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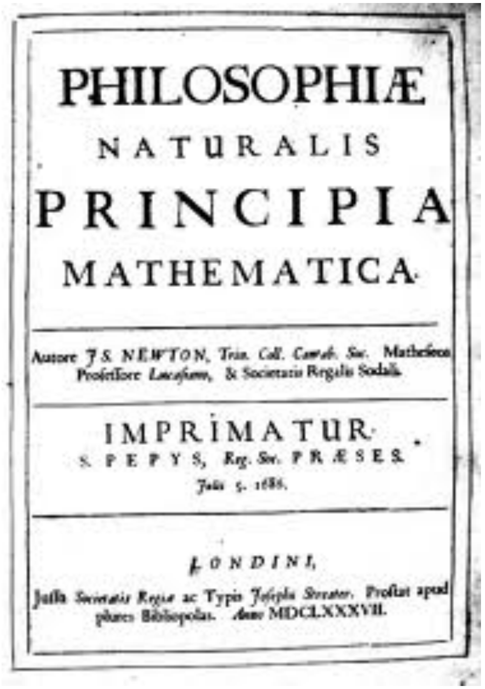
PHY121 Physics I

Chapter6 Force and Motion 2

Assoc. Prof. Dr. Eyüp DUMAN
A.U. Faculty of Engineering
Department of Energy Engineering

Bölüm 6 Kuvvet ve Hareket 2

6.1. Friction
6.2. Drag Force and Terminal Speed
6.3. Unifrom Circular Motion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S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

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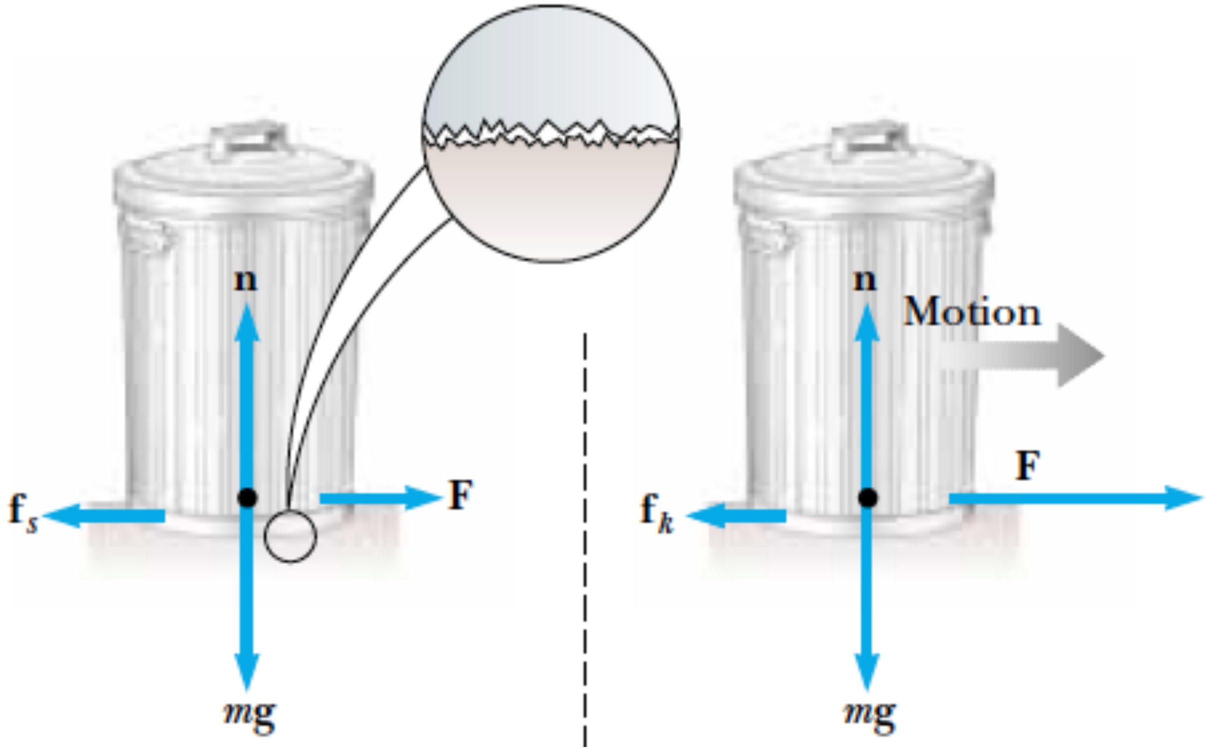
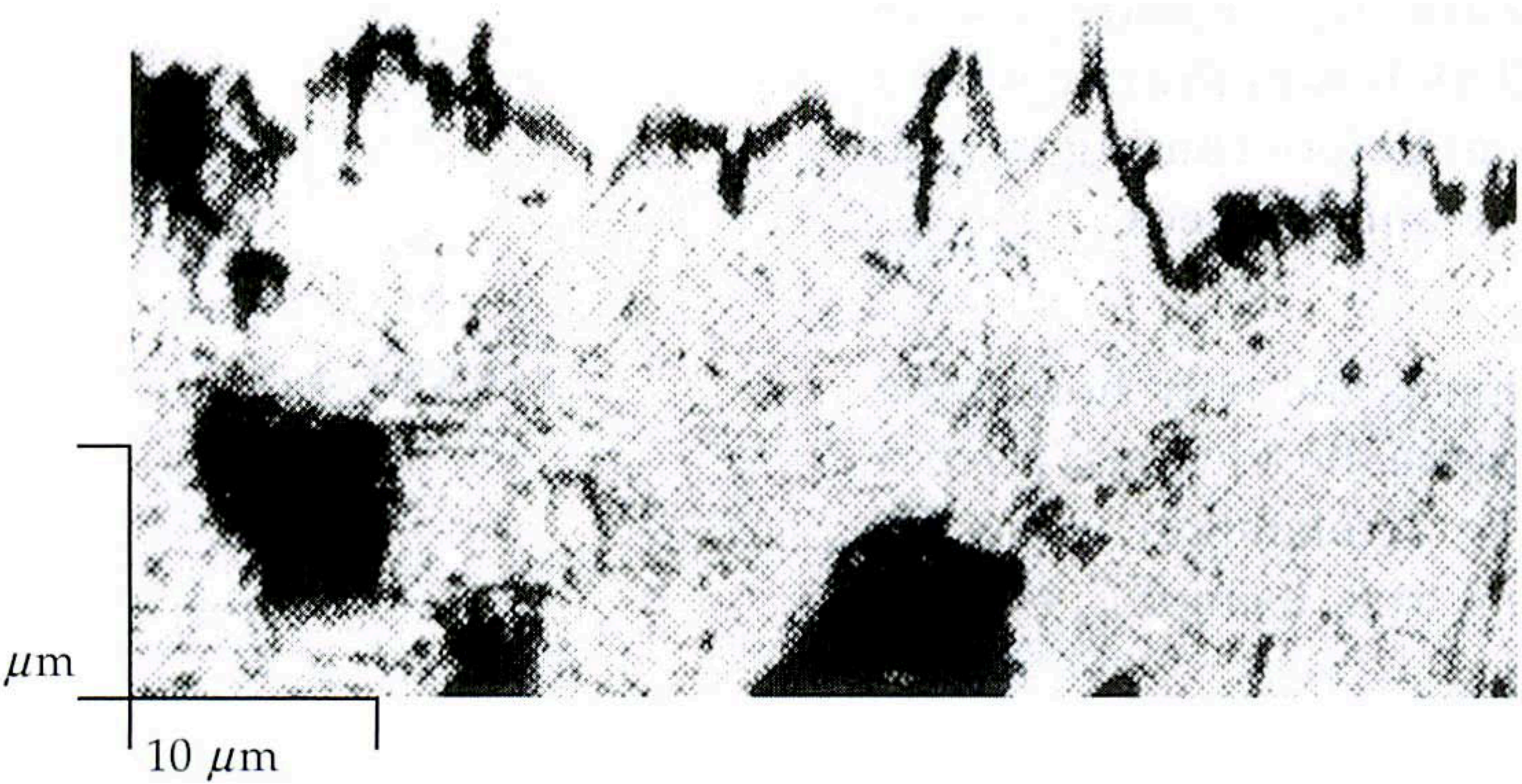
6.1. Friction



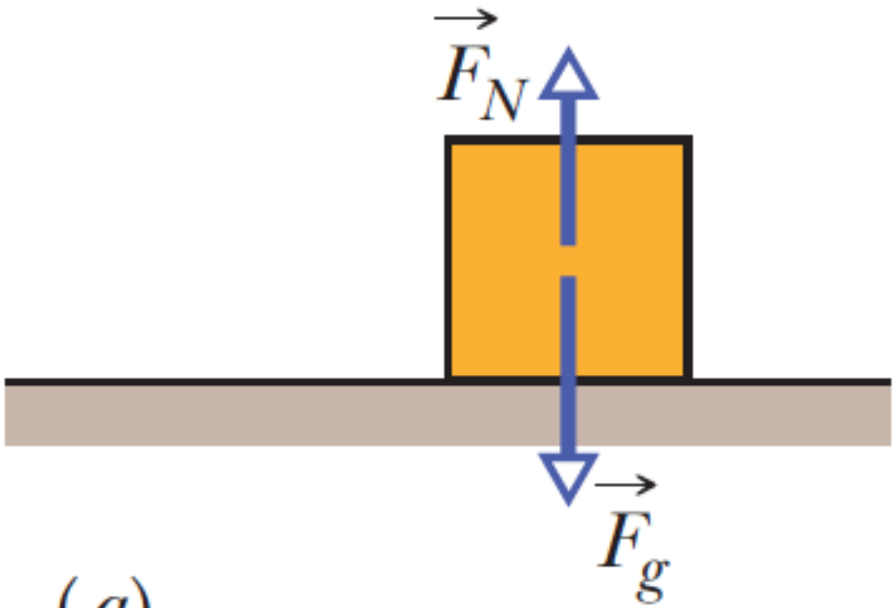
static friction f_s kinetic friction f_k

6.1. Friction



6.1. Friction

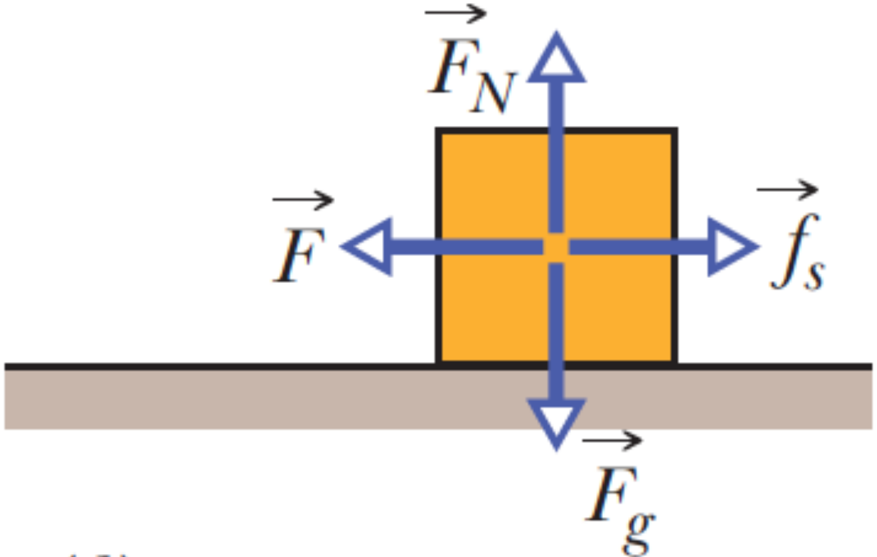
There is no attempt at sliding. Thus, no friction and no motion.



(a)

6.1. Friction

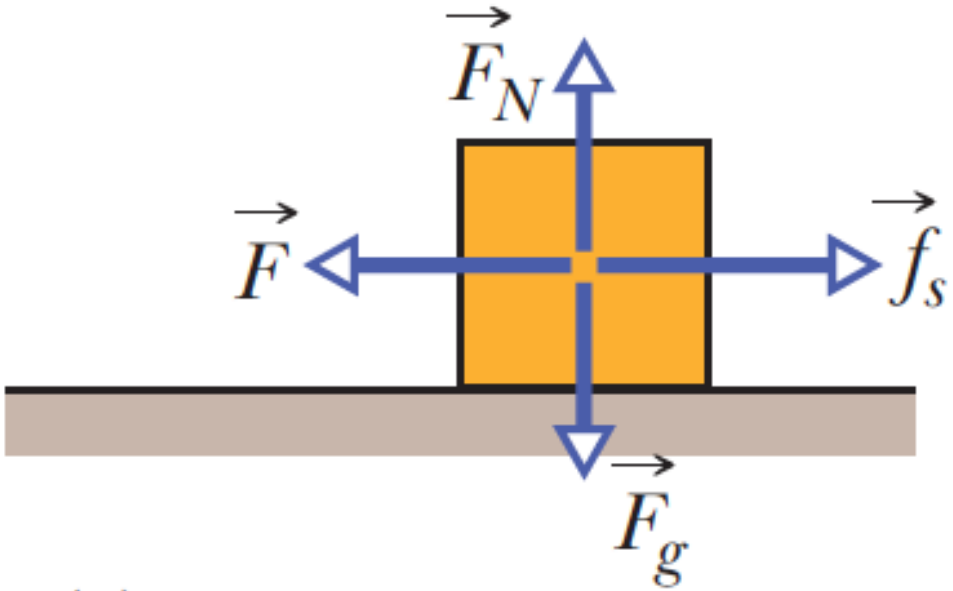
Force $\vec{F}$ attempts sliding but is balanced by the frictional force. No motion.



(b)

6.1. Friction

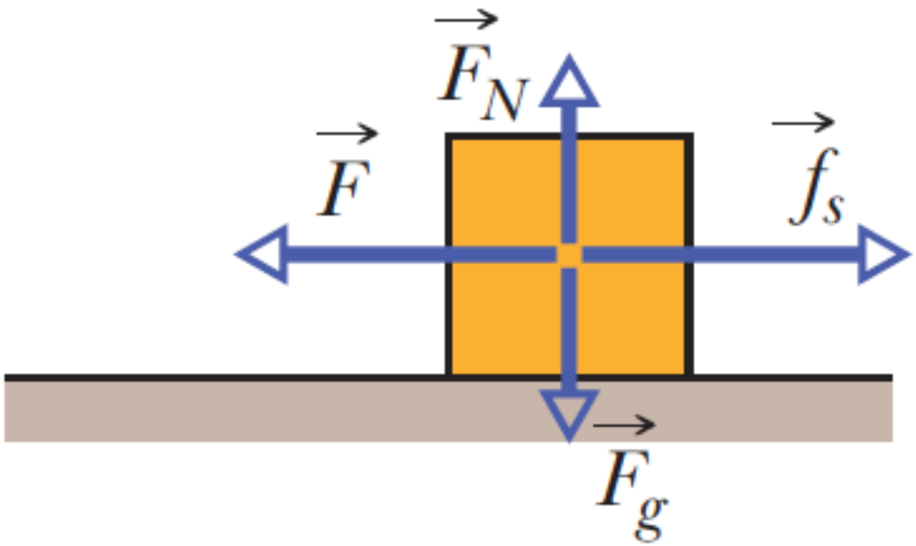
Force $\vec{F}$ is now stronger but is still balanced by the frictional force. No motion.



(c)

6.1. Friction

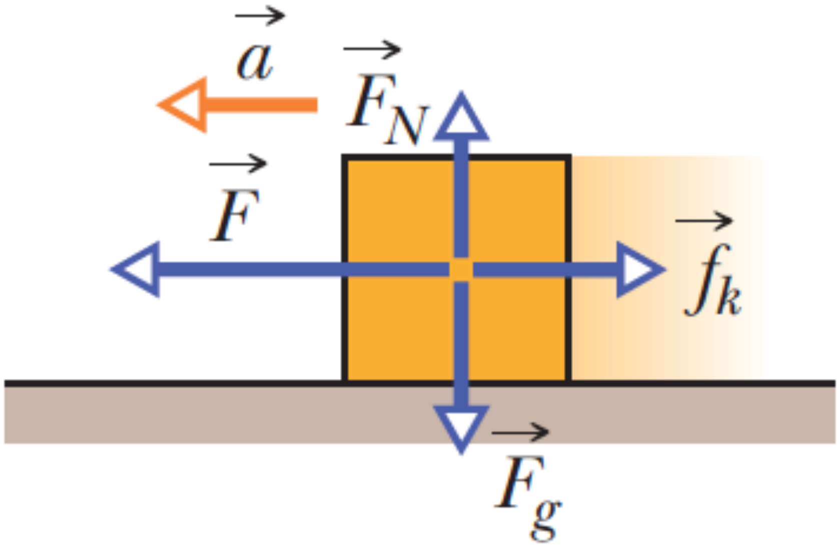
Force $\vec{F}$ is now even stronger but is still balanced by the frictional force. No motion.



(*d*)

6.1. Friction

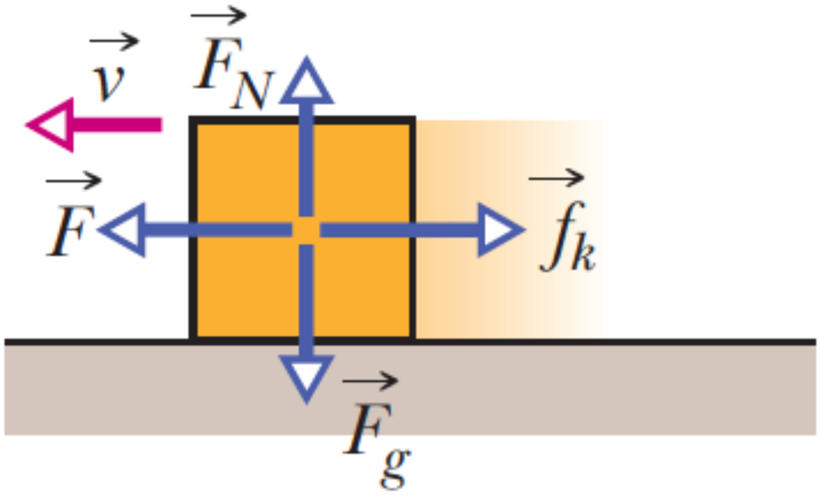
Finally, the applied force has overwhelmed the static frictional force. Block slides and accelerates.



(e)

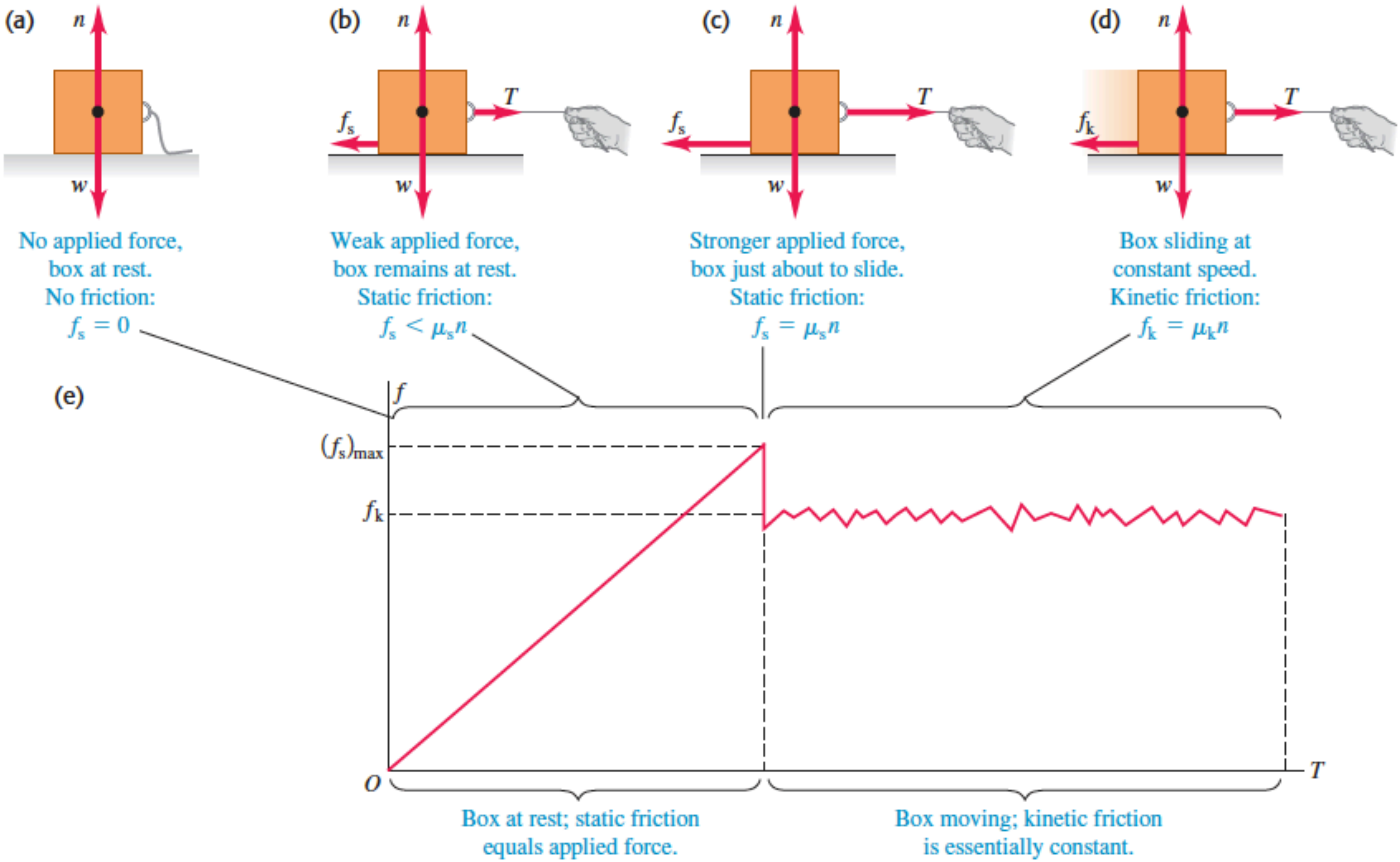
6.1. Friction

To maintain the speed, weaken force $\vec{F}$ to match the weak frictional force.



(f)

6.1. Friction



(a)
n
w
No applied force, box at rest. No friction: $f_s = 0$
(b)
n
T
f_s
w
Weak applied force, box remains at rest. Static friction: $f_s < \mu_s n$
(c)
n
T
f_s
w
Stronger applied force, box just about to slide. Static friction: $f_s = \mu_s n$
(d)
n
T
f_k
w
Box sliding at constant speed. Kinetic friction: $f_k = \mu_k n$
(e)
f
$(f_s)_{max}$
f_k
O
T
Box at rest; static friction equals applied force.
Box moving; kinetic friction is essentially constant.

6.1. Friction

Some propertied of friction forces:

- **If the body does not move, then the static frictional force and the component of applied force that is parallel to the surface balance each other. They are equal in magnitude, and fs is directed opposite that component of F .**

- **The magnitude of friction force fs,max that is given by**

$$f_{s,mak} = \mu_s F_N$$

Where μ_s is static friction coefficient and F_N is the magnitude of the normal force on the body from the surface.

- **If the body begins to slide along the surface, the magnitude of the kinetic frictional force**

$$f_k = \mu_k F_N$$

where μ_k kinetic friction coefficient and F_N is the magnitude of the normal force on the body from the surface..

6.1. Friction

Some propertied of friction forces:

Coefficients of Friction[a]

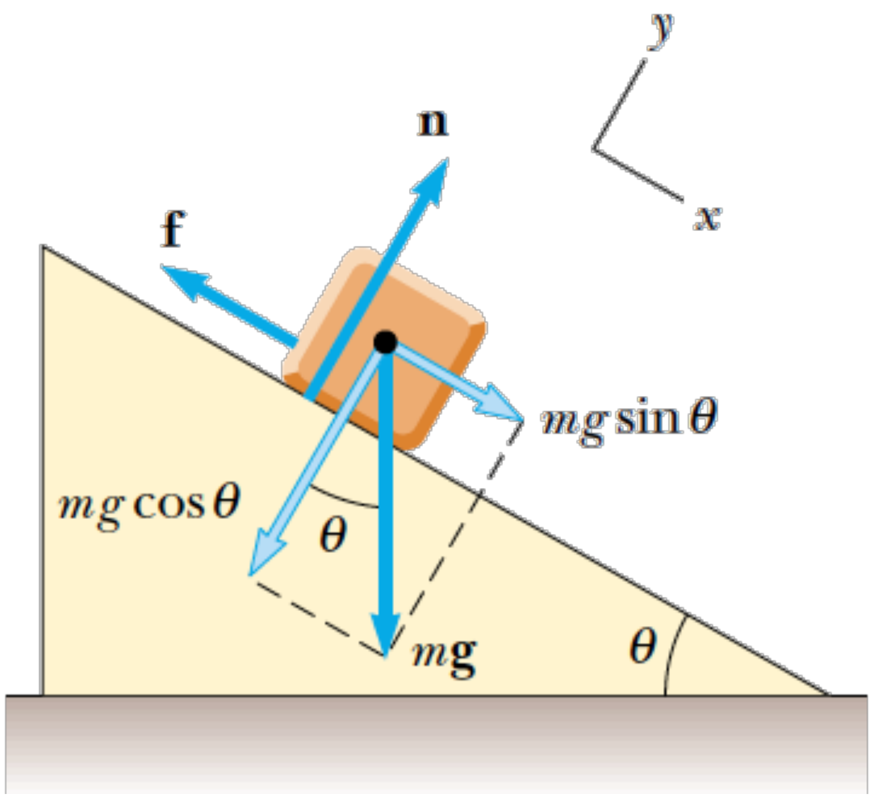
	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

- **μ_s ve μ_k depends on surfaces. Generally $\mu_k < \mu_s$ dir.**
- **The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion.**
- **The coefficients of friction are nearly independent of the area of contact between the surfaces.**

6.1. Friction

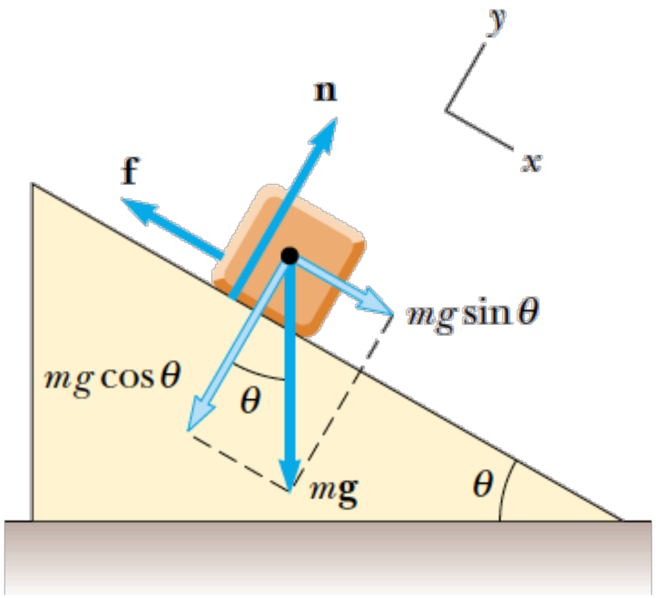
Example:1

Measuring coefficients of friction: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Figure below. The incline angle is increased until the block starts to move. Show that by measuring the critical angle θ_c at which this slipping just occurs, we can obtain μ_s .



6.1. Friction

Answer:1



x-axis

$$(1)\quad mg\sin\theta - f_s = ma_x = 0$$

y-axis

$$(2)\quad n - mg\cos\theta = ma_y = 0$$

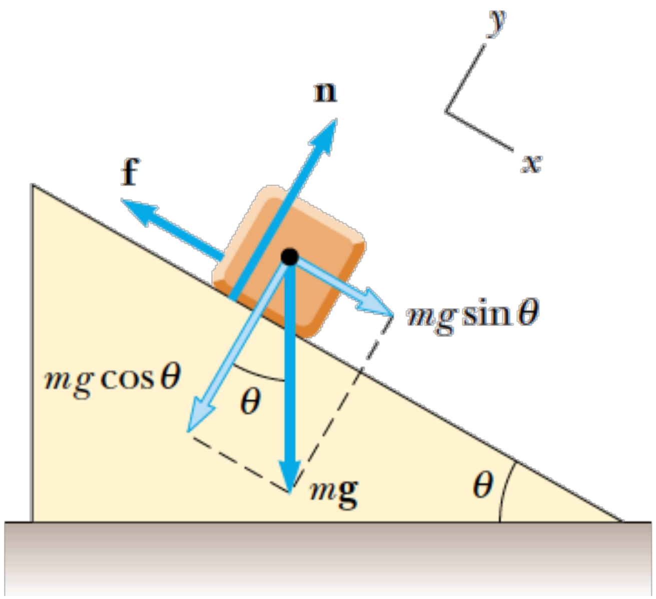
from equation 2

$$n / \cos\theta = mg$$

By substituting of this in to equation 1

6.1. Friction

Answer:1



$$mg\sin\theta - f_s = 0$$

$$f_s = \frac{n}{\cos\theta}\sin\theta$$

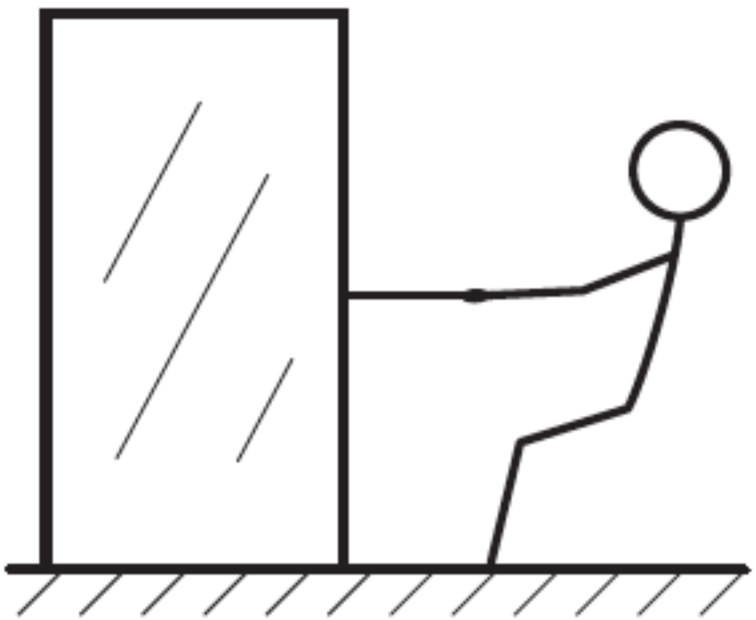
$$\mu_s n = n\tan\theta$$

$$\mu_s = \tan\theta$$

6.1. Friction

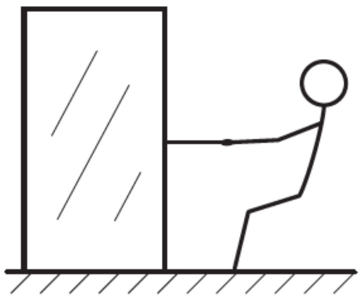
Example:2

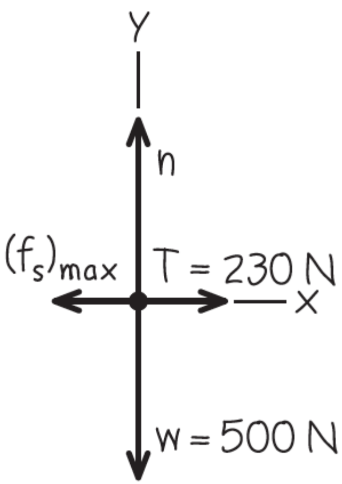
You want to move a 500-N crate across a level floor. To start the crate moving, you have to pull with a 230-N horizontal force. Once the crate “breaks loose” and starts to move, you can keep it moving at constant velocity with only 200 N. What are the coefficients of static and kinetic friction?



6.1. Friction

Answer:2





Free-body diagram for crate just before it starts to move

on x-axis

$$\sum F_x = T - f_{s,mak} = 0 \Rightarrow f_{s,mak} = T = 230\,N$$

on y-axis

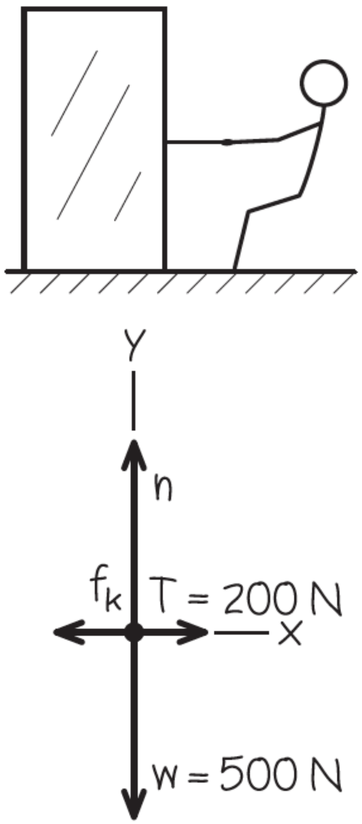
$$\sum F_y = n - w = 0 \Rightarrow n = w = 500\,N$$

so the static coefficient of friction

$$f_{s,mak} = \mu_s n \Rightarrow \mu_s = f_{s,mak} / n = 0.46$$

6.1. Friction

Answer:2



Free-body diagram for crate moving at constant speed

on x-axis

$$\sum F_x = T - f_k = 0 \Rightarrow f_k = T = 200N$$

on y-yönünde

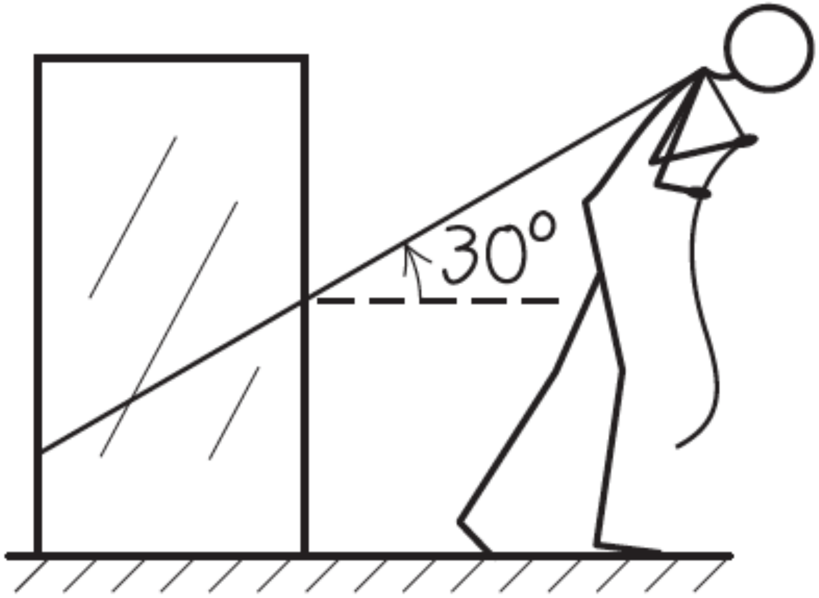
$$\sum F_y = n - w = 0 \Rightarrow n = w = 500N$$

So the kinetic coefficient of friction:

$$f_k = \mu_k n \Rightarrow \mu_k = f_k / n = 0.40$$

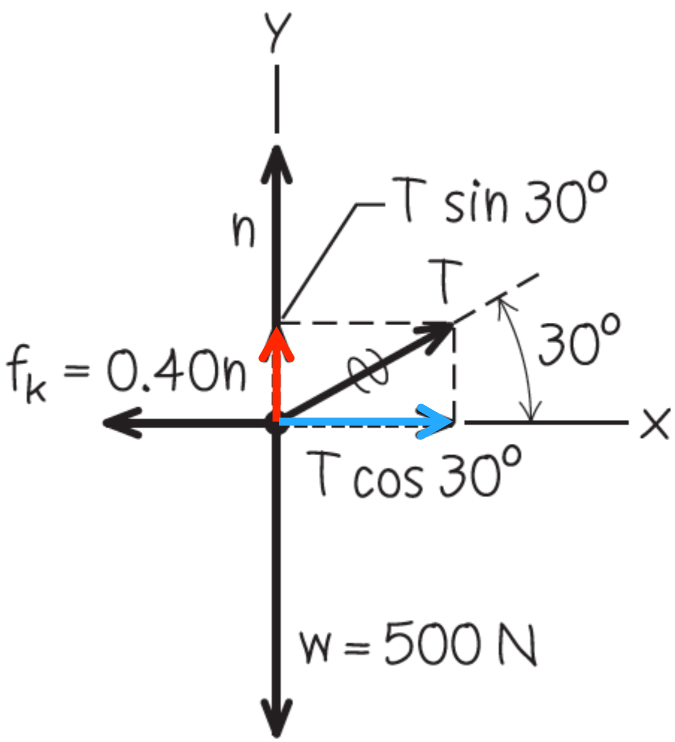
Example:3

In example 2 suppose you move the crate by pulling upward on the rope at an angle of above the horizontal. How hard must you pull to keep it moving with constant velocity? Assume that $\mu_k = 0.40$.



6.1. Friction

Answer:3



on x-axis

$$T\cos 30 - f_k = 0$$

$$f_k = \mu_k n = T\cos 30$$

on y-axis

$$T\sin 30 + n - w = 0$$

$$n = w - T\sin 30$$

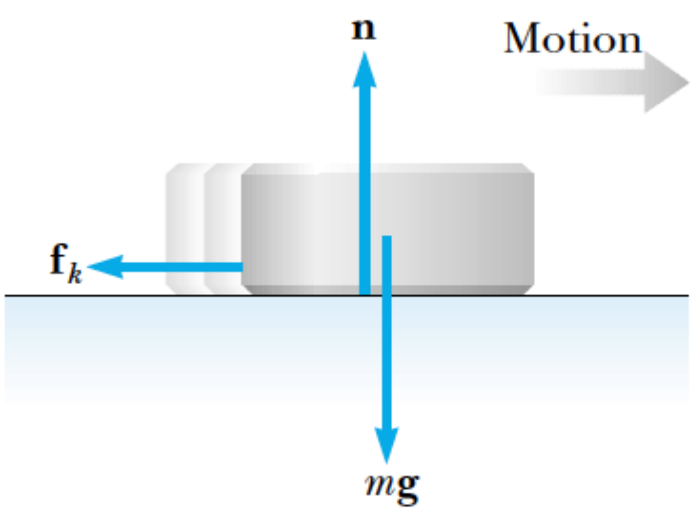
$$T\cos 30 = \mu_k (w - T\sin 30)$$

$$T = \frac{\mu_k w}{\cos 30 + \mu_k \sin 30} = 188N$$

6.1. Friction

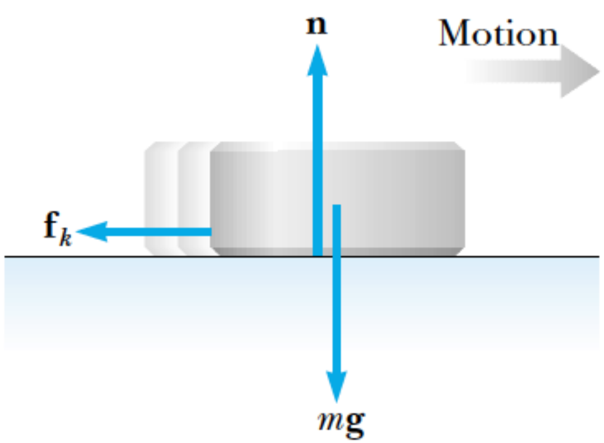
Example:4

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.



6.1. Friction

Answer:4



x-axis

$$\sum F_x = -f_k = ma_x$$

$$-\mu_k n = ma_x$$

y-axis

$$n - mg = 0 \Rightarrow n = mg$$

$$-\mu_k n = ma_x$$

$$-\mu_k mg = ma_x \Rightarrow a_x = -\mu_k g$$

6.1. Friction

Answer:4

$$v_s^2 = v_i^2 + 2a_x \Delta x$$

$$0 = v_i^2 - 2\mu_k g \Delta x$$

$$\mu_k = \frac{v_i^2}{2g\Delta x} = \frac{(20m/s)^2}{2(9.8m/s^2)(115m)} = 0.17$$

6.2. Drag Force and Terminal Speed

Resistive Force Proportional to Object Speed

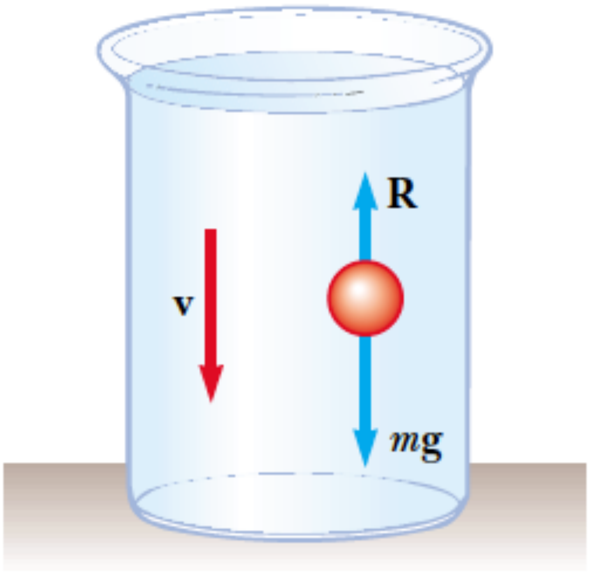
The magnitude of the fluid resistance force usually increases with the speed of the body through the fluid.

$$\vec{R} = -b\vec{v}$$

where v is the velocity of the object and b is a constant whose value depends on the properties of the medium and on the shape and dimensions of the object. **The negative sign indicates that R is in the opposite direction to v.**

6.2. Drag Force and Terminal Speed

Resistive Force Proportional to Object Speed



Applying Newton's second law to the vertical motion

$$\sum F_y = mg - bv = ma = m\frac{dv}{dt}$$

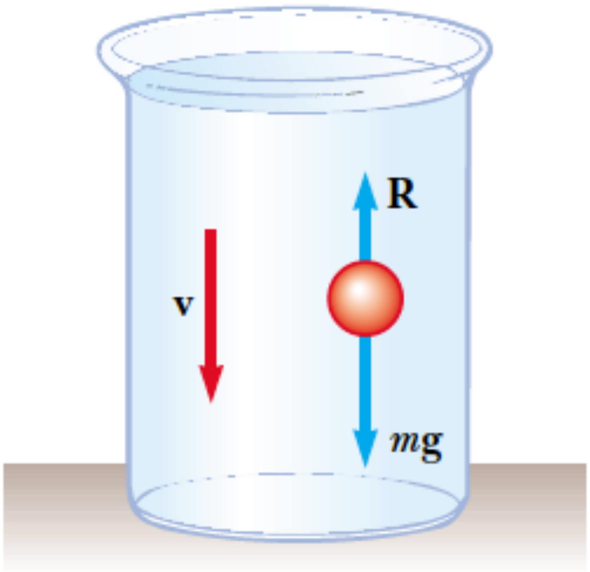
Solving this expression for the acceleration gives:

$$\frac{dv}{dt} = g - \frac{b}{m}v$$

The acceleration approaches zero when the magnitude of the resistive force approaches the sphere's weight. In this situation, the speed of the sphere approaches its terminal speed v_T.

6.2. Drag Force and Terminal Speed

Resistive Force Proportional to Object Speed



We can obtain the terminal speed from acceleration equation by setting a=dv/dt=0. This gives as terminal speed:

$$v_T = \frac{mg}{b}$$

In reality, the sphere only approaches terminal speed but never reaches terminal speed.

6.2. Drag Force and Terminal Speed

Resistive Force Proportional to Object Speed

$$\frac{dv}{dt} = g - \frac{b}{m} v$$

This equation is called a differential equation and the the expression for v that satisfies this equation:

$$v = \frac{mg}{b}(1 - e^{-bt/m})$$

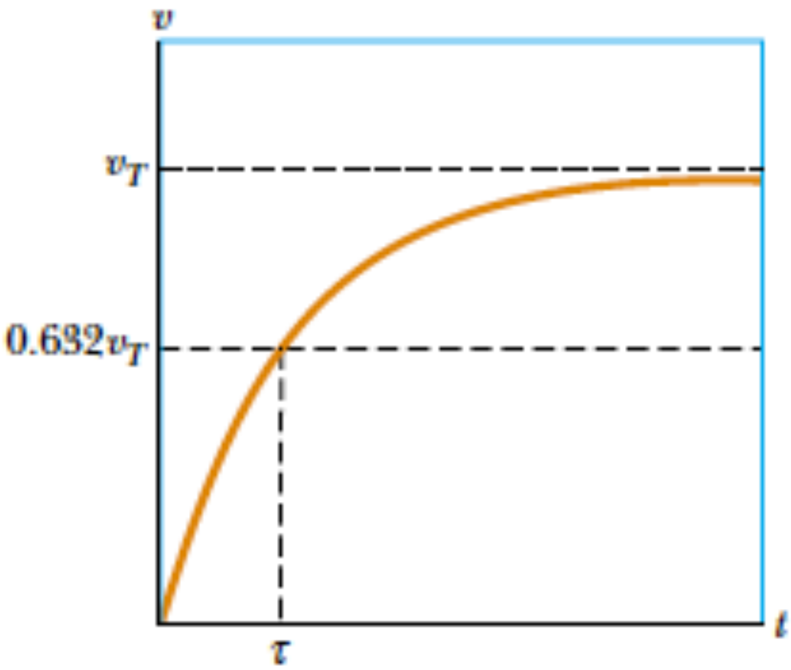
By using definition of terminal speed:

$$v = v_T(1 - e^{-bt/m})$$

$$v = v_T(1 - e^{-t/\tau})$$

6.2. Drag Force and Terminal Speed

Resistive Force Proportional to Object Speed



$$v = v_T(1 - e^{-t/\tau})$$

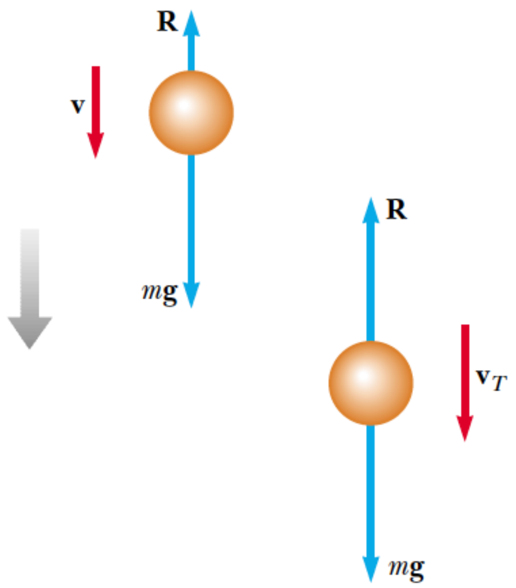
The symbol e represents the base of the natural logarithm, and is also called Euler's number: e =2.71828. The time constant

$$\tau = \frac{m}{b}$$

is the time at which the sphere released from rest reaches 63.2% of its terminal speed

6.2. Drag Force and Terminal Speed

Air Drag at High Speeds



For objects moving at high speeds through air, the resistive force is approximately proportional to the square of the speed.

$$R = \frac{1}{2} D \rho A v^2$$

where ρ is the density of air, A is the cross-sectional area of the moving object measured in a plane perpendicular to its velocity, and D is a dimensionless empirical quantity called the drag coefficient.

Applying Newton's 2. Law to this object

$$\sum F_y = mg - \frac{1}{2} D \rho A v^2 = ma$$

6.2. Drag Force and Terminal Speed

Air Drag at High Speeds

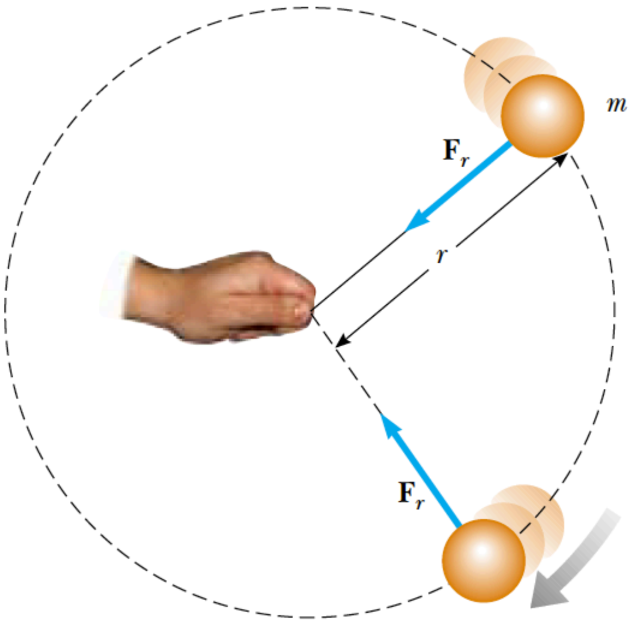
So the magnitude of downward acceleration is given by:

$$a = g - \left(\frac{D\rho A}{2m}\right)v^2$$

Terminal speed is given by:

$$v_T = \sqrt{\frac{2mg}{D\rho A}}$$

6.3. Uniform Circular Motion



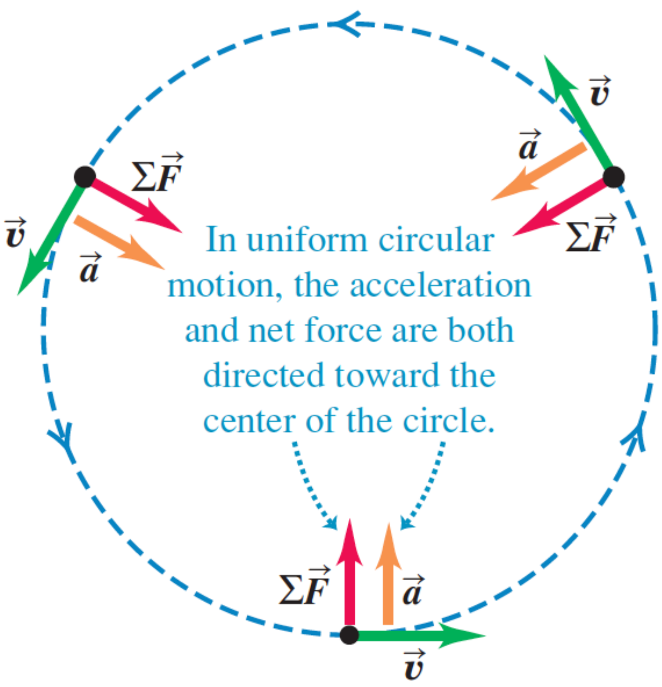
The centripetal acceleration (directed toward the center of the circle) of an object, which moves in a circle (or a circular arc) at constant speed v, is given by:

$$a_c = \frac{v^2}{r}$$

where r is the radius of the circle.

Why does the ball move in a circle?

6.3. Uniform Circular Motion

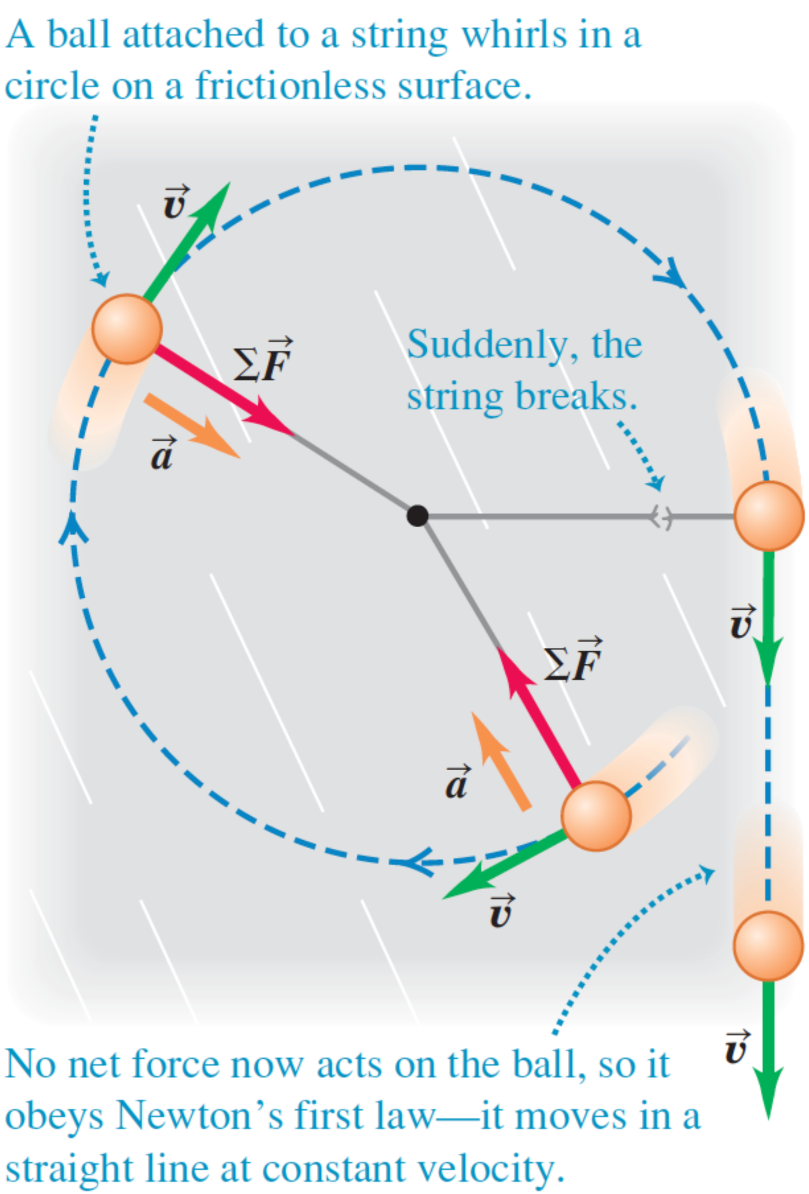


If we apply Newton's second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated:

$$\sum F = ma_c = m\frac{v^2}{r}$$

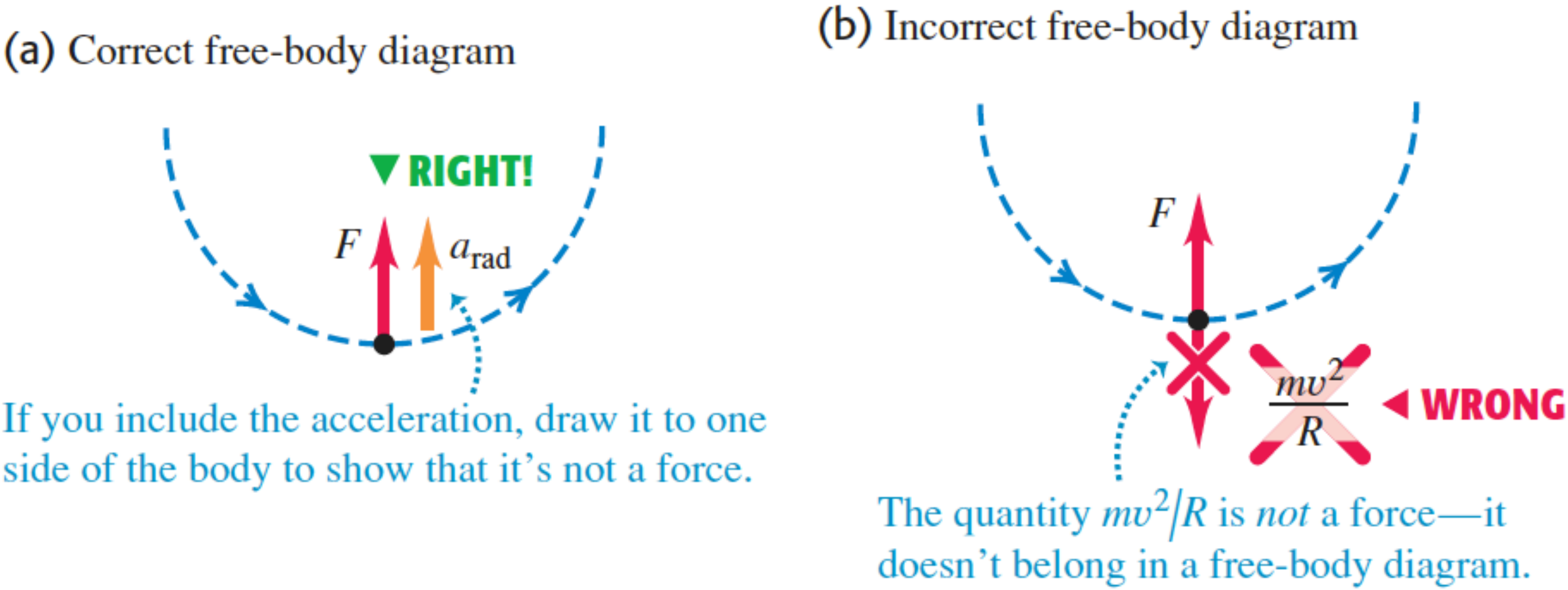
A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

6.3. Uniform Circular Motion



A force causing a centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle.

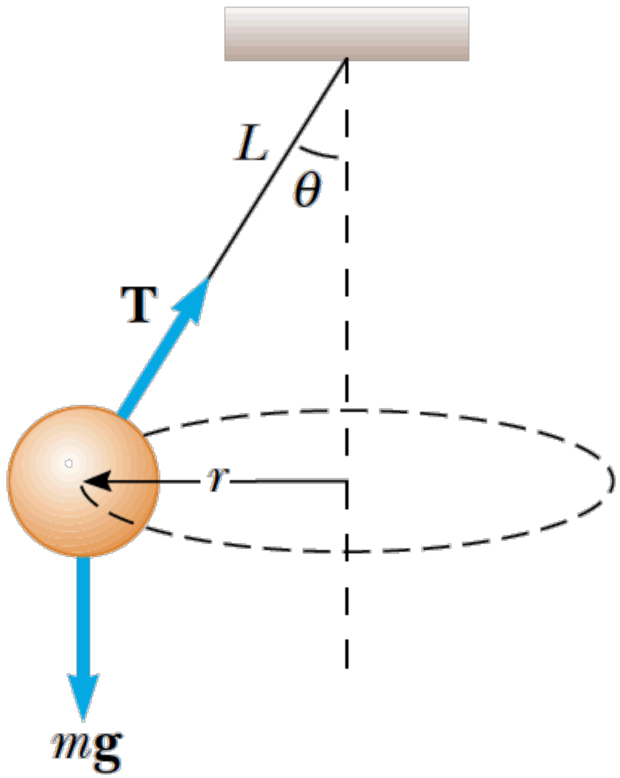
6.3. Uniform Circular Motion



Avoid using "centrifugal force"

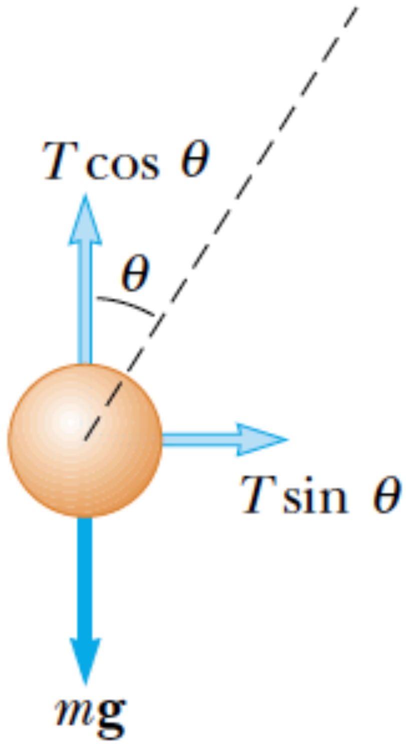
6.3. Uniform Circular Motion

Example:5



A tether ball of mass m is suspended from a length of rope and travels at constant speed v in a horizontal circle of radius r as shown. The rope makes an angle θ with the vertical. Find (a) the direction of the acceleration, (b) the tension in the rope, and (c) the speed of the ball.

Answer:5



a) The acceleration is horizontal and directed from the ball toward the center of the circle it is moving in.

b) Tension in rope:

$$(1) \qquad T\cos\theta = mg \Rightarrow T = {}^{mg}\!/_{\cos\theta}$$

$$(2) \qquad T\sin\theta = ma_c = m\frac{v^2}{r}$$

c) The speed of the ball can be found from Eg 1 and 2.

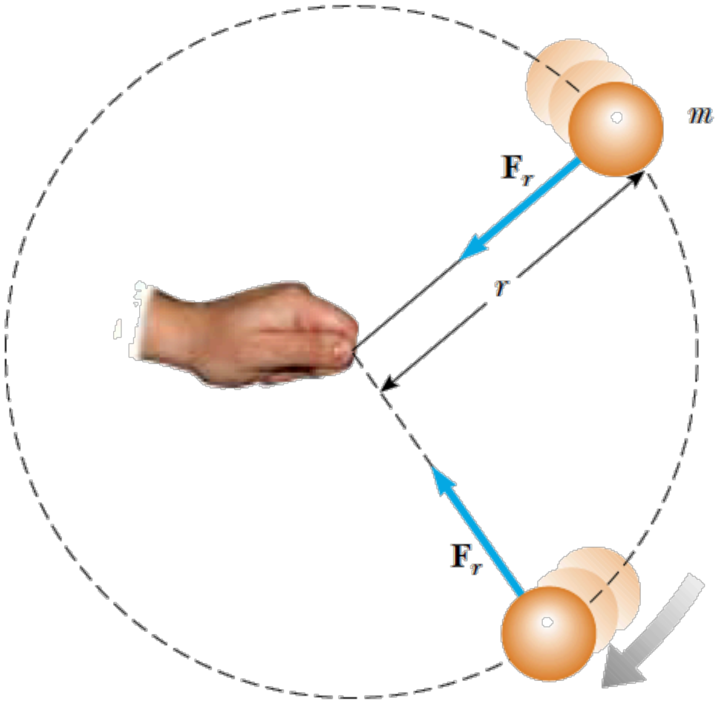
$$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$$

by using r=Lsinθ

$$v = \sqrt{Lg\sin\theta\tan\theta}$$

6.3. Uniform Circular Motion

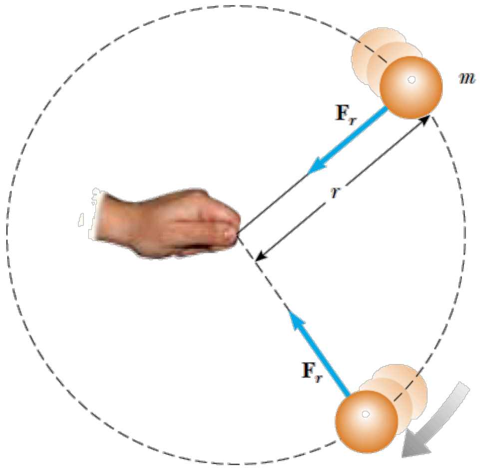
Example:6



A ball of mass 0.500 kg is attached to the end of a cord 1.50 m long. The ball is whirled in a horizontal circle as shown in Figure. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the ball can be whirled before the cord breaks?

6.3. Uniform Circular Motion

Answer:6



Topu dairesel yörüngede tutan kuvvet ipteki gerilme. Newton'un 2. yasasını uygulayacak olursak:

$$T = m\frac{v^2}{r} \Rightarrow v = \sqrt{\frac{Tr}{m}}$$

İpte maksiumum gerilme oluştuğundaki hız:

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50N)(1.5m)}{0.5kg}} \approx 12.2m/s$$