4.1 Potential Difference and Electric Potential
4.2 Potential Differences in a Uniform Electric Field
4.3 Electric Potential and Potential Energy Due to Point Charges
4.4 Equipotential Surfaces
4.5 Obtaining the Value of the Electric Field from the Electric Potential
4.6 Electric Potential Due to Continuous Charge Distributions
4.7 Electric Potential Due to a Charged Conductor
4.8 Applications of Electrostatics

25.1 Elektrik Potansiyeli ve Potansiyel Farkı

Klasik mekanikte, kuvvet yaklaşımını kullanarak çözemediğimiz ya da çözümü çok zor olan birçok problemi enerjinin korunumunu kullanarak kolaylıkla çözebiliyorduk. Geçen dönem tanımladığımız potansiyel enerji kavramı elektromanyetizmada da kullanışlı bir araçtır. Elektriksel kuvvet korunumlu bir kuvvet olduğundan birçok elektrostatik olay elektrik potansiyeli olarak tanımlayacağımız skaler bir büyüklükle tanımlanabilir.

Bir elektrik alan içine koyduğumu q_o test yüküne etki eden $F_E=q_oE$ kuvveti korunumlu bir kuvvet. Diyelim ki q_o yükü bu alan içinde bir dış etken tarafından hareket ettirilirse, yük üzerinde alan tarafından yapılan iş yer değiştirmeye neden olan dış etken tarafından yapılan işin eksi işaretli değerine eşittir.

Bu olay kütle çekim alanı içinde bir m kütlesini yukarı kaldırmaya eşdeğerdir: Bu durumda m kütlesini kaldıran dış etkenin yaptığı iş mgh iken, kütle çekim kuvvetinin yaptığı iş ise –mgh.

Diyelim ki q_o yükü bir elektrik alan içinde $d\vec{\mathbf{s}}$ yerdeğiştirmesini yapmış olsun. Elektrik alanın yük üzerinde yapmış olduğu iş

$$W_{\text{int}} = \vec{\mathbf{F}}_e \cdot d\vec{\mathbf{s}} = q_0\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

4.1 Potential Difference and Electric Potential

When a force **F** acts on a particle that moves from point a to point b , the work done by the force is given by a line integral

$$W_{a\to b} = \int_a^b \vec{F}.d\vec{l} = \int_a^b F\cos\phi\, dl$$

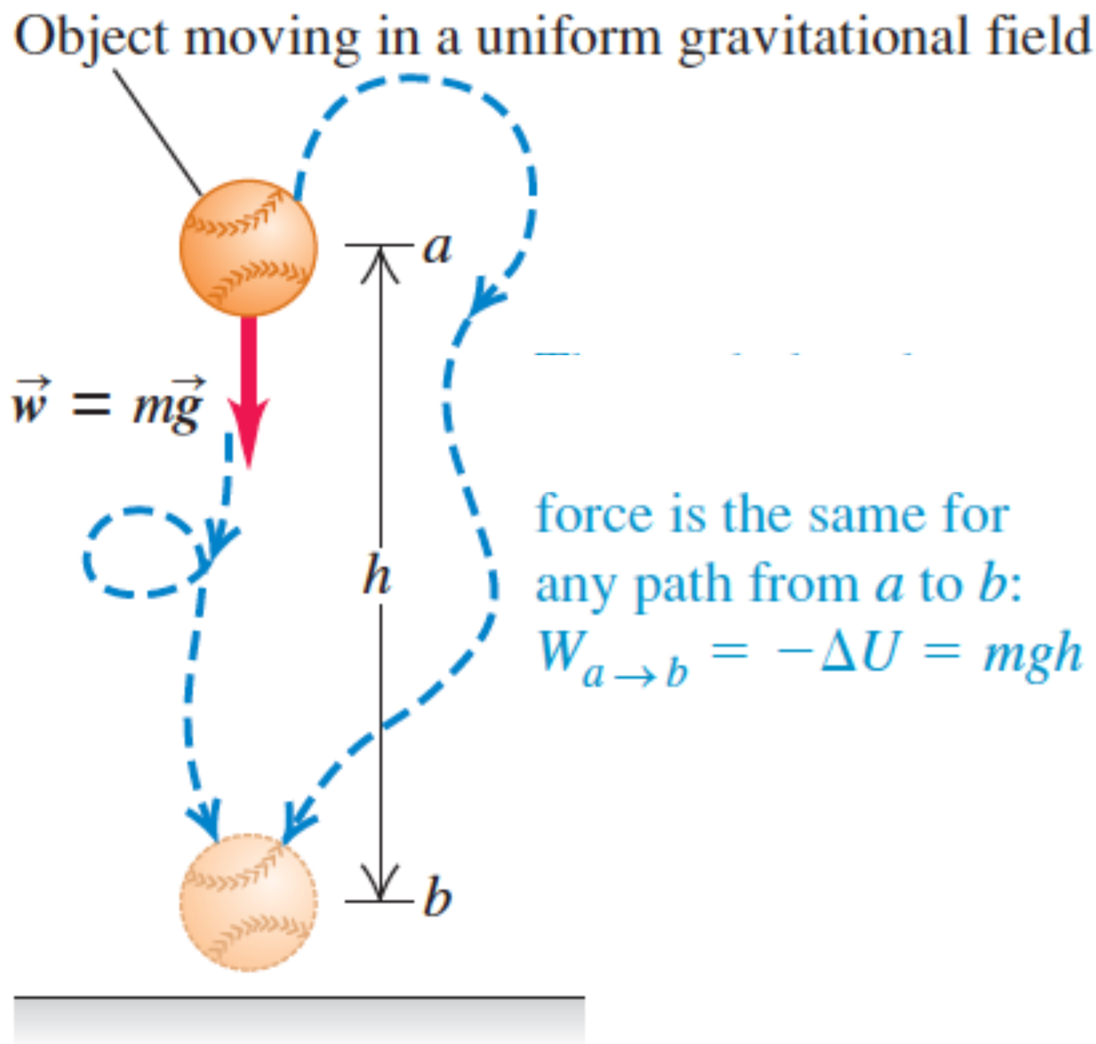
If the force **F** is conservative, the work done by this force can always be expressed in terms of a potential energy U.

$$\boxed{W_{a\to b} = U_a - U_b = -\Delta U}$$

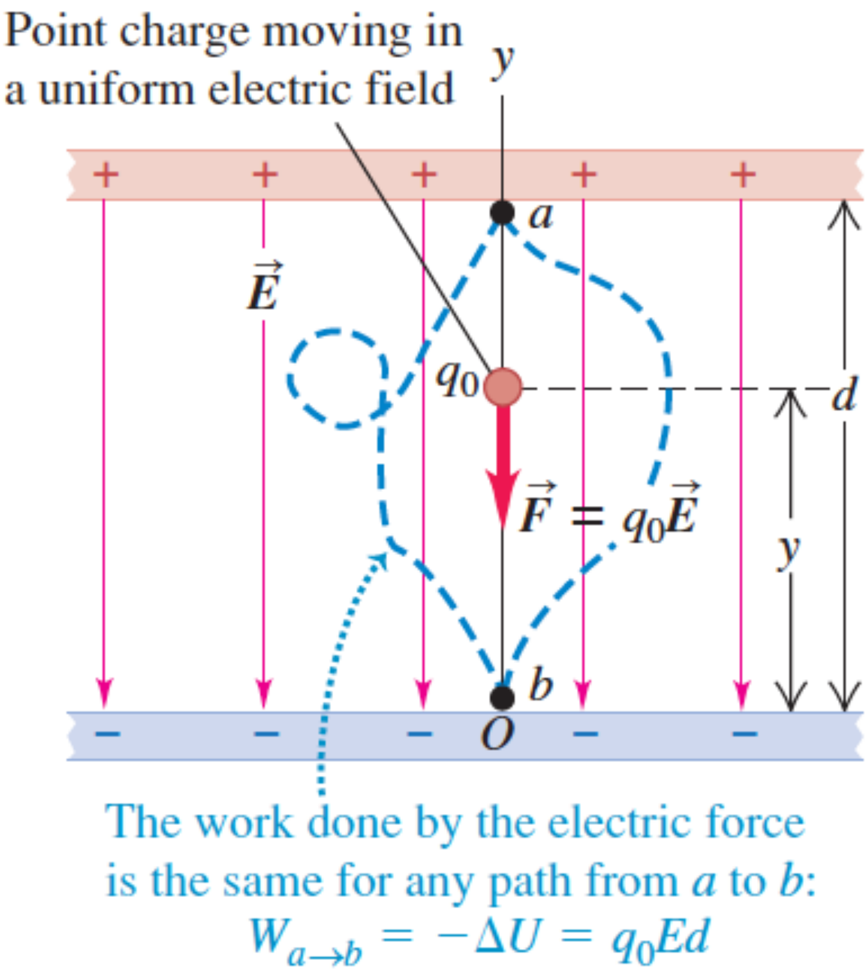
The work–energy theorem says that the change in kinetic energy during a displacement equals the total work done on the particle.

$$\boxed{K_a + U_a = K_b + U_b}$$

4.1 Potential Difference and Electric Potential



4.1 Potential Difference and Electric Potential

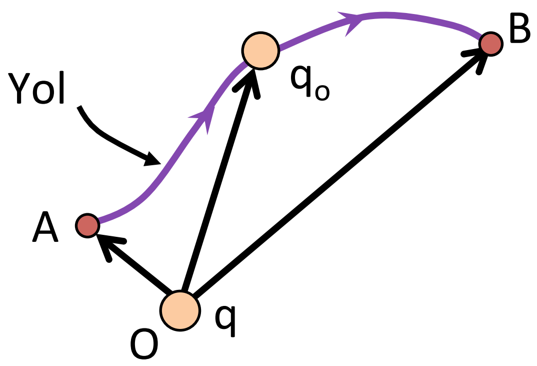


$$W_{a\to b} = -\Delta U = q_0 E d$$

4.1 Potential Difference and Electric Potential

Potansiyel enerji, q_o yükü
Hareket ettikçe değişir

Yükü bir A noktasından B noktasına taşıdığımızı düşünelim.
Bu durumda sistemin potansiyel enerjisindeki değişim



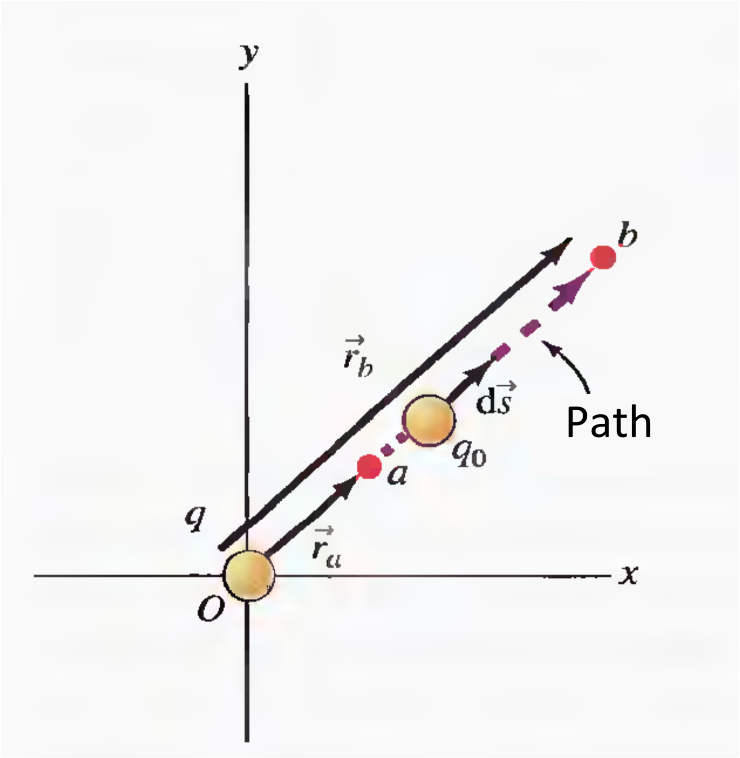
$$\Delta U = U_B - U_A$$

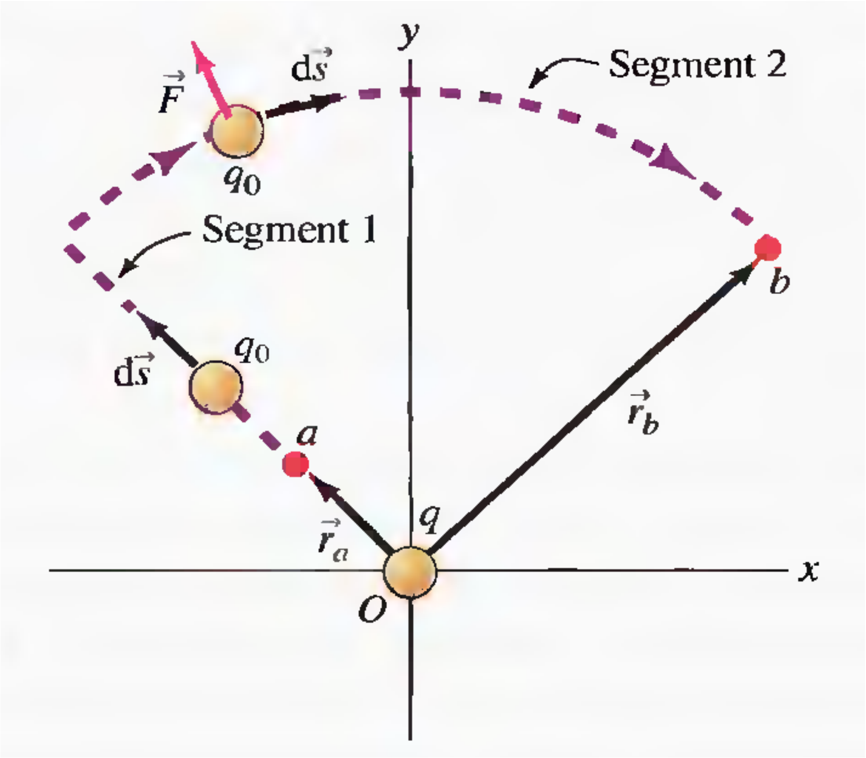
$$\Delta U = -q_o \int_A^B \vec{E} . d\vec{s}$$

$q_0\vec{E}$ korunumlu bir kuvvet olduğundan bu integral yoldan bağımsızdır !

4.1 Potential Difference and Electric Potential

$q_0\vec{E}$ korunumlu bir kuvvet olduğundan bu integral yoldan bağımsızdır !





4.1 Potential Difference and Electric Potential

Thus, the potential energy per unit charge, which can be symbolized as U /q , is independent of the charge q of the particle we happen to use and is characteristic only of the electric field we are investigating.The potential energy per unit charge at a point in an electric field is called the electric potential V (or simply the potential) at that point. Thus:

$$V = \frac{U}{q_o}$$

Electric potential is a scalar, not a vector.

The SI unit for potential is the joule per coulomb. This combination occurs so often that a special unit, the volt (abbreviated V), is used to represent it.Thus

$$1V = 1J / C$$

4.1 Potential Difference and Electric Potential

The electric potential difference V between any two points i and f in an electric field is equal to the difference in potential energy per unit charge between the two points:

$$\Delta V = V_f - V_i = \frac{\Delta U}{q} = \frac{U_f - U_i}{q}$$

Since ΔU=-W,

$$q\Delta V = -W$$

A potential difference can be positive, negative, or zero, depending on the signs and magnitudes of q and W .

The SI unit for potential is the joule per coulomb. This combination occurs so often that a special unit, the volt (abbreviated V), is used to represent it.Thus

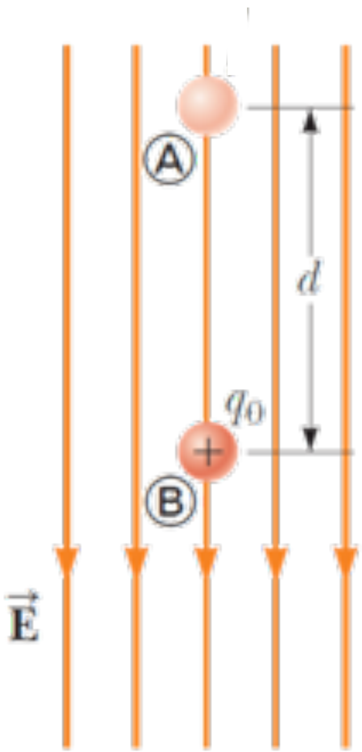
$$1V = 1J / C$$

This new unit allows us to adopt a more conventional unit for the electric field which we have measured up to now in newtons per coulomb. With two unit conversions, we obtain

$$1N / C = 1V / m$$

4.2 Potential Differences in a Uniform Electric Field

We want to calculate the potential differences btween the point A and B.



$$V_B - V_A = \Delta V = -\frac{W}{q}$$

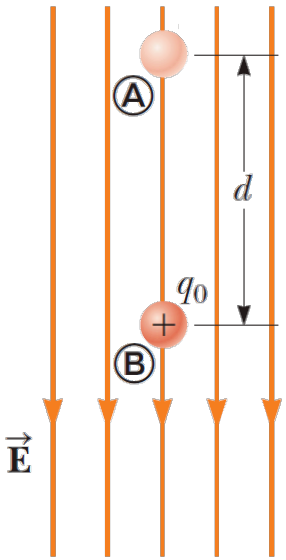
$$\Delta V = -\frac{qEd}{q}$$

$$\Delta V = -Ed$$

The negative sign indicates that the electric potential at point B is lower than at point A; that is, $V_B<V_A$

Electric field lines always point in the direction of decreasing electric potential.

24.2 Düzgün Bir Elektrik Alandaki Potansiyel Farkı



Şimdi bir q_o test yükünün A'dan B'ye hareket ettiğini düşünelim.
Yük-alan sisteminin potansiyel enerjisindeki değişim

$$\Delta U = q_0 \, \Delta V = -q_0 E d$$

Buna göre q_o pozitif ise elektrik alan yönünde hareket eden yük nedeniyle sistemin potansiyel enerjisi azalır. NEDEN?

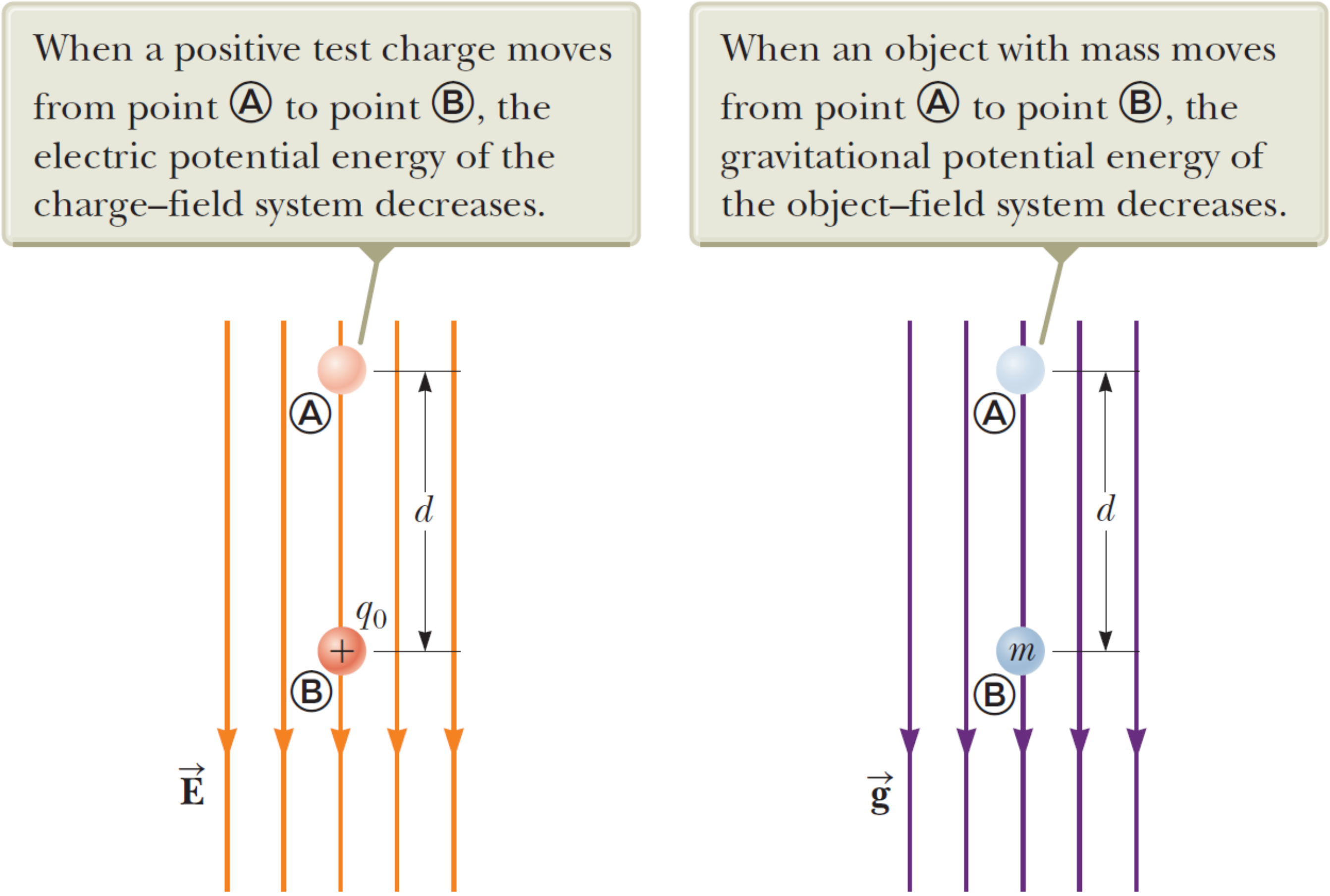
A noktasında pozitif q_o yükünü serbest bıraksak yük elektrik alan doğrultusunda hızlanır. Yani KE artar. Mekanik enerjinin korunumundan sistemin potansiyel enerjisinin azalması gerekir.

q_o negatif olsa ve bu yükü A'dan B'ye hareket ettirsek potansiyel enerji nasıl değişir ?

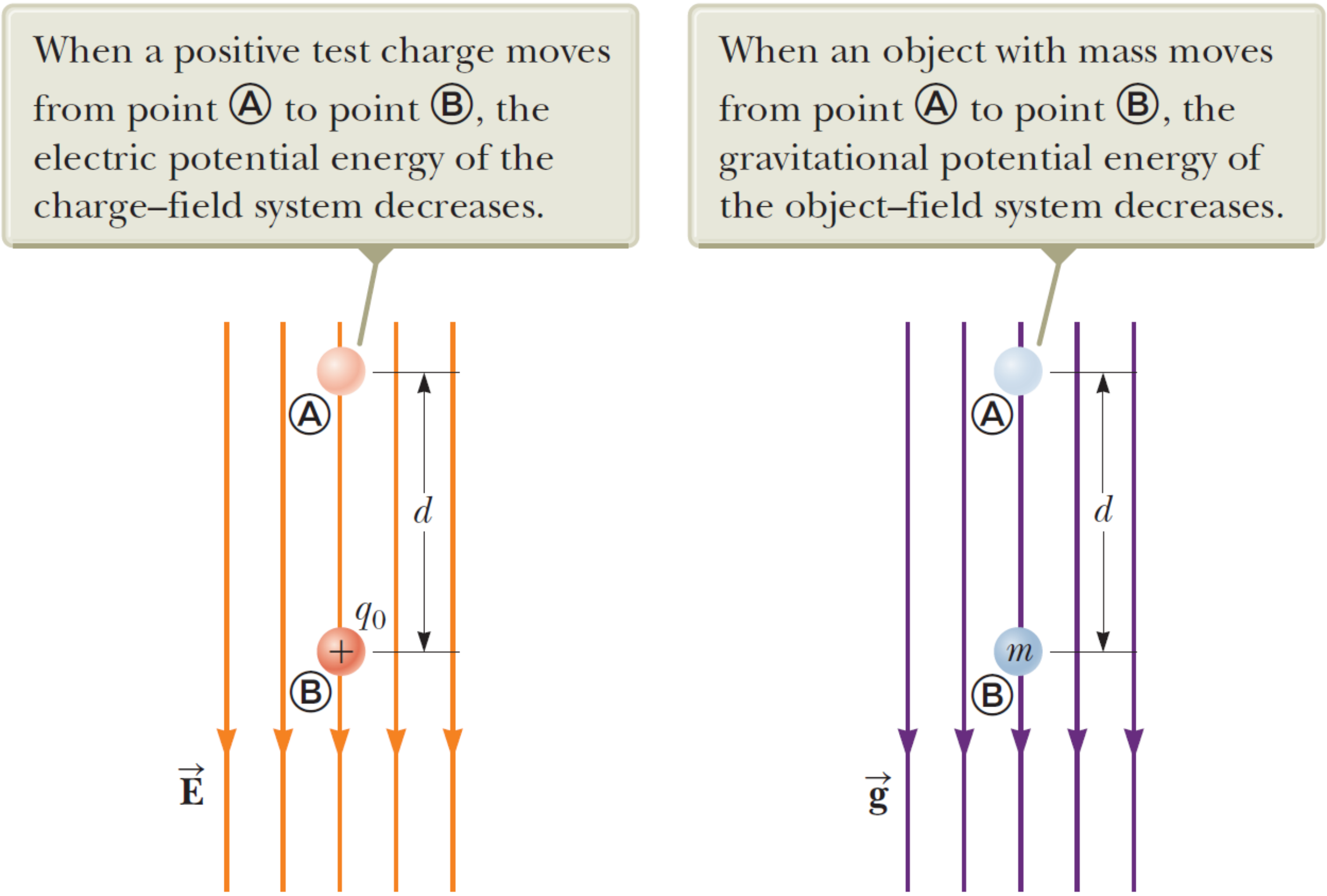
Şekildeki elektrik alan içinde negatif q_o yüküne alana zıt yönde bir kuvvet etki ettiğinden A'dan B'ye hareket ettirebilmek için dışarıdan bir iş yapmak gerekir. Bu durumda negatif bir yük elektrik alan çizgileri yönünde hareket ettirilirse sistemin potansiyel enerjisi artar.

A noktasında negatif yükü serbest bırakırsak yük elektrik alan çizgilerine ters yönde hareket eder ve yükün KE'si artar. Bu durumda negatif bir yük elektrik alan çizgilerine ters yönde hareket ederse sistemin potansiyel enerjisi azalır.

4.2 Potential Differences in a Uniform Electric Field

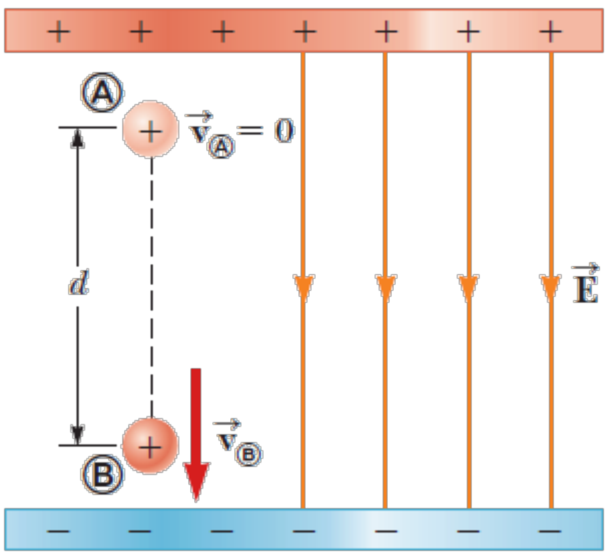


4.2 Potential Differences in a Uniform Electric Field



Example: Motion of a Proton in a Uniform Electric Field

A proton is released from rest in a uniform electric field that has a magnitude of 8×10^4 V/m. The proton undergoes a displacement of 0.50 m in the direction of E.

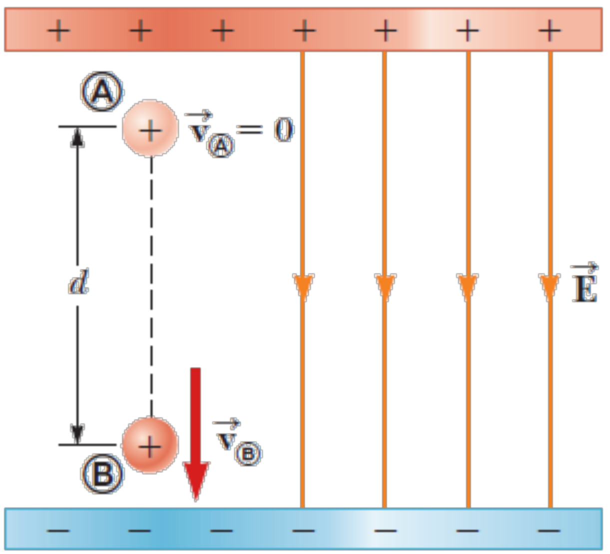


a) Find the change in electric potential between points A and B.

b) Find the change in potential energy of the proton–field system for this displacement.

c) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

Solution: Motion of a Proton in a Uniform Electric Field



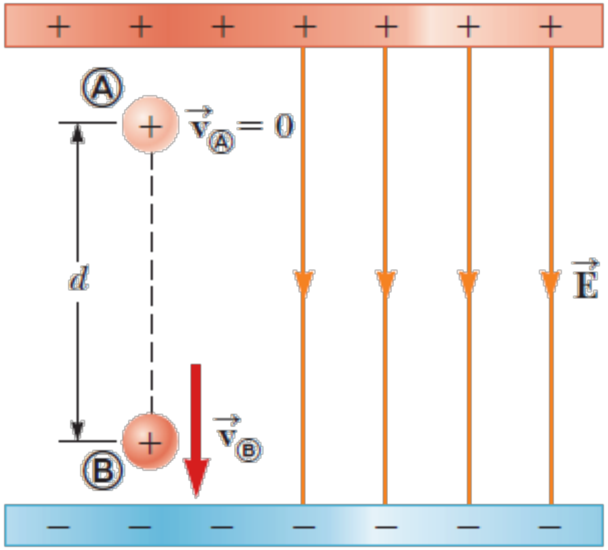
a) Find the change in electric potential between points A and B.

$$\Delta V = -Ed$$

$$\Delta V = -(8.0x10^4 V/m)(0.50m)$$

$$\Delta V = -4.0x10^4 V$$

Solution: Motion of a Proton in a Uniform Electric Field



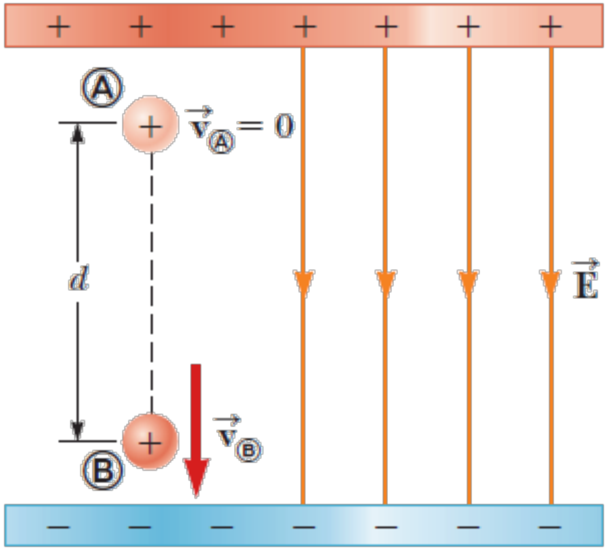
b) Find the change in potential energy of the proton–field system for this displacement.

$$\Delta U = -q\Delta V$$

$$\Delta U = -(1.6x10^{-19}C)(-4.0x10^{4}V)$$

$$\Delta U = -6.4x10^{-15}J$$

Solution: Motion of a Proton in a Uniform Electric Field



c) Find the change in potential energy of the proton–field system for this displacement.

$$\Delta K + \Delta U = 0$$

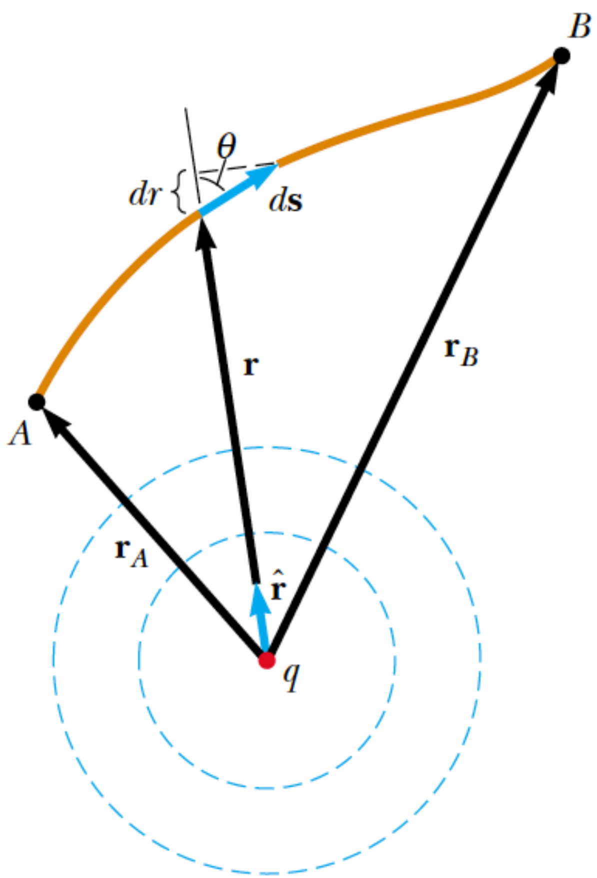
$$(K_f - K_i) + q\Delta V = 0$$

$$(\frac{1}{2}mv^2 - 0) + q\Delta V = 0$$

$$v = \sqrt{\frac{-(2q\Delta V)}{m}} = 2.8x10^6 m/s$$

4.3 Electric Potential and Potential Energy Due to Point Charges

The potential difference V_b- V_a between any two points i and f in an electric field



$$V_B - V_A = -\int_a^b \vec{E}.d\vec{s}$$

By using the electric field due to a point charge:

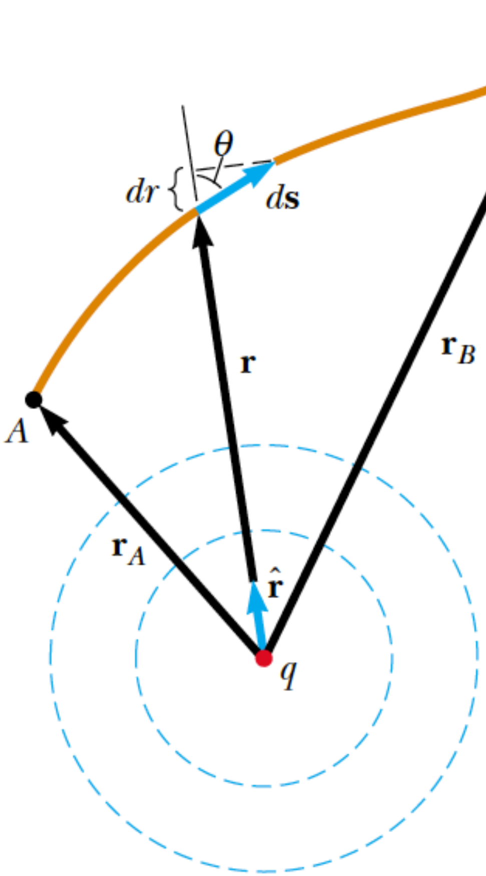
$$\vec{E}.d\vec{s} = k\frac{q}{r^2}\hat{r}.d\vec{s}$$

$$\hat{r}.d\vec{s} = ds\cos\theta$$

$$ds\cos\theta = dr$$

Any displacement ds along the path from point A to point B produces a change dr in the magnitude of r, the position vector of the point relative to the charge creating the field.

4.3 Electric Potential and Potential Energy Due to Point Charges

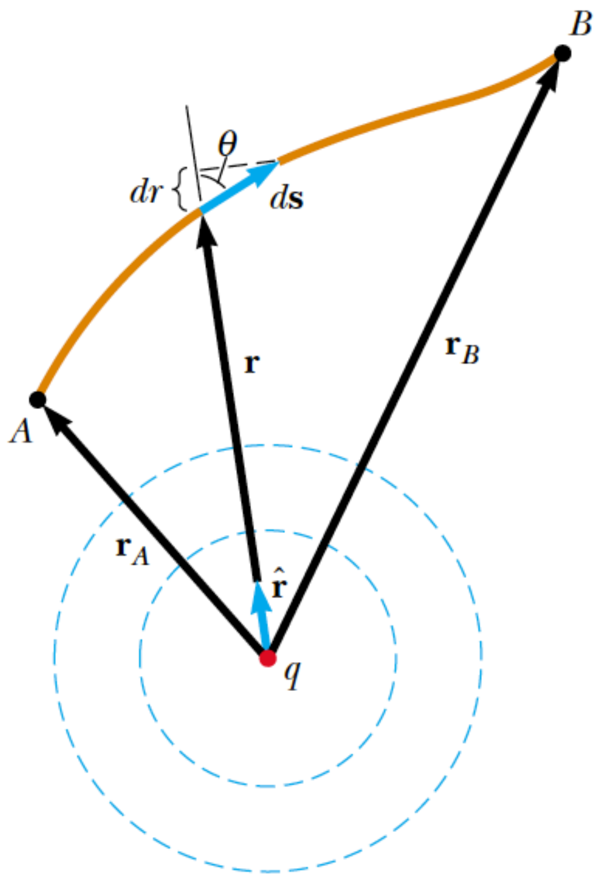


$$V_B - V_A = -\int_a^b \vec{E}.d\vec{s}$$

$$V_B - V_A = -kq\int_{r_a}^{r_b} \frac{dr}{r^2} = k\frac{q}{r}\Bigg|_{r_a}^{r_b}$$

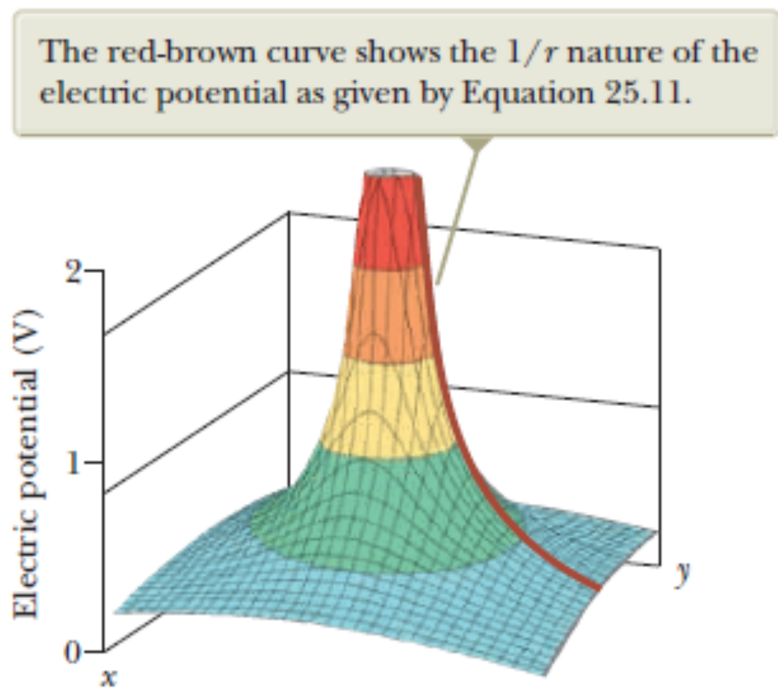
$$V_B - V_A = kq\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

4.3 Electric Potential and Potential Energy Due to Point Charges



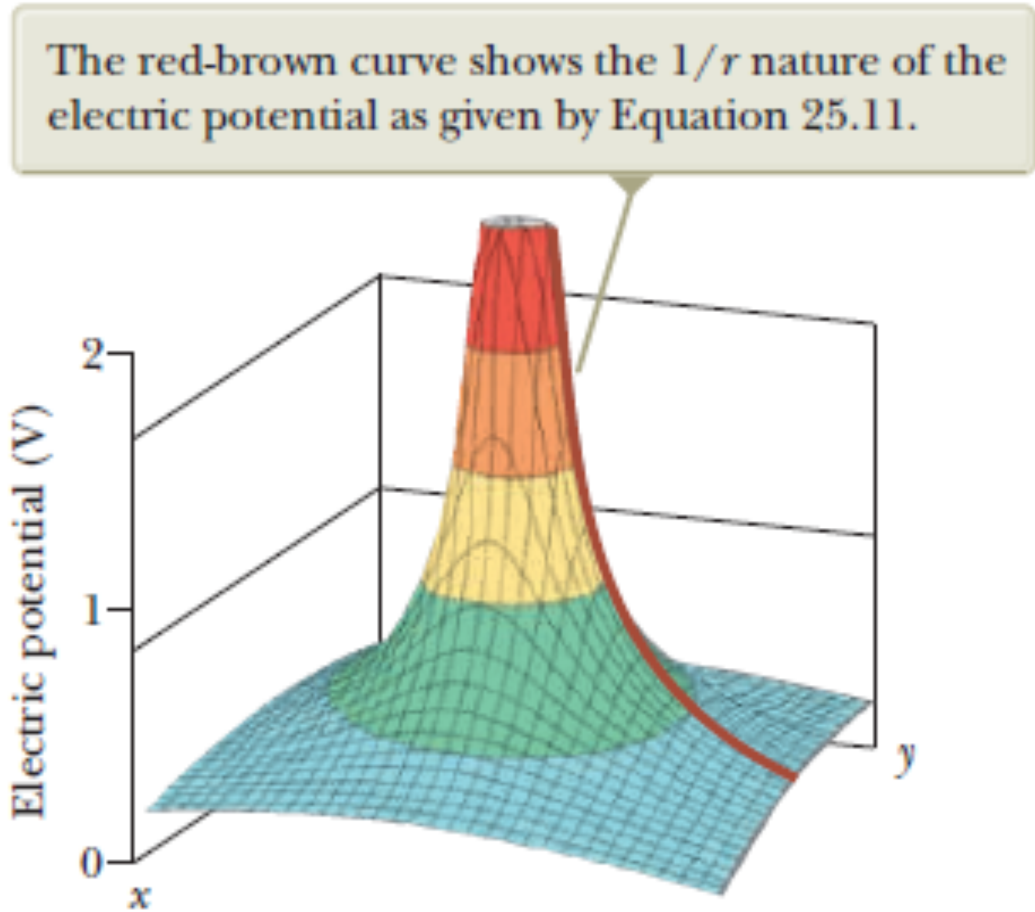
It is customary to choose the reference of electric potential for a point charge to be V = 0 at $r_a=\infty$

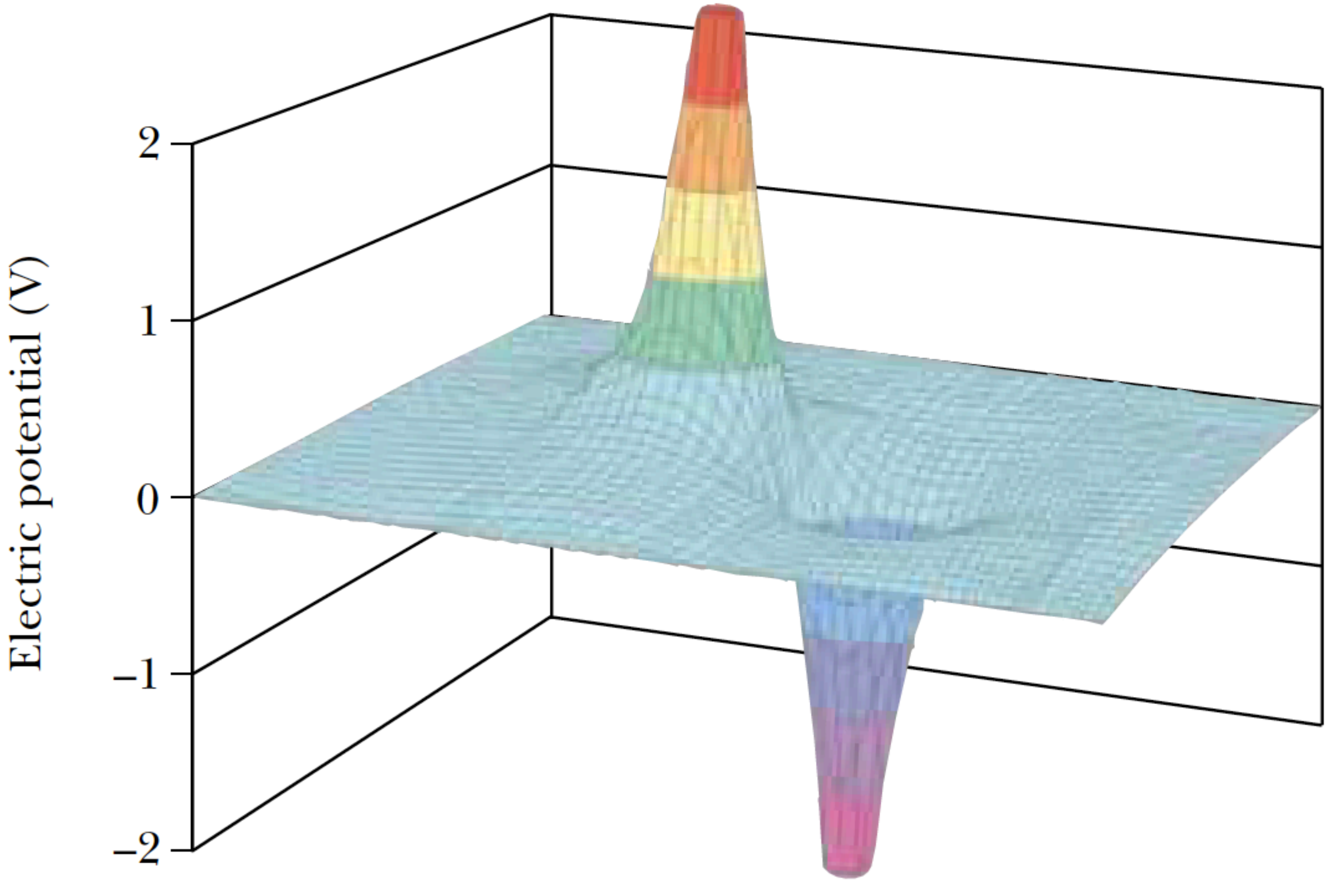
$$V = k\frac{q}{r}$$



4.3 Electric Potential and Potential Energy Due to Point Charges

$$V = k\frac{q}{r}$$




2
1
0
–1
–2
Electric potential (V)

4.3 Electric Potential and Potential Energy Due to Point Charges

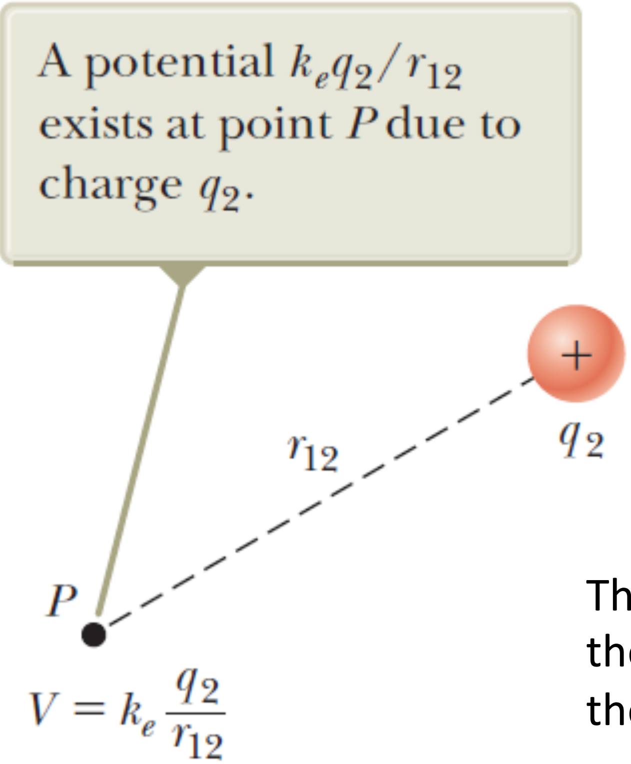
The electric potential resulting from two or more point charges can b obtained by applying the superposition principle. the total electric potential at some point P due to several point charges is the sum of the potentials due to the individual charges.

$$V = k\sum_i \frac{q_i}{r_i}$$

The potential is again taken to be zero at infinity and r_i is the distance from the point P to the charge q_i.

Note that the sum in this equation is an algebraic sum of scalars rather than a vector sum (which we use to calculate the electric field of a group of charges). Thus, it is often much easier to evaluate V than to evaluate E

4.3 Electric Potential and Potential Energy Due to Point Charges



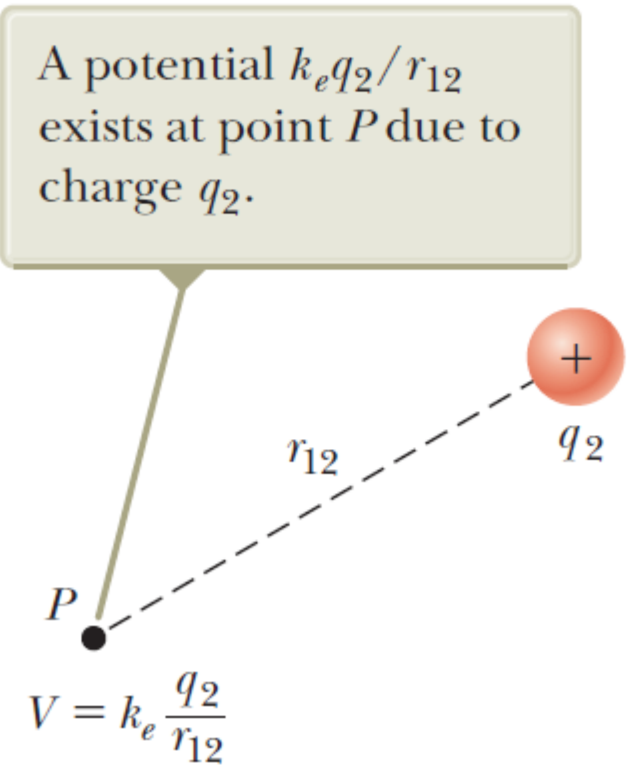
If V_2 is the electric potential at a point P due to charge q_2, then the work an external agent must do to bring a second charge q_1 from infinity to P without acceleration is :

$$q_1 V_2$$

This work represents a transfer of energy into the system and the energy appears in the system as potential energy U when the particles are separated by a distance r_{12}:

$$U = k\frac{q_1 q_2}{r_{12}}$$

4.3 Electric Potential and Potential Energy Due to Point Charges

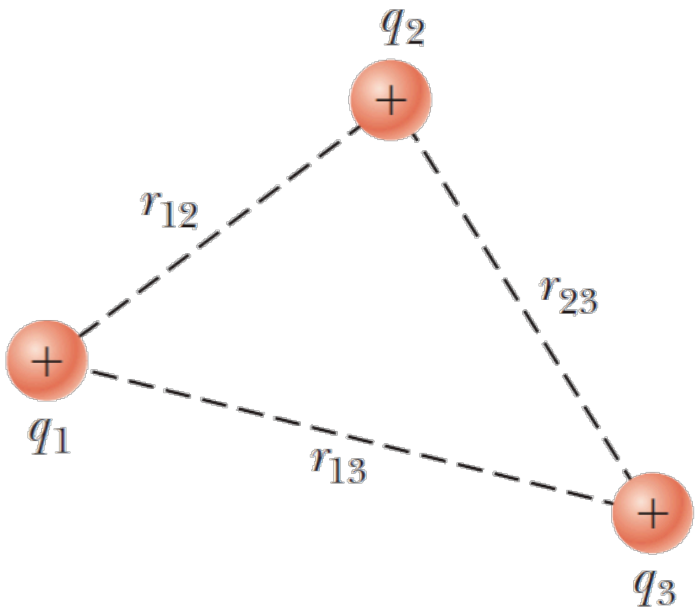


$$U = k\frac{q_1 q_2}{r_{12}}$$

if the charges are of the same sign, U is positive.

If the charges are of opposite sign, U is negative

4.3 Electric Potential and Potential Energy Due to Point Charges

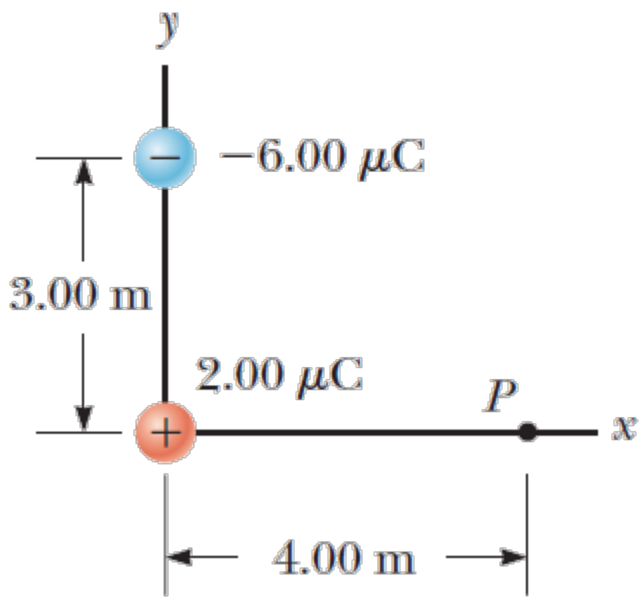


If the system consists of more than two charged particles, we can obtain the total potential energy by calculating U for every pair of charges and summing the terms algebraically.

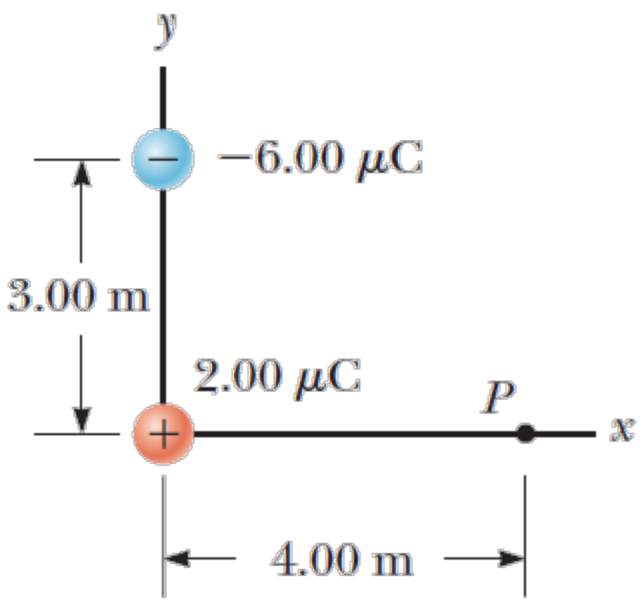
$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Example: The Electric Potential Due to Two Point Charges

Find the total electric potential due to charges in Figure at the point P, whose coordinates are (4.00, 0) m.



Solution: The Electric Potential Due to Two Point Charges



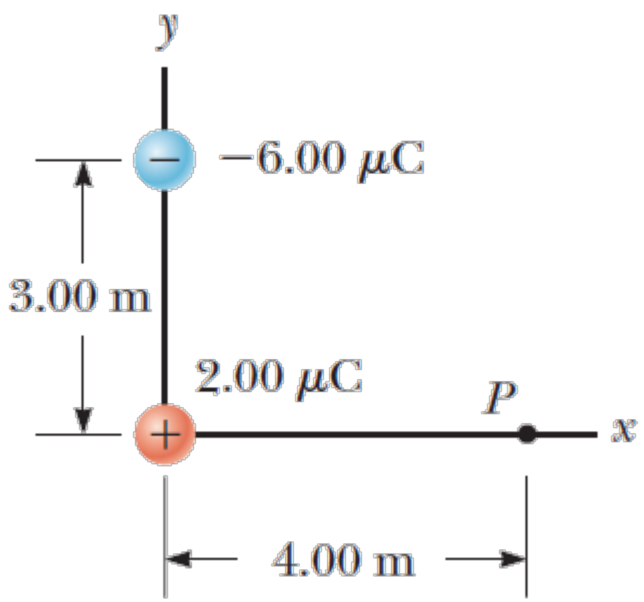
$$V_p = k\sum_i \frac{q_i}{r_i}$$

$$V_p = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

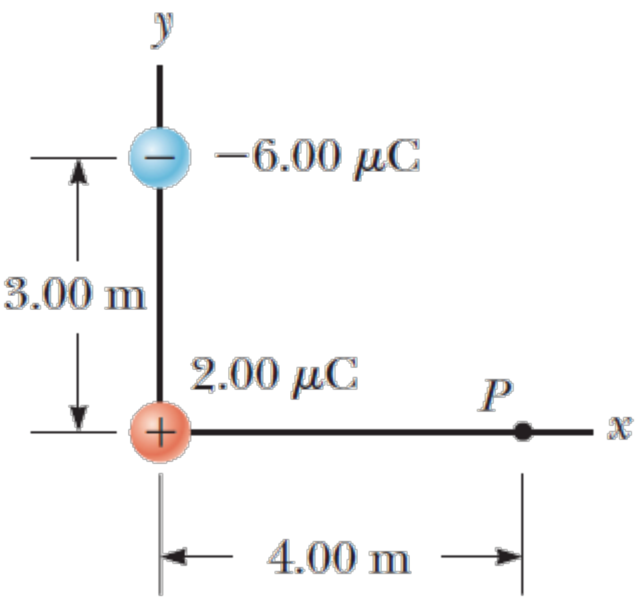
$$V_p = -6.3x10^3 V$$

Example: The Electric Potential Due to Two Point Charges

Find the change in potential energy of the system of two charges plus a charge q_3 = 3 μC as the latter charge moves from infinity to point P.



Solution: The Electric Potential Due to Two Point Charges



The work done by external agent to bring the point charge q_3 from infinity to point P is equal to change in potential eergy:

$$\Delta U = q_3 V_P - 0 = -1.89x10^{-2}\,J$$

Example: Nuclear force

Find the potential energy of two protons in nucleus, which have a distance of $2x10^{-15}$ m each other.

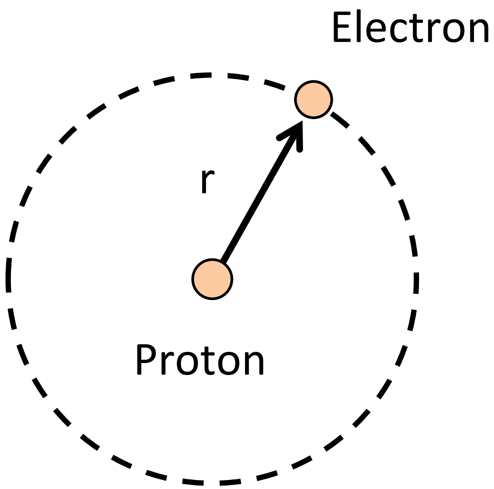
$$U = k\frac{e^2}{r} = 10^{-13} J$$

Example: Hydrogen Atom

The distance between the proton end electron in hydrogen atom is 5.3×10^{-11} m.

a) Find the electric potential due to proton at position of electron
b) Calculate the potential energy of system

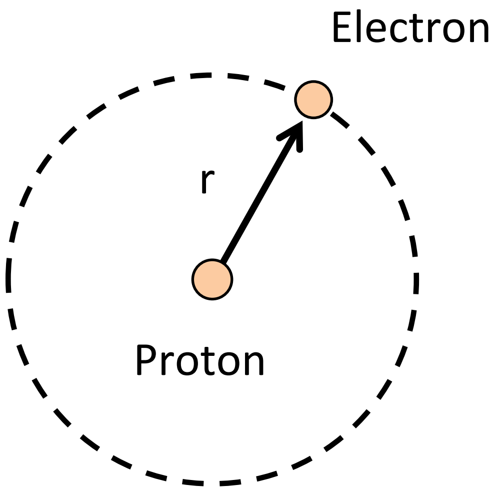
Solution: Hydrogen Atom



a) The electric potential due to proton at position of electron

$$V_p = k\frac{q}{r} = 27V$$

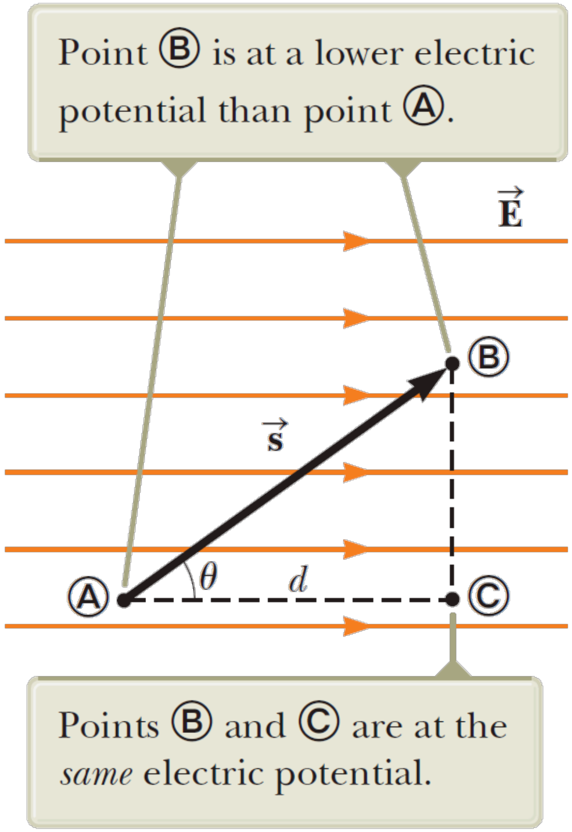
Solution: Hydrogen Atom



b) The potential energy of system

$$U = -eV_p = -4.3x10^{-18}J$$

4.4 Equipotential Surfaces



A charged particle that moves between A and B in a uniform electric field as shon in Figure, then we obtain

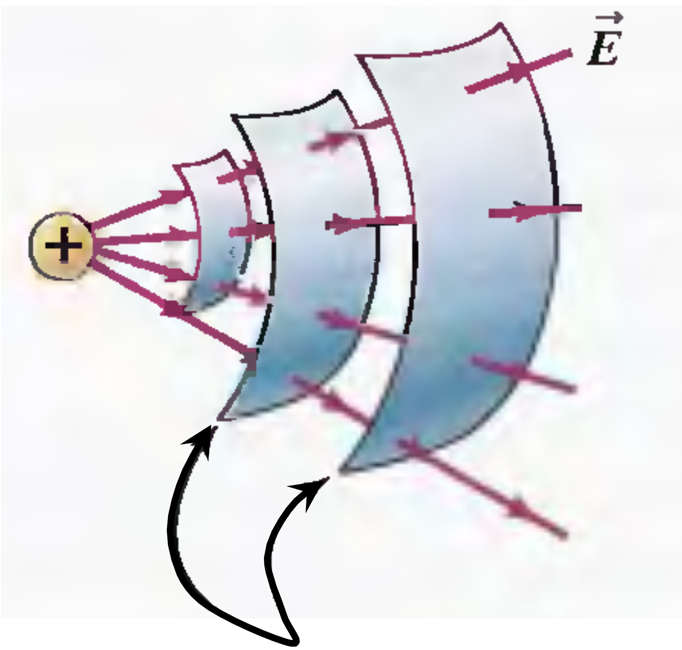
$$\Delta V = -\int_a^b \vec{E}.d\vec{s} = -\vec{E}.\int_a^b d\vec{s} = -\vec{E}.\vec{s}$$

All points in a plane perpendicular to a uniform electric field are at the same electric potential.

$$\vec{E}.\vec{s}_{A\to B} = \vec{E}.\vec{s}_{A\to C}$$

Therefore the point B and C are in same potential: $V_B=V_C$.

4.4 Equipotential Surfaces



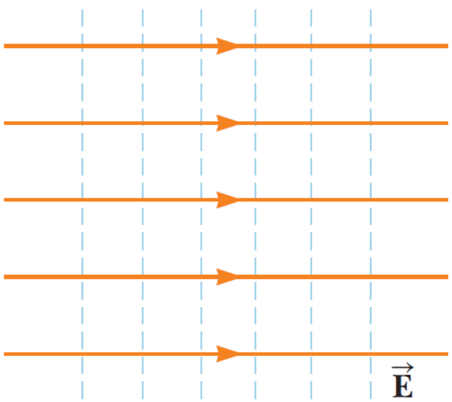
equipotential surfaces

The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.

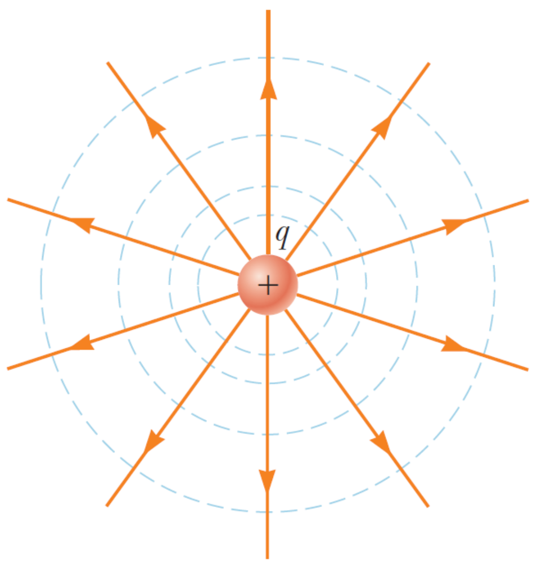
Equipotential surfaces must always be perpendicular to the electric field lines passing through them.

4.4 Equipotential Surfaces

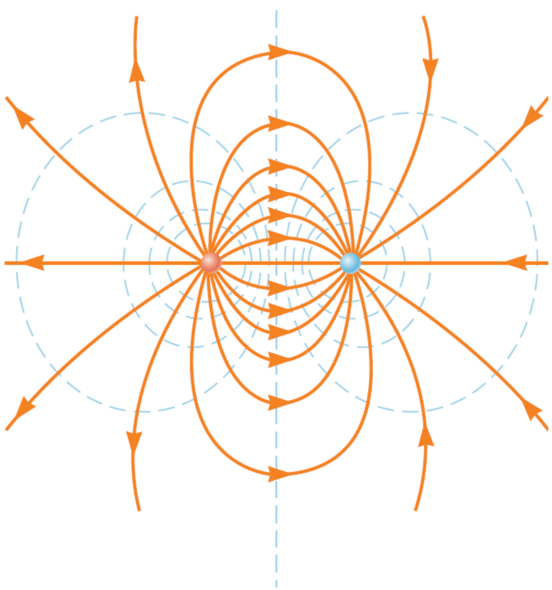
A uniform electric field produced by an infinite sheet of charge



A spherically symmetric electric field produced by a point charge



An electric field produced by an electric dipole



4.5 Obtaining the Value of the Electric Field from the Electric Potential

For a uniform electric field, the potential difference dV between two points a distance ds apart as

$$dV = -\vec{E}.d\vec{s}$$

If the electric field has only one component It's say x-component, then

$$\vec{E}.d\vec{s} = E_x dx$$

So the x-component of electric field can be calculated as:

$$E_x = -\frac{dV}{dx}$$

That is, the x component of the electric field is equal to the negative of the derivative of the electric potential with respect to x.

Similar statements can be made about the y and z components.

4.5 Obtaining the Value of the Electric Field from the Electric Potential

In general, the electric potential is a function of all three spatial coordinates. In Cartesian coordiates

$$E_x = -\frac{dV}{dx} \qquad E_y = -\frac{dV}{dy} \qquad E_z = -\frac{dV}{dz}$$

This is the mathematical statement of the fact that the electric field is a measure of the rate of change with position of the electric potential

4.5 Obtaining the Value of the Electric Field from the Electric Potential

If the charge distribution creating an electric field has spherical symmetry

$$\vec{E}.d\vec{s} = E_r dr$$

so the electric field can be calculated as:

$$E_r = -\frac{dV}{dr}$$

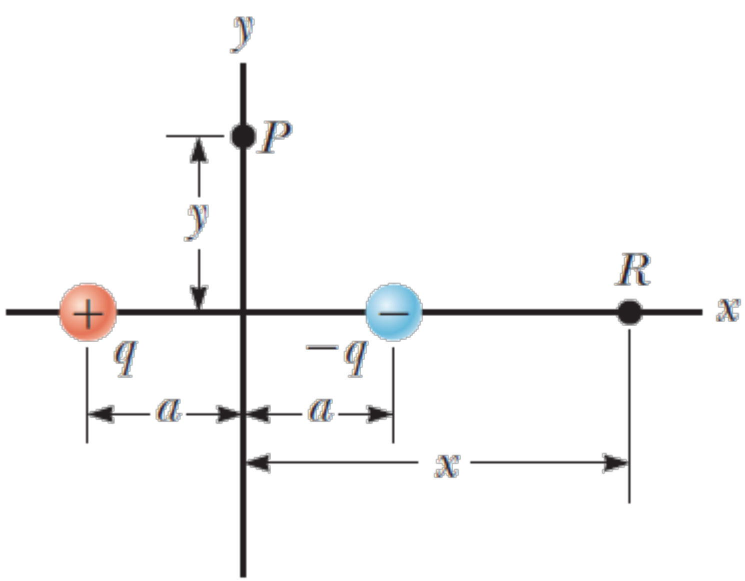
Homework: The Electric Potential Due to a Dipole

An electric dipole is along the x axis and is centered at the origin as shon in Figure.

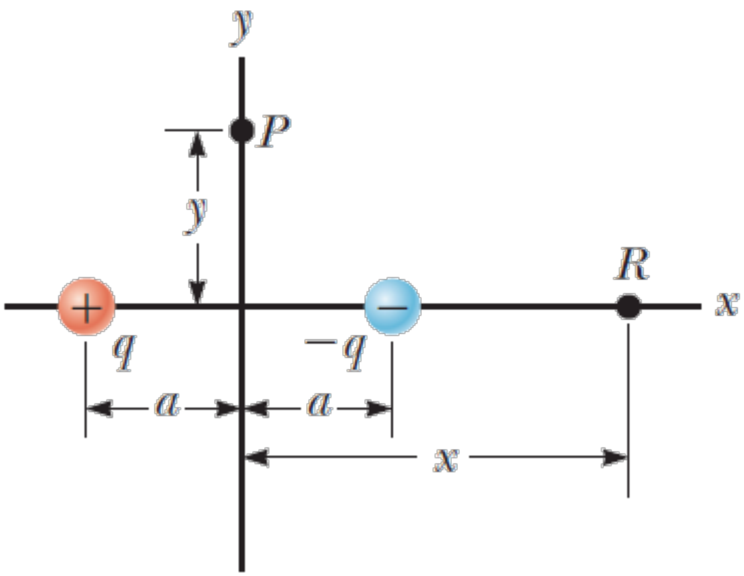
a) Calculate the electric potential at point P.

b) Calculate the electric potential at point R.

c) Calculate V and E_x at a point far from the dipole..



Solution of HM: The Electric Potential Due to a Dipole



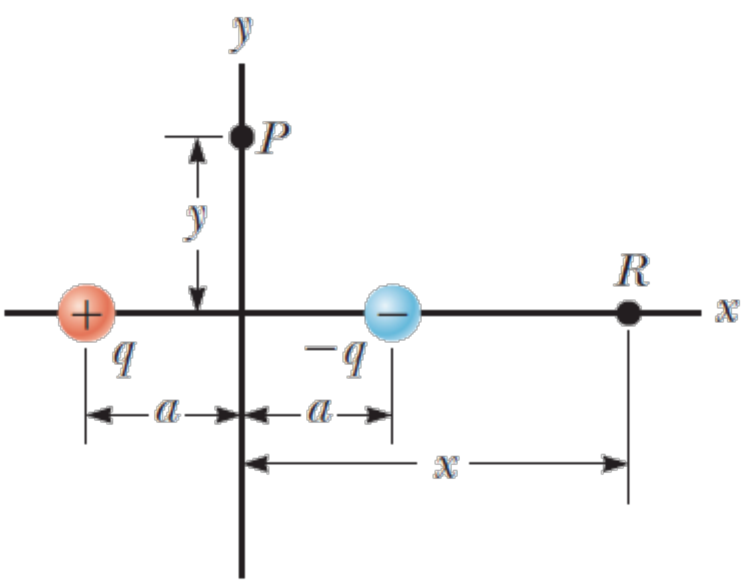
a) The electric potential at point P

$$V_P = k_e \sum_i \frac{q_i}{r_i}$$

$$V_P = k_e\left(\frac{q}{\sqrt{a^2 + y^2}} + \frac{-q}{\sqrt{a^2 + y^2}}\right) = \boxed{0}$$

Solution of HM: The Electric Potential Due to a Dipole

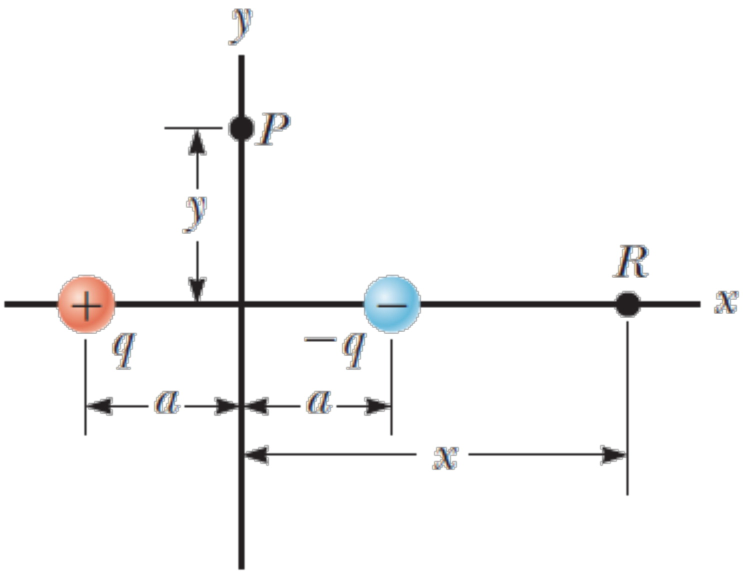
b) The electric potential at point R



$$V_R = k_e \sum_i \frac{q_i}{r_i}$$

$$V_R = k_e\left(\frac{-q}{x-a} + \frac{q}{x+a}\right) = \boxed{-\frac{2k_e qa}{x^2 - a^2}}$$

Solution of HM: The Electric Potential Due to a Dipole

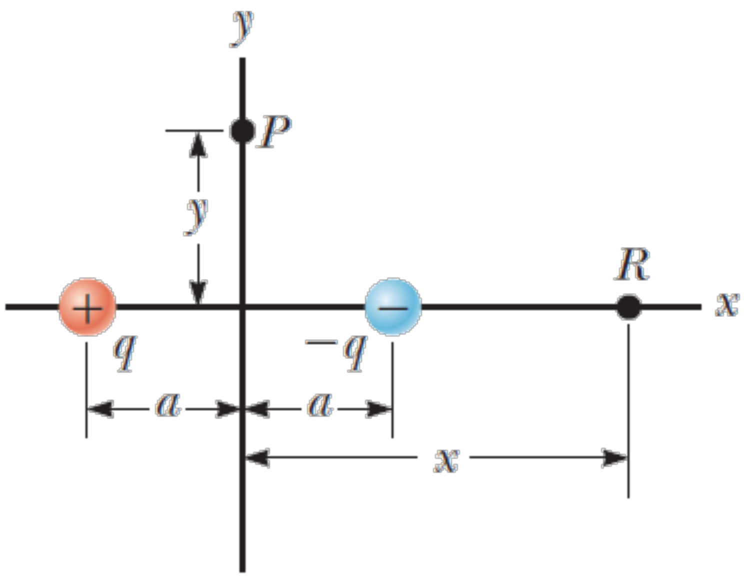


c) The electric potential for x>>a

$$V_R = \lim_{x>>a}\left(-\frac{2k_e qa}{x^2 - a^2}\right)$$

$$V_R \approx \boxed{-\frac{2k_e qa}{x^2} \quad (x >> a)}$$

Solution of HM: The Electric Potential Due to a Dipole



c) The electric field for x>>a

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qa}{x^2}\right)$$

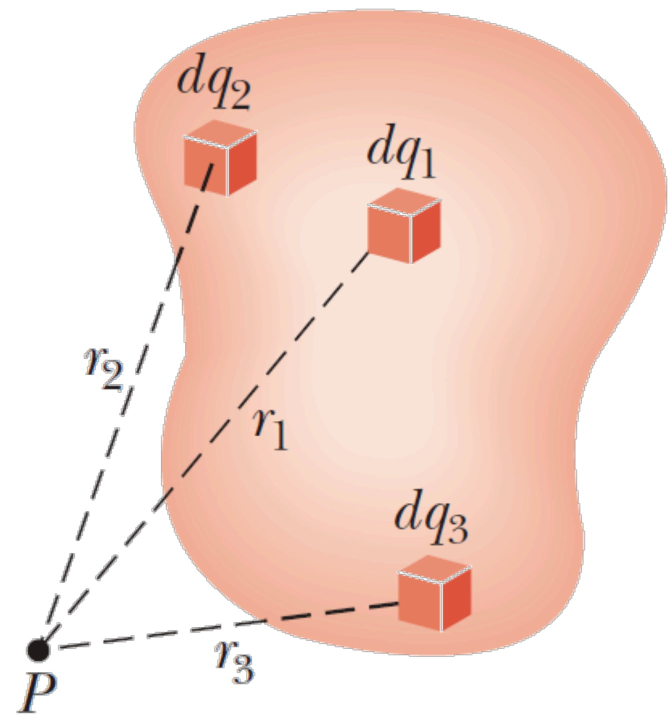
$$= 2k_e qa \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= \boxed{-\frac{4k_e qa}{x^3} \quad (x >> a)}$$

4.6 Electric Potential Due to Continuous Charge Distributions

The electric potential dV at some point P due to the charge element dq is

$$dV = k\frac{dq}{r}$$

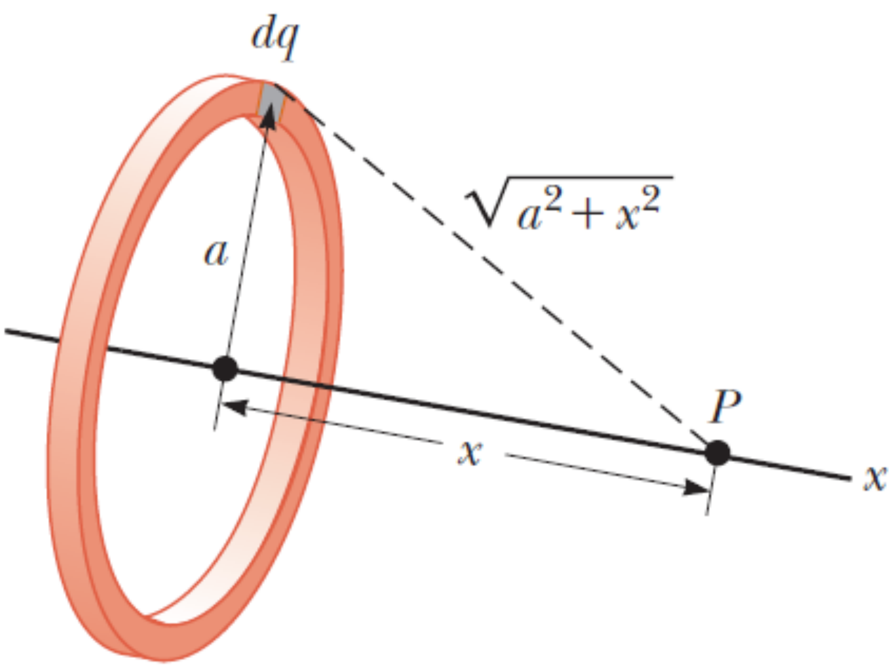


To obtain the total potential at point P, we integrate this Equation to include contributions from all elements of the charge distribution.

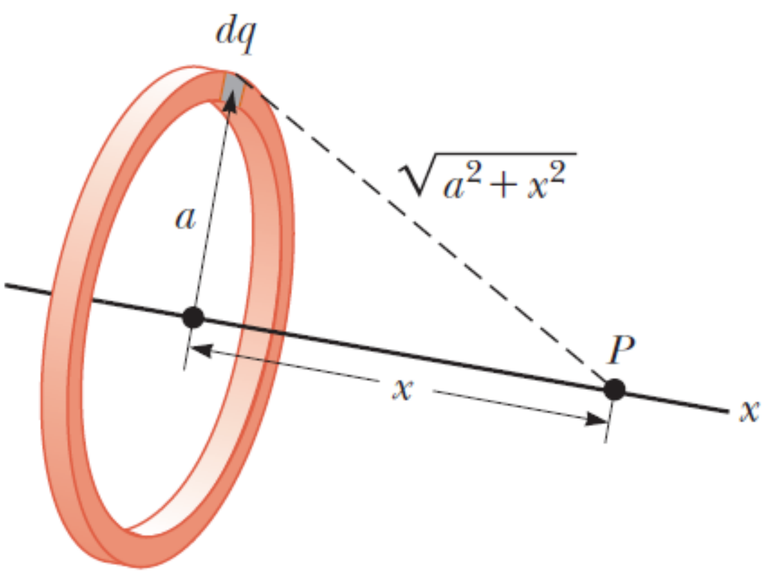
$$V = k\int\frac{dq}{r}$$

Example: Electric Potential Due to a Uniformly Charged Ring

Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.



Solution: Electric Potential Due to a Uniformly Charged Ring



The electric potential of the ring, consist of continuous distribution of charge eleements, at point P

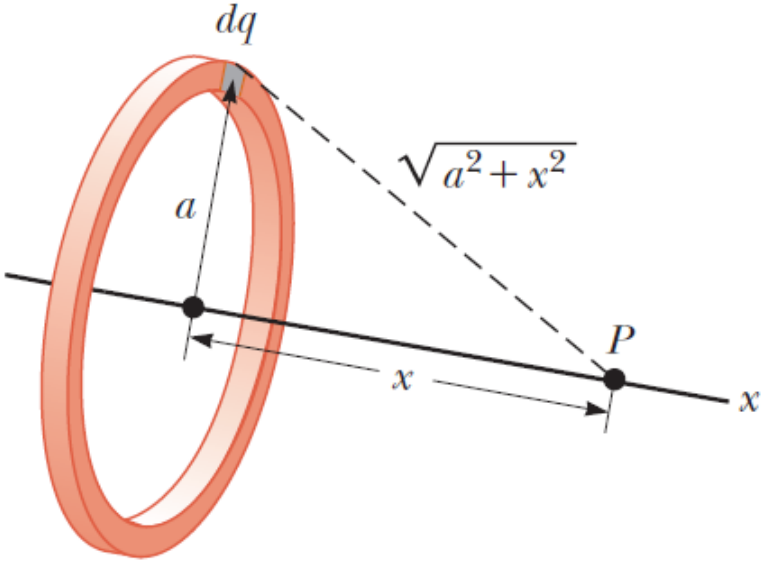
$$V = k\int \frac{dq}{r}$$

$$V = k\int \frac{dq}{\sqrt{a^2+x^2}}$$

$$V = k\frac{Q}{\sqrt{a^2+x^2}}$$

4.6 Electric Potential Due to Continuous Charge Distributions

By using the electric potential, we can find an expression for the magnitude of the electric field at point P..

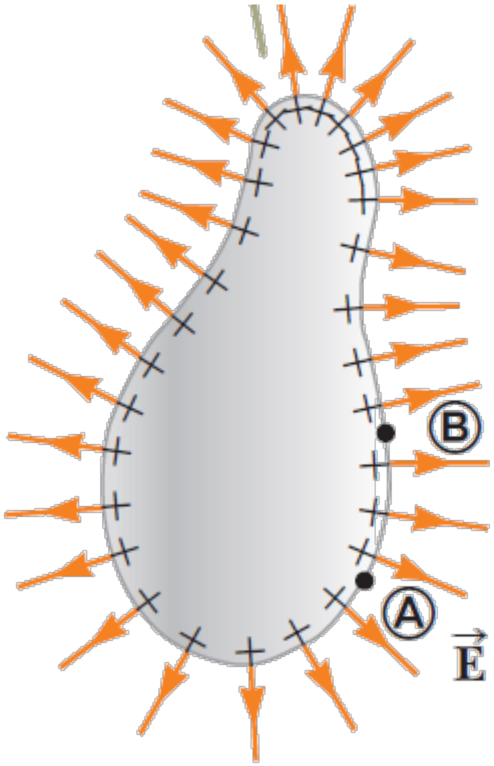


$$E_x = -\frac{dV}{dx} = kQ\frac{d}{dx}\left(\frac{1}{\sqrt{a^2+x^2}}\right)$$

$$E_x = -kQ\left(-\frac{1}{2}\right)(a^2+x^2)^{-3/2}(2x)$$

$$E_x = \frac{kx}{(a^2+x^2)^{3/2}}Q$$

4.7 Electric Potential Due to a Charged Conductor



Consider two points A and B on the surface of a charged conductor, as shown in Figure. The potential difference between A and B is given by:

$$V_B - V_A = \Delta V = -\int_a^b \vec{E}.d\vec{s}$$

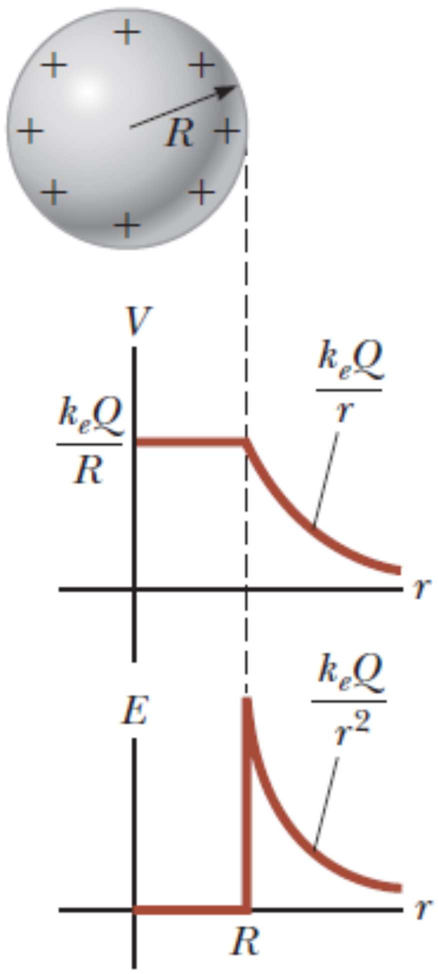
Since the electric field at any point on the surface is perpendicular to the surface:

$$\vec{E}.d\vec{s} = 0$$

Therefore, the potential difference between A and B is necessarily zero:

$$\Delta V = -\int_a^b \vec{E}.d\vec{s} = 0$$

4.7 Electric Potential Due to a Charged Conductor

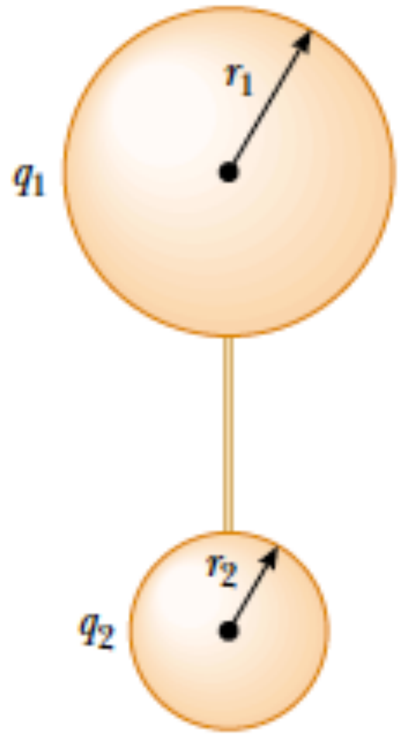


$$\Delta V = -\int_a^b \vec{E}.d\vec{s} = 0$$

The surface of any charged conductor in electrostatic equilibrium is an equipotential surface. Furthermore, because the electric field is zero inside the conductor, we conclude that the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

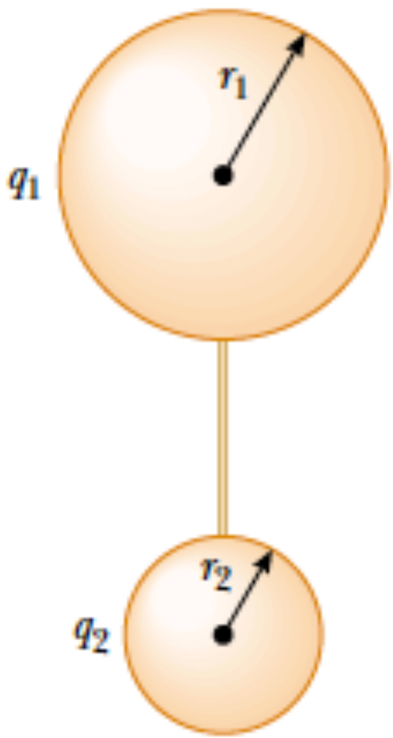
Example: Two Connected Charged Spheres

Two spherical conductors of radii r_1 and r_2 are separated by a distance much greater than the radius of either sphere. The spheres are connected by a conducting wire, as shown in Figure. The charges on the spheres in equilibrium are q_1 and q_2, respectively, and they are uniformly charged. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres.



4.7 Electric Potential Due to a Charged Conductor

Solution: Two Connected Charged Spheres



Because the spheres are connected by a conducting wire, they must both be at the same electric potential

$$V = k\frac{q_1}{r_1} = k\frac{q_2}{r_2} \Rightarrow \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

Because the spheres are very far apart and their surfaces uniformly charged, we can express the magnitude of the electric fields at their surfaces as

$$E_1 = k\frac{q_1}{r_1^2} \qquad E_2 = k\frac{q_1}{r_2^2}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

4.8 Application of Electrostatic