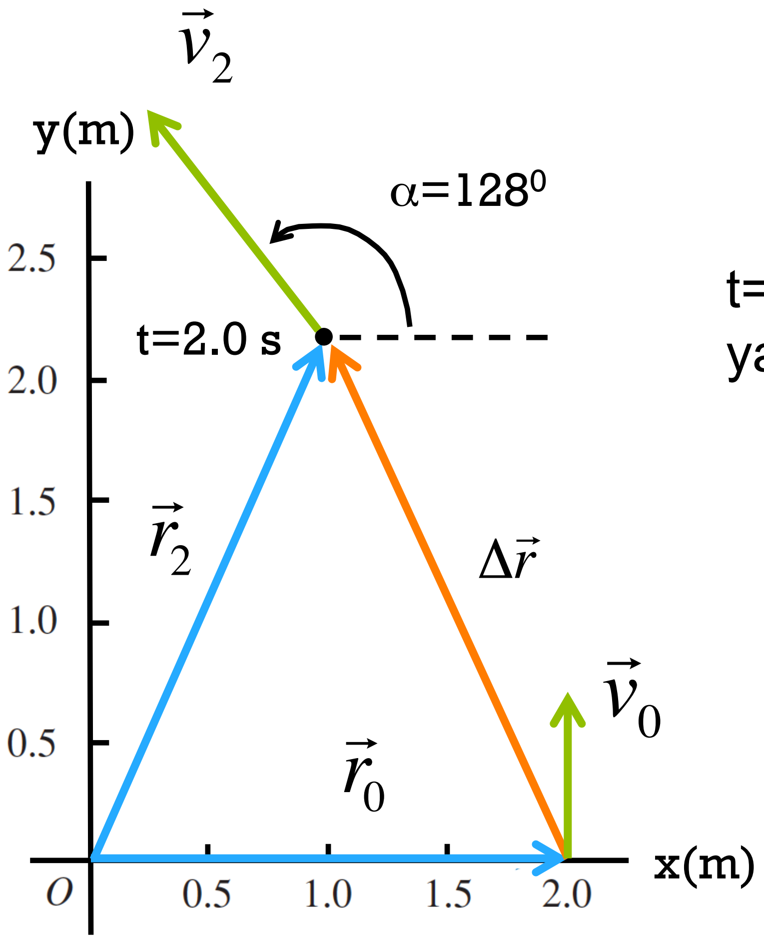
t=2.0 için anlık hız böylece

$$\vec{v} = -1.0\hat{i} + 1.3\hat{j}$$

bulunur.

4.2. Average and Instantaneous Velocity

Çözüm:3c



t=2.0 sn'deki anlık hızın x ekseni ile yaptığı açı

$$\tan\theta = \frac{v_y}{v_x} = \left(\frac{1.3}{(-1.0)}\right) \Rightarrow \theta = 128^0$$

4.4. Two-Dimensional Motion with Constant Accel.

The position vector for a particle moving in the xy plane:

$$\vec{r} = x\hat{i} + y\hat{j}$$

If the position vector is known, the velocity of the particle:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

The acceleration vector is given by:

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

The accelaration is constant, which means, that x and y component of it are constant !

4.4. Two-Dimensional Motion with Constant Accel.

The final velocity vector at any time t,.

$$\vec{v}_f = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j}$$

$$\vec{v}_f = (v_{xi}\hat{i} + v_{yi}\hat{j}) + (a_x\hat{i} + a_y\hat{j})$$

$$\boxed{\vec{v}_f = \vec{v}_i + \vec{a}t}$$

This result states that the velocity of a particle at some time t equals **the vector sum** of its initial velocity $\mathbf{v}_i$ and the additional velocity **a**t acquired at time t as a result of constant acceleration.

4.4. Two-Dimensional Motion with Constant Accel.

x and y coordinates of the particle moving with constant acceleration :

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Position vector in two dimension:

$$\vec{r}_f = x_f\hat{i} + y_f\hat{j}$$

$$\vec{r}_f = (x_i + v_{xi}t + \frac{1}{2}a_x t^2)\hat{i} + (y_i + v_{yi}t + \frac{1}{2}a_y t^2)\hat{j}$$

$$\vec{r}_f = (x_i\hat{i} + y_i\hat{j}) + (v_{xi}\hat{i} + v_{yi}\hat{j})t + \frac{1}{2}(a_x\hat{i} + a_y\hat{j})t^2$$

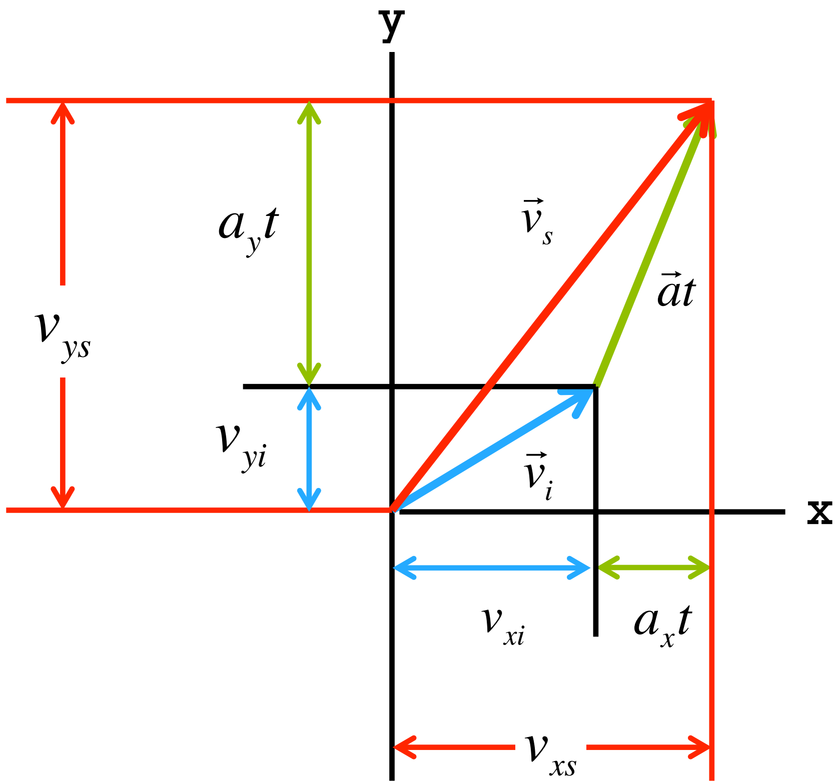
$$\boxed{\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2}$$

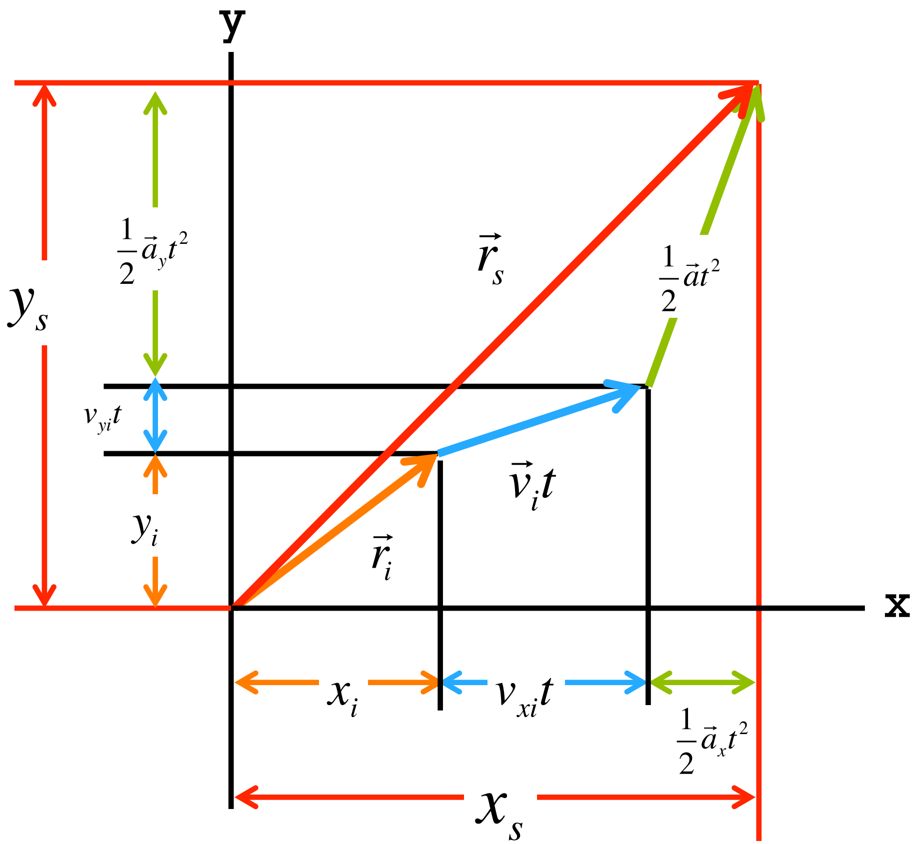
the position vector $\mathbf{r}_f$ is the vector sum of the original position $\mathbf{r}_i$, a displacement $\mathbf{v}_i t$ arising from the initial velocity of the particle and a displacement $1/2at^2$ resulting from the constant acceleration of the particle.

4.4. Two-Dimensional Motion with Constant Accel.

$$\vec{v}_s = \vec{v}_i + \vec{a}t$$

$$\vec{r}_s = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$



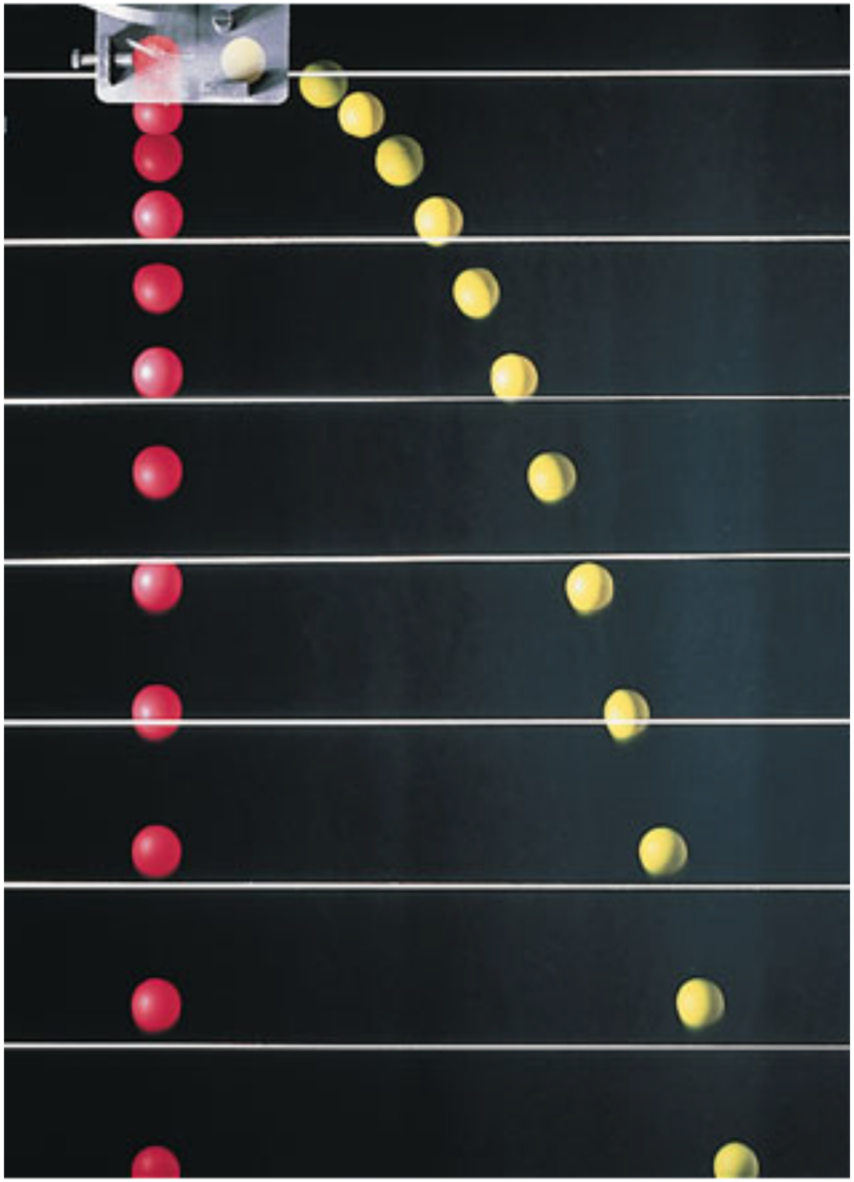


4.4. Two-Dimensional Motion with Constant Accel.

$$\vec{v}_s = \vec{v}_i + \vec{a}t \begin{cases} v_{xs} = v_{xi} + a_x t \\ v_{ys} = v_{yi} + a_y t \end{cases}$$

$$\vec{r}_s = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \begin{cases} r_{xs} = r_{xi} + v_{xi}t + \frac{1}{2}a_x t^2 \\ r_{ys} = r_{yi} + v_{yi}t + \frac{1}{2}a_y t^2 \end{cases}$$

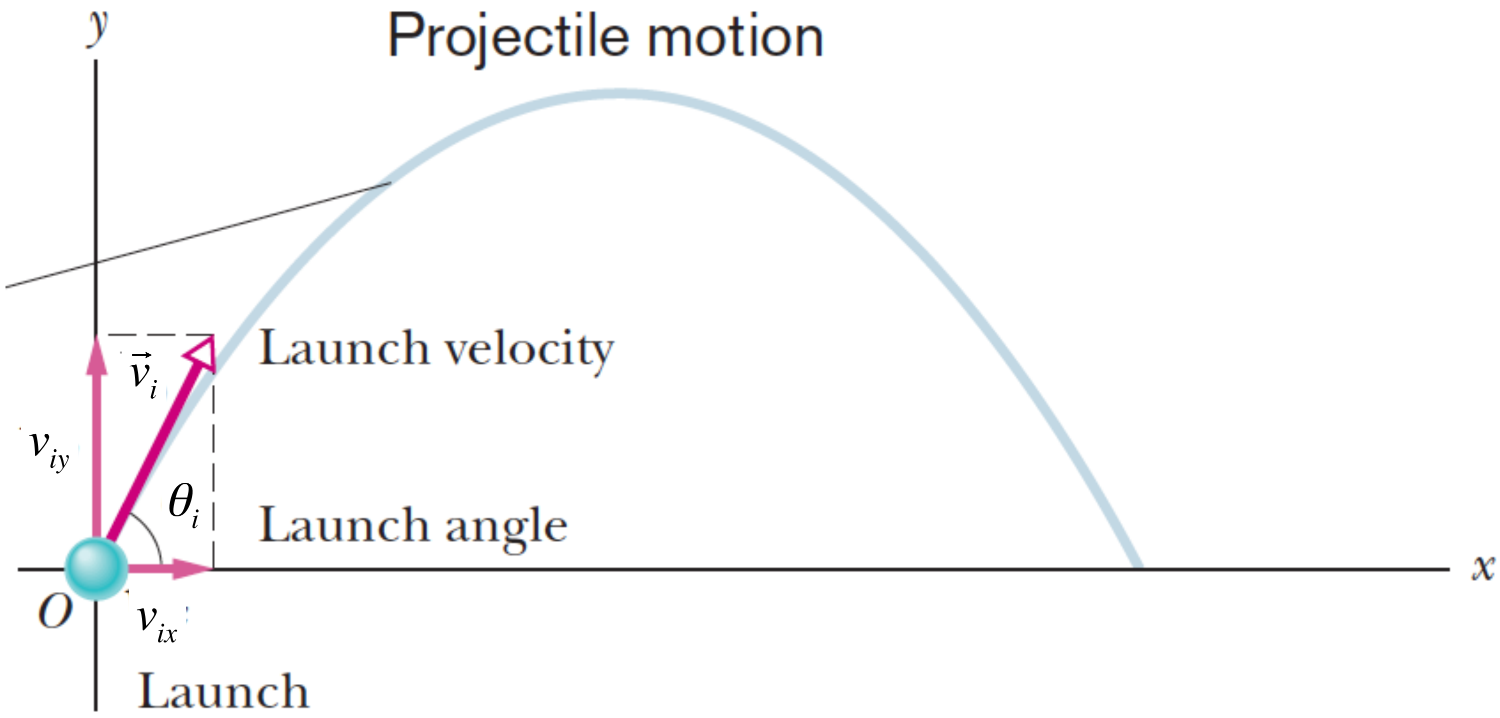
4.5.Projectile Motion



Two assumptions:

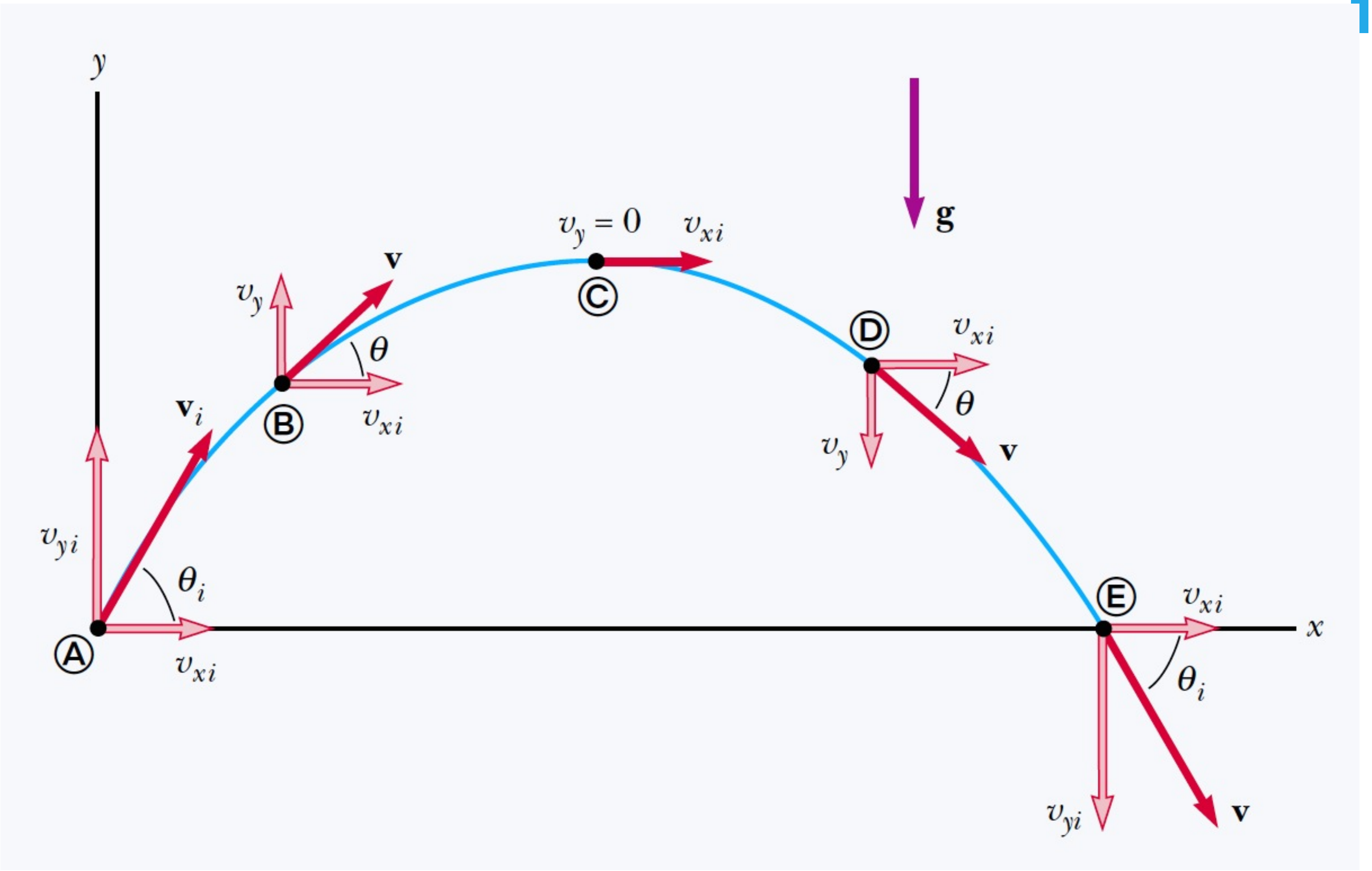
- the free-fall acceleration g is constant over the range of motion and is directed downward
- the effect of air resistance is negligible.

4.5.Projectile Motion



$$v_{ix} = v_i \cos\theta_i \qquad v_{iy} = v_i \sin\theta_i$$

4.5.Projectile Motion



1. constant-velocity motion in the horizontal direction
2. free-fall motion in the vertical direction

4.5.Projectile Motion

Horizontal motion

Since there is no acceleration in the horizontal direction, the horizontal component V_x of the projectile's velocity remains unchanged from its initial value. At any time t , the projectile horizontal position is given by :

$$x_f = x_i + v_{ix} t = x_i + (v_i \cos\theta_i) t$$

4.5.Projectile Motion

Vertical Motion

The vertical motion is the motion for a particle in free fall. The vrtical positon of projectille at any given time:

$$y_f = y_i + v_{iy}t + \frac{1}{2}at^2 = y_i + (v_i \sin\theta_i)t - \frac{1}{2}gt^2$$

Vertical velocity component at any given time:

$$v_{fy} = v_{iy} - gt = v_i \sin\theta_i - gt$$

4.5.Projectile Motion

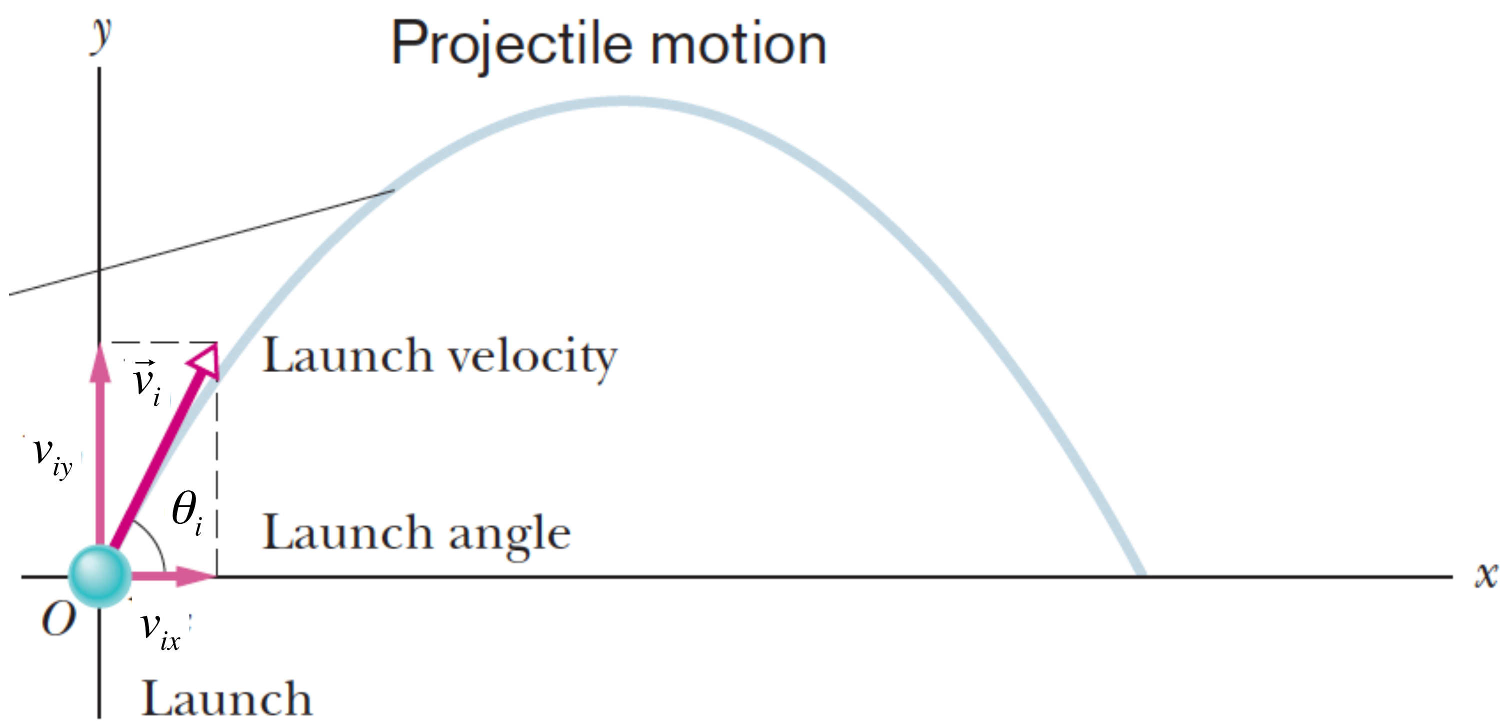
Vertical Motion

Zamansız hız formülü için yine dikey hareket için:

$$v_{fy}^2 = v_{iy}^2 - 2g(y_s - y_i)$$

Bu denklemlerden görüyoruzki, dikey hız bileşeni top sanki yukarı fırlatılmış gibi davranıyor. Önce giderek azalıyor. Sıfıra ulaştıktan sonra yön değiştirir ve sonra aşağıya doğru artar.

The Equation of the Path



4.5.Projectile Motion

The Equation of the Path

Yörüngeyi matemataiksel olarak ifade etmeye çalışalım. Hesaplamaların kolay olması için cismin harekete başladığı andaki konumunu orijin olarak seçelim. Bu durumda x_i ve y_i sıfır olacaktır.

$$x_f = (v_i \cos\theta_i)t \Rightarrow t = \frac{x_s}{v_i \cos\theta_i}$$

Bulduğumuz bu t'yi konumun y bileşeni için elde ettiğimiz denklemde yerine yazalım:

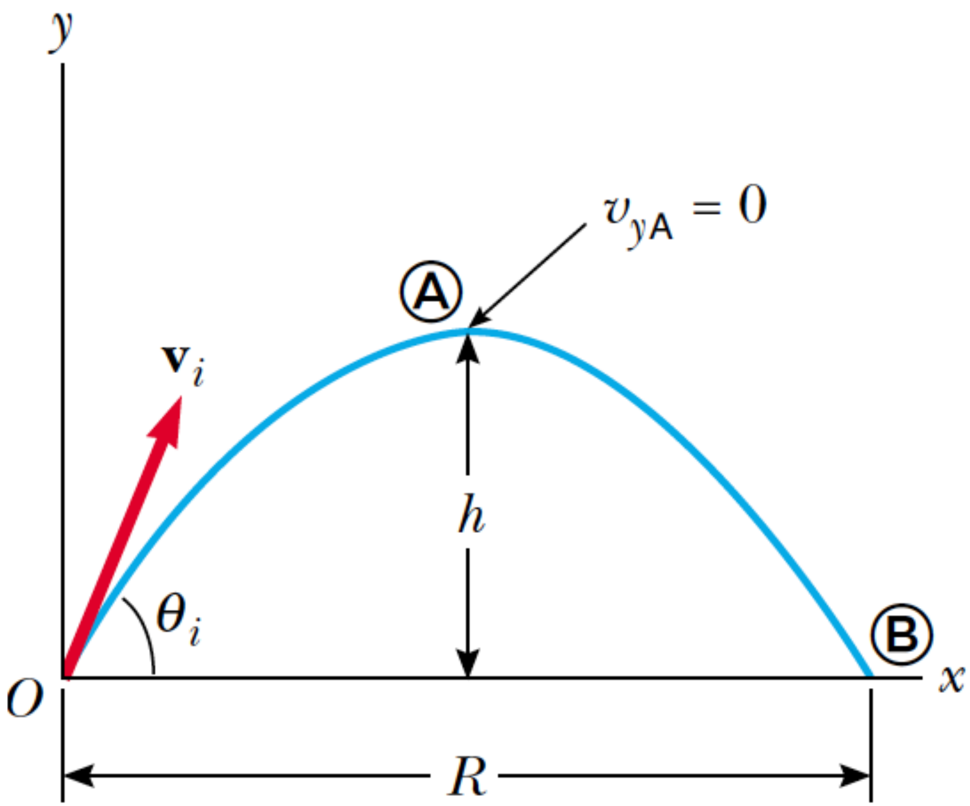
$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

Bu denkleme yörünge denklemi denir ve matematiksel olarak bu bir parabol denklemidir.

4.5.Projectile Motion

The Equation of the Path

We can determine h by noting that at the peak, Vfy=0. By using this we can determine time t_A, at which the projectile reaches the peak:



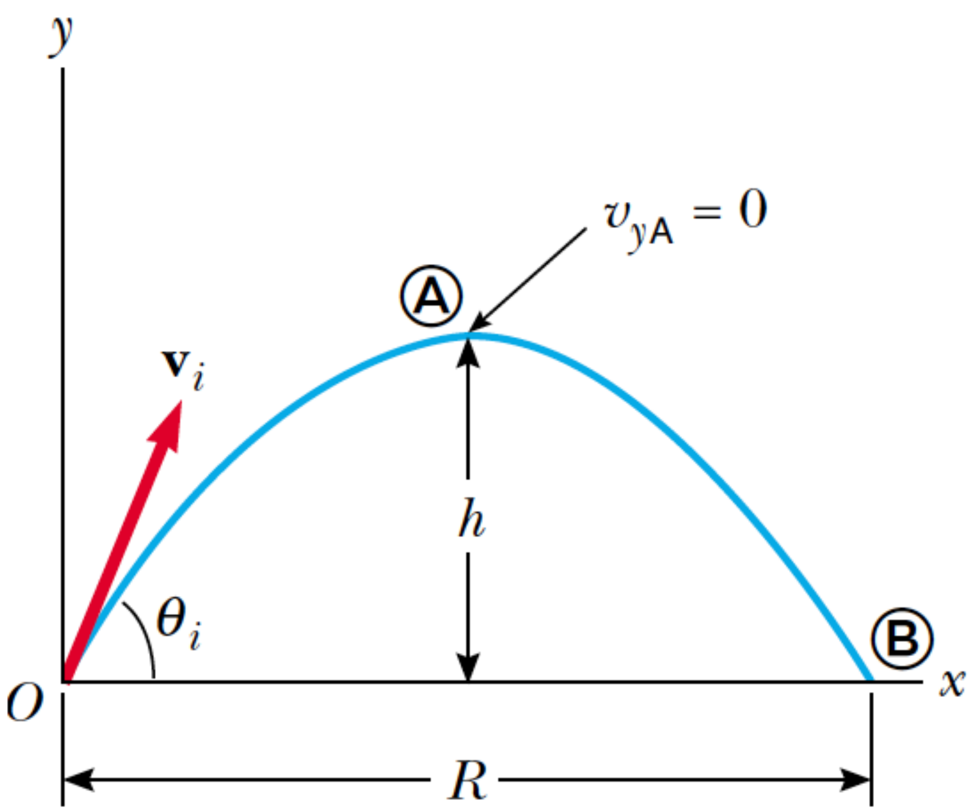
$$v_{sy} = v_{iy} - gt$$

$$0 = v_i \sin\theta_i - gt_A$$

$$t_A = \frac{v_i \sin\theta_i}{g}$$

4.5.Projectile Motion

Makximum height



we obtain an expression for h in terms of the magnitude and direction of the initial velocity vector

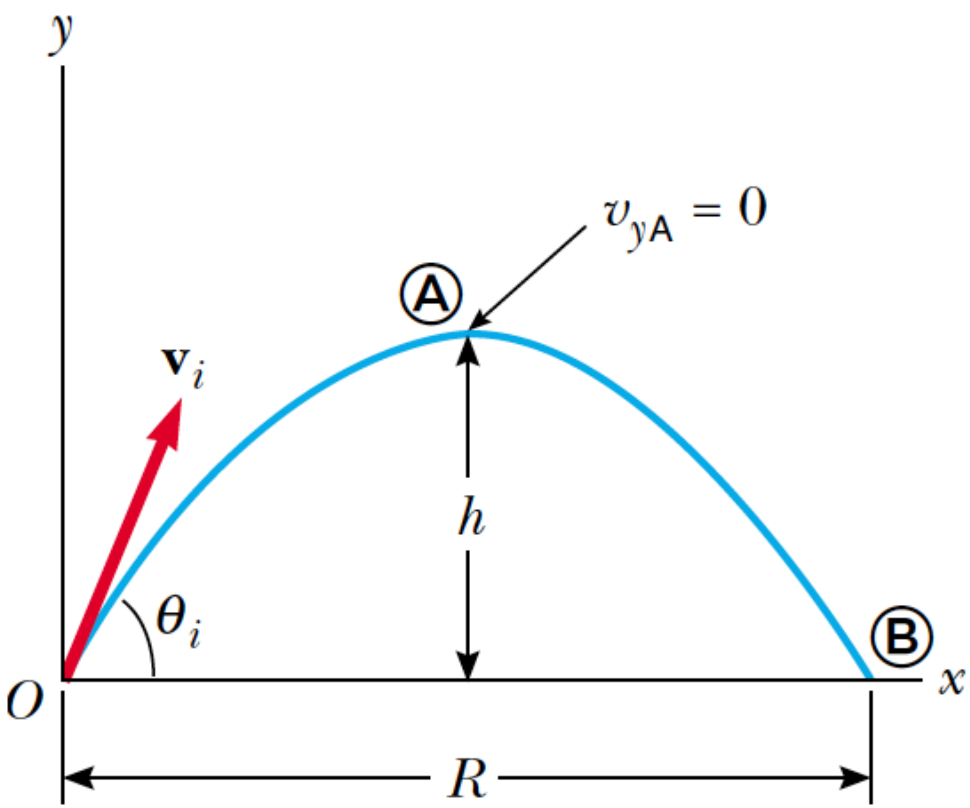
$$y_s = v_{iy}t - \frac{1}{2}gt^2$$

$$h = (v_i \sin\theta_i)t_A - \frac{1}{2}gt_A^2$$

$$h = \frac{v_i^2 \sin^2\theta_i}{2g}$$

4.5.Projectile Motion

Horizontal Range



$$\sin 2\theta = 2\sin\theta\cos\theta$$

The range R is the horizontal position of the projectile at a time that is twice the time at which it reaches its peak. Therefore:

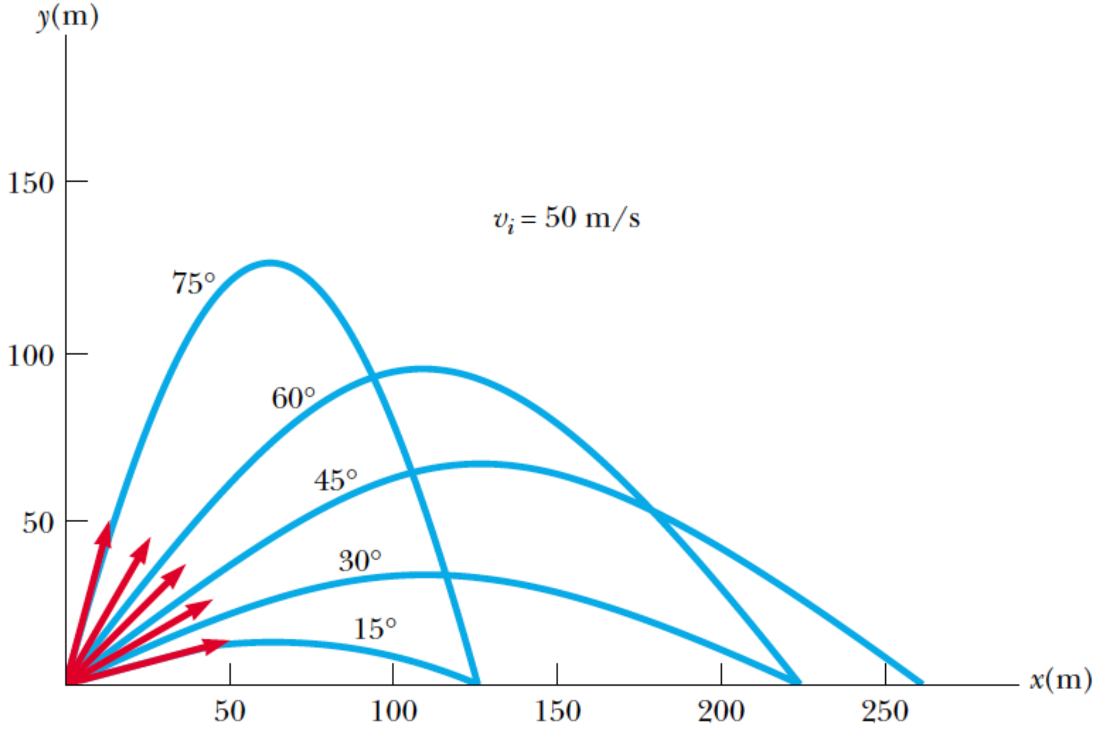
$$x_s = (v_i \cos\theta_i)t$$

$$R = (v_i \cos\theta_i)2t_A$$

$$R = \frac{v_i^2 \sin 2\theta_i}{g}$$

4.5.Projectile Motion

Yatay Erim (Menzil)



$$R = \frac{v_i^2 \sin 2\theta_i}{g}$$

$$\sin 2\theta_i = 1 \Rightarrow \theta_i = 45^0$$

4.5.Projectile Motion

Yatay Erim (Menzil)

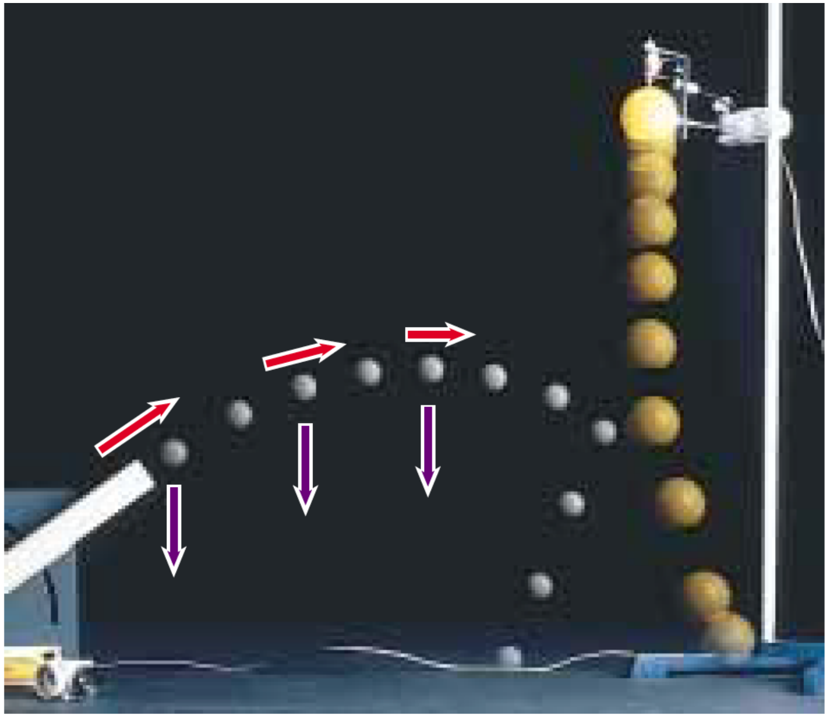
$$R = \frac{v_i^2 \sin 2\theta_i}{g}$$

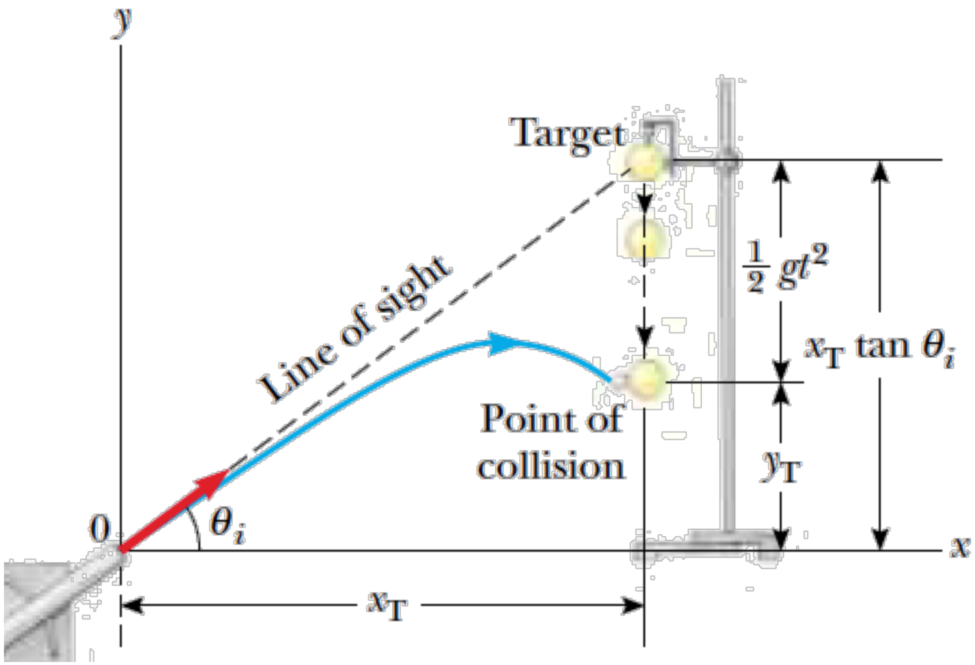
$$\sin 2\theta_i = 1 \Rightarrow \theta_i = 45^0$$

4.5.Projectile Motion

HM2

a projectile is fired at a target T in such a way that the projectile leaves the gun at the same time the target is dropped from rest, as shown in Figure. Show that if the gun is initially aimed at the stationary target, the projectile hits the target.





4.5.Projectile Motion

Answer HM2

the y coordinate of the target at any moment after release is

$$y_T = x_T \tan\theta_i - \frac{1}{2}gt^2$$

y coordinate of the projectile at any moment,

$$y_P = (v_{iP}\sin\theta_i)t - \frac{1}{2}gt^2$$

At collision $X_P = X_T$

$$x_P = (v_{iP}\cos\theta_i)t \qquad\qquad x_T = x_T$$

4.5.Projectile Motion

Answer HM2

Lt's consider that the collision takes place at $t_{C.}$ At this time the x-coordinate of projectile is:

$$x_P = x_T = x_T = (v_{iP}\cos\theta_i)t_C \Rightarrow t_C = \frac{x_T}{v_i\cos\theta_i}$$

By using this, we can find the y position of projectile

$$y_P = (v_{iP}\sin\theta_i)t_C - \frac{1}{2}gt_C^2 = x_T\tan\theta_i - \frac{1}{2}gt_C^2$$

we see that the y coordinates of the projectile and target are the same, when the posiiton in x direction ar same:

$$y_T = x_T\tan\theta_i - \frac{1}{2}gt_C^2 \qquad y_P = x_T\tan\theta_i - \frac{1}{2}gt_C^2$$