ELE 321
Linear System Analysis

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Signals and Systems

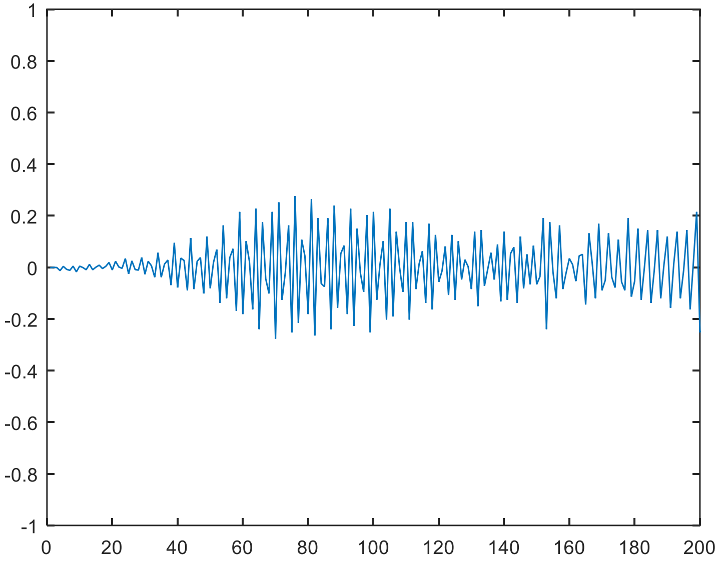
ELE321 Linear System Analysis

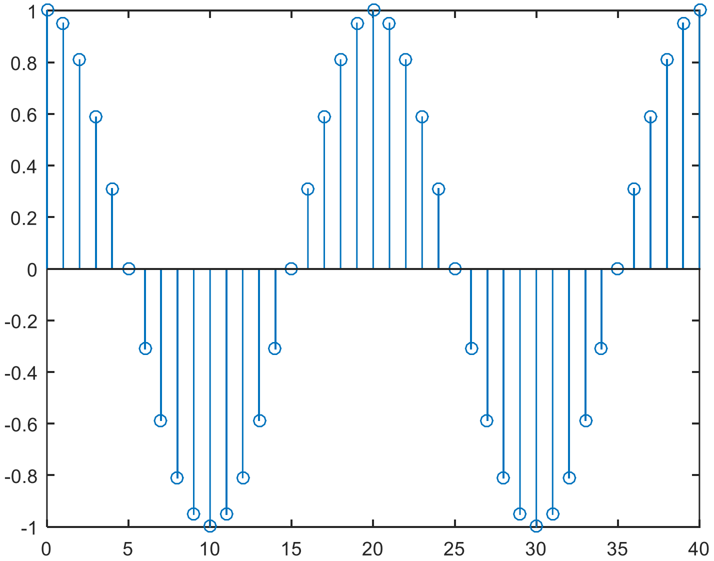
Lecture 1

Agenda

- Signals
- Energy
- Power
- Important classes of signals

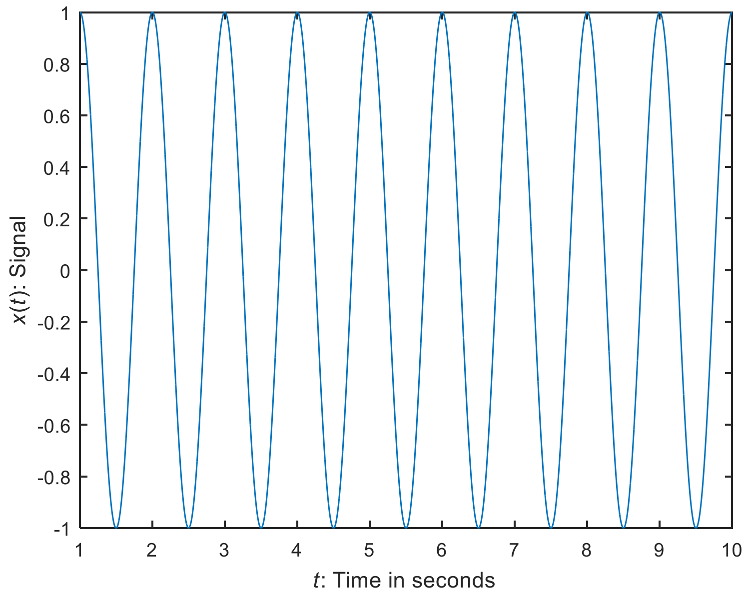
Signal is a gesture, sound, action or a perceptional phenomenon, which conveys information or instruction. In electronics, signal is electrical form or electromagnetic field that is used to contain or convey data from one place to another one.



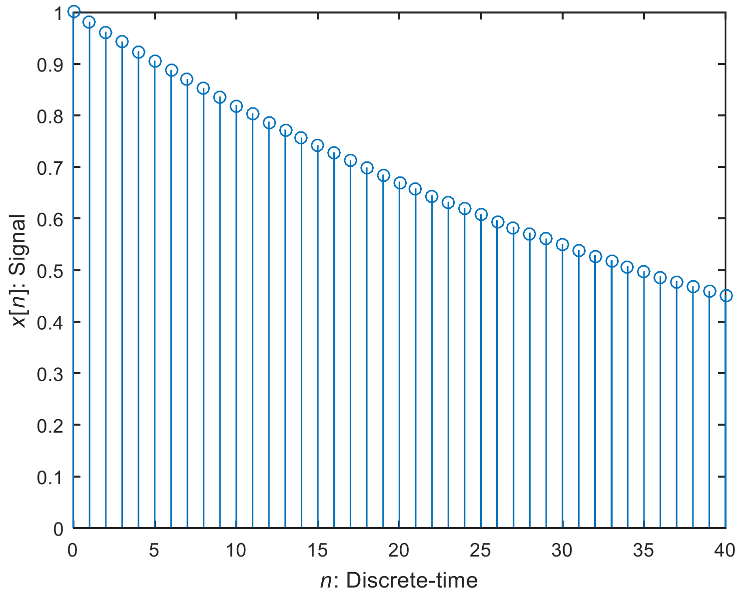


- Signals are represented in mathematical functions of one or more independent variables.
  - A music signal can be represented as a function with one independent variable, time. Dependent variable is amplitude of waveform of the music signal.
  - A digital photograph can be represented as a function with two independent variables, horizontal and vertical spatial coordinates. Dependent variable is pixel value of the photograph at spatial coordinates.
  - In the concept of this course, signals with one independent variable are considered; continuous time t for countinuous-time signals $x(t)$ and discrete time n for discrete-time signals $x[n]$.

Continuous-time signal: $x(t)$

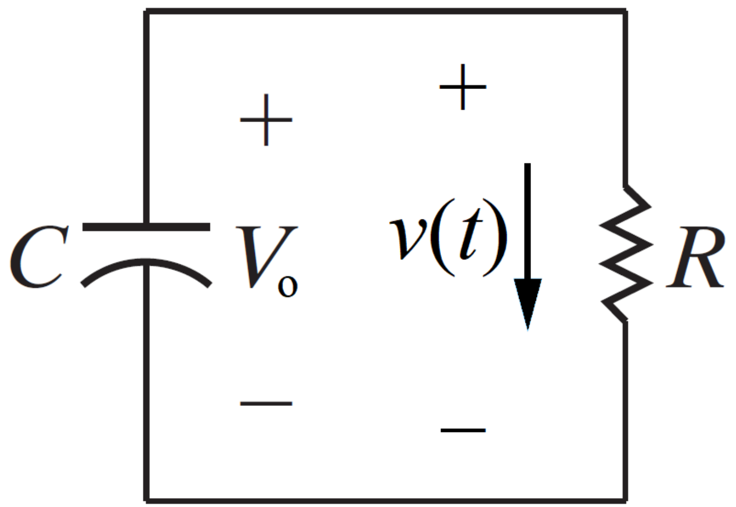


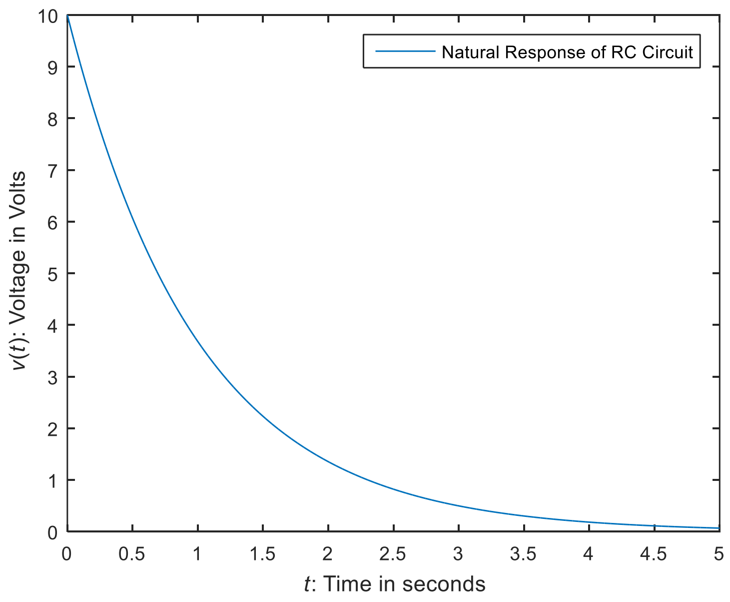
Discrete-time signal: $x[n]$



Natural Response of an RC Circuit

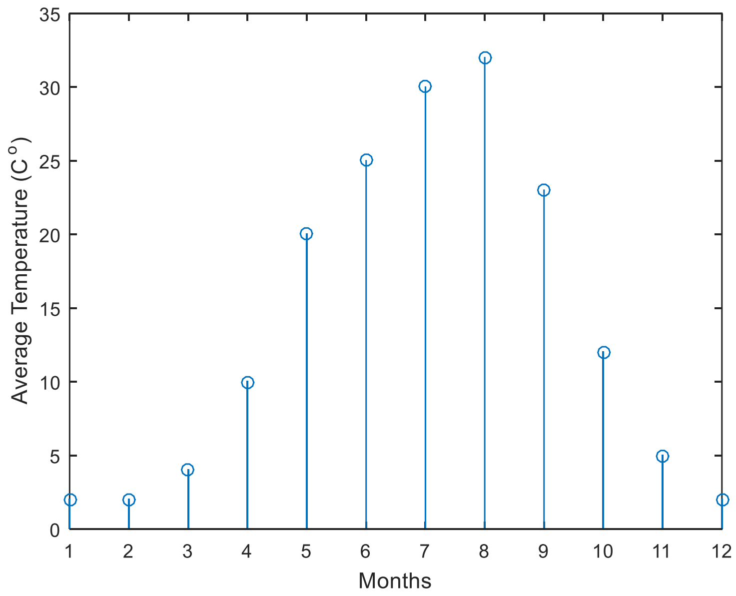
- For instance, natural response of an RC circuit will have a voltage signal, which is represented with mathematical continuous-time function $v(t)=V_o e^{-t/RC}$, $t \geq 0$ (V_o: Initial voltage of the capacitor).





Average Temperature per Month

- An example for discrete-time signal is average temperature per month.



Signals with two Independent Variables

- A gray-scale image is an example of two dimensional signal (with two independent variables).

Instantaneous power of a resistor

$p(t)=v(t)i(t)=i(t)^2R=v(t)^2/R$

$v(t)$: voltage (V)

$i(t)$: current (A)

R: resistance (Ω)

$p(t)$: instantaneous power (W)

Energy and Power

- Total energy expended over the interval $t_1 \leq t \leq t_2$:

$$E_{t_2 - t_1} = \int_{t_1}^{t_2} p(t)dt$$

- Average power over the same time interval $t_1 \leq t \leq t_2$:

$$P_{t_2 - t_1} = \frac{1}{t_1 - t_2} \int_{t_1}^{t_2} p(t)dt$$

Total Energy

- In general, total energy over the continuous time interval $t_1 \leq t \leq t_2$ for a continuous-time signal $x(t)$ is given as

$$E_{t_2 - t_1} = \int_{t_1}^{t_2} |x(t)|^2 dt$$

- Similarly, total energy over the discrete time interval $n_1 \leq n \leq n_2$ for a discrete-time signal $x[n]$ is described by

$$E_{n_2 - n_1} = \sum_{n=n_1}^{n_2} |x[n]|^2$$

Energy over Infinite Time Interval

- Energy over infinite time inverval in continuous time is

$$E_\infty \triangleq \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

and in discrete time it is given as

$$E_\infty \triangleq \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Total Power

- Total power expended over infinite time interval for continuous time signals is

$$P_\infty \triangleq \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2$$

- and for discrete time, it is

$$P_\infty \triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Important classes of signals

- There are three important classes of signals considering power and energy definitions:

1. Signals with finite total energy, namely $E_\infty < \infty$. Such a signal will have zero average power, $P_\infty = 0$. It is
$$P_\infty = \lim_{T \to \infty} \frac{E_\infty}{2T} = 0.$$

2. Signals with finite average power, $P_\infty < \infty$. Here, since P_∞>0 then total energy is required to be $E_\infty = \infty$.

3. Signals with $P_\infty = \infty$ and $E_\infty = \infty$.

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab