ELE 321
Linear System Analysis

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Periodic Signals

ELE321 Linear System Analysis

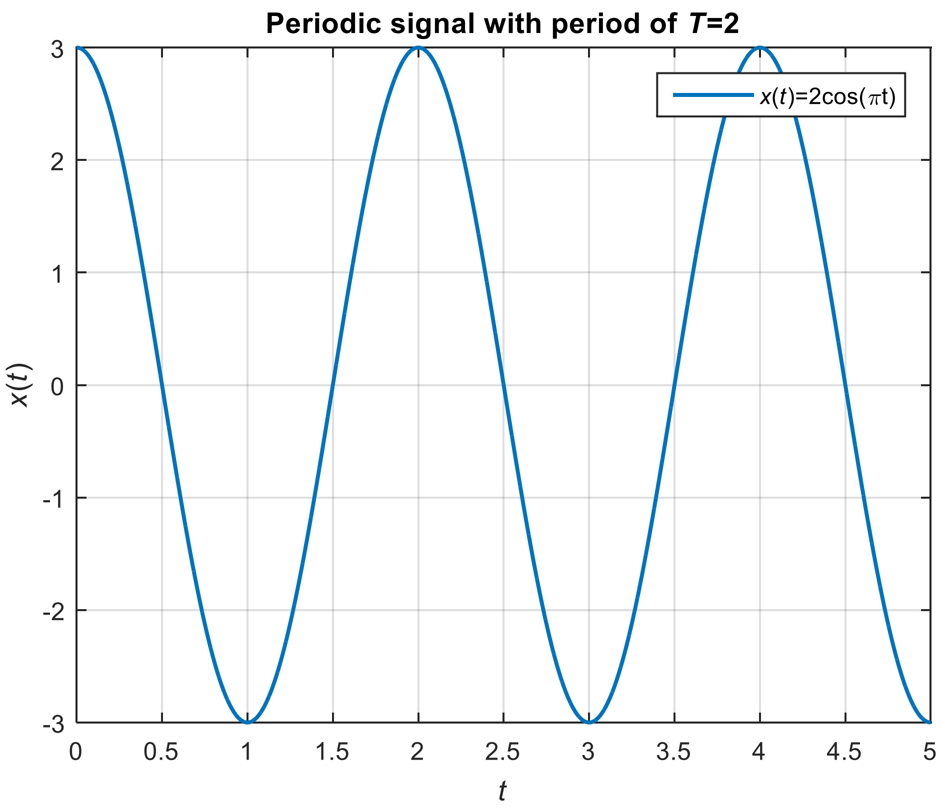
Lecture 3

Agenda

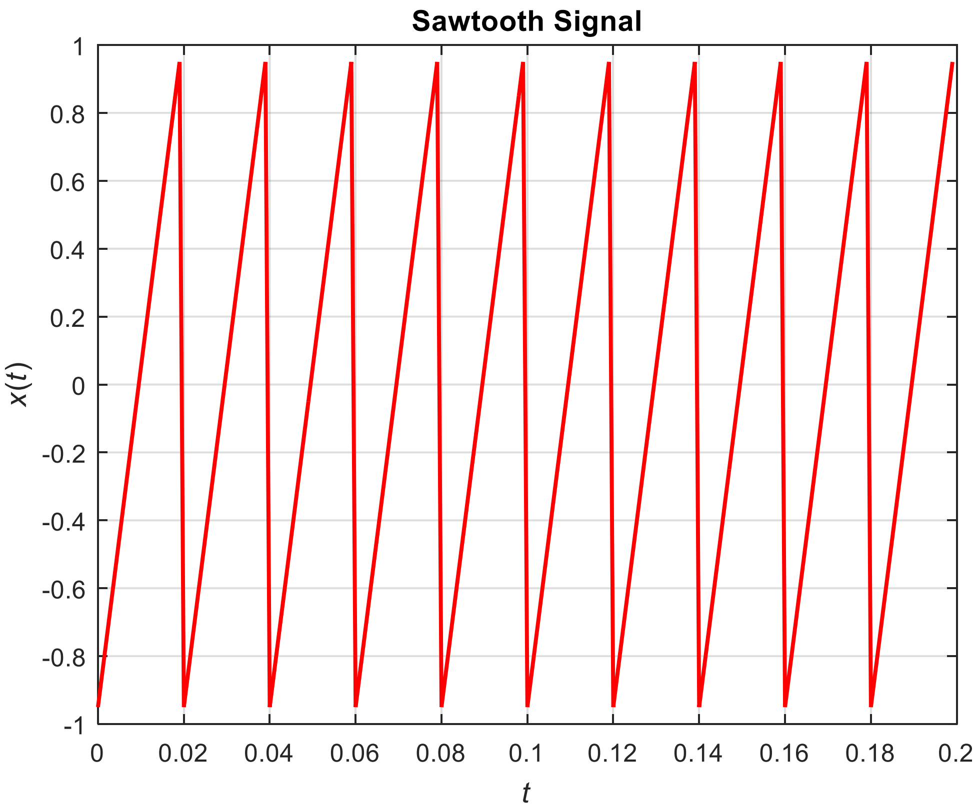
- Periodic signals
- Even and odd signals
- Complex exponentials
- Sinusoidal signals

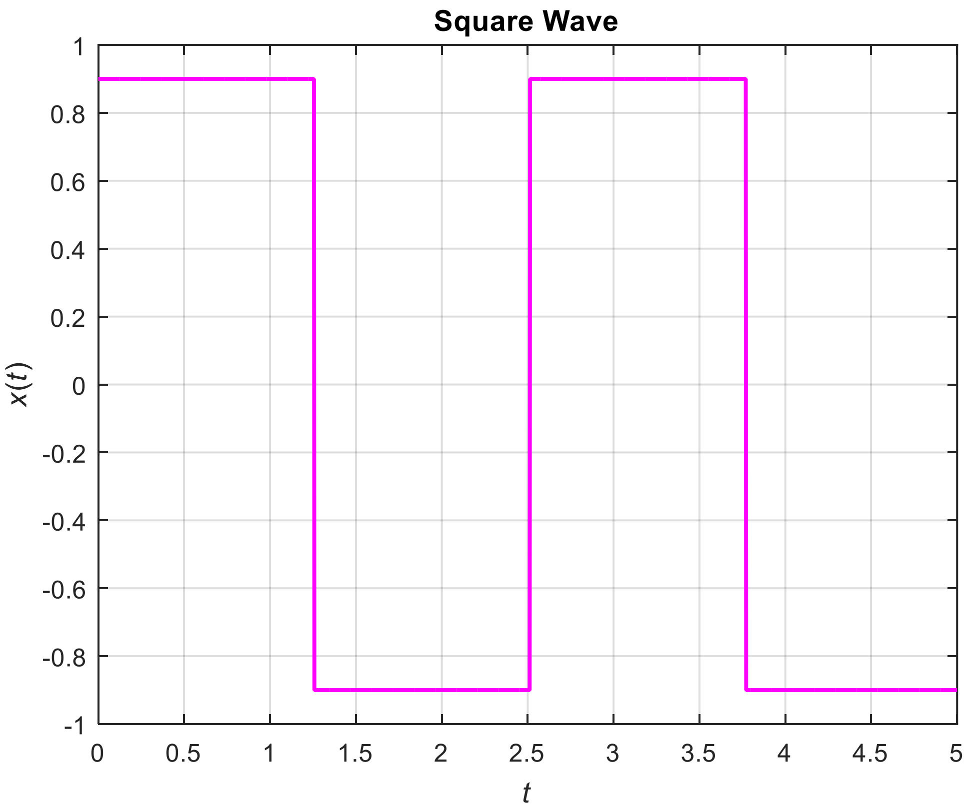
Periodic Signals

- Continuous-time Periodic Signal: $x(t)=x(t+T)$, T: Period

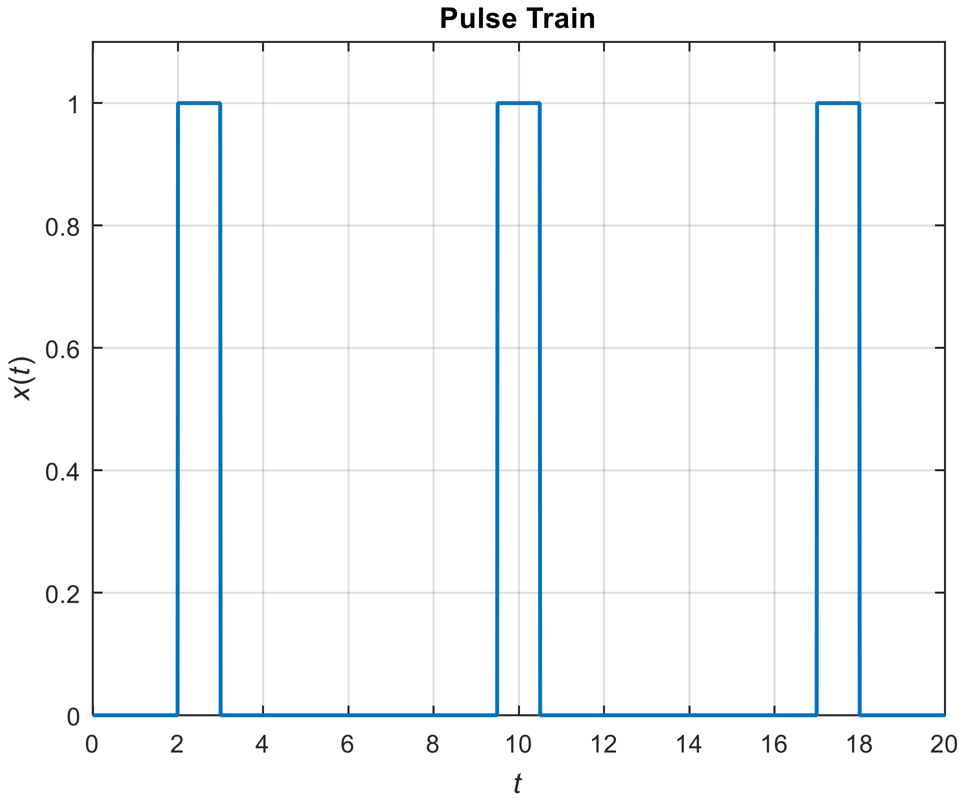


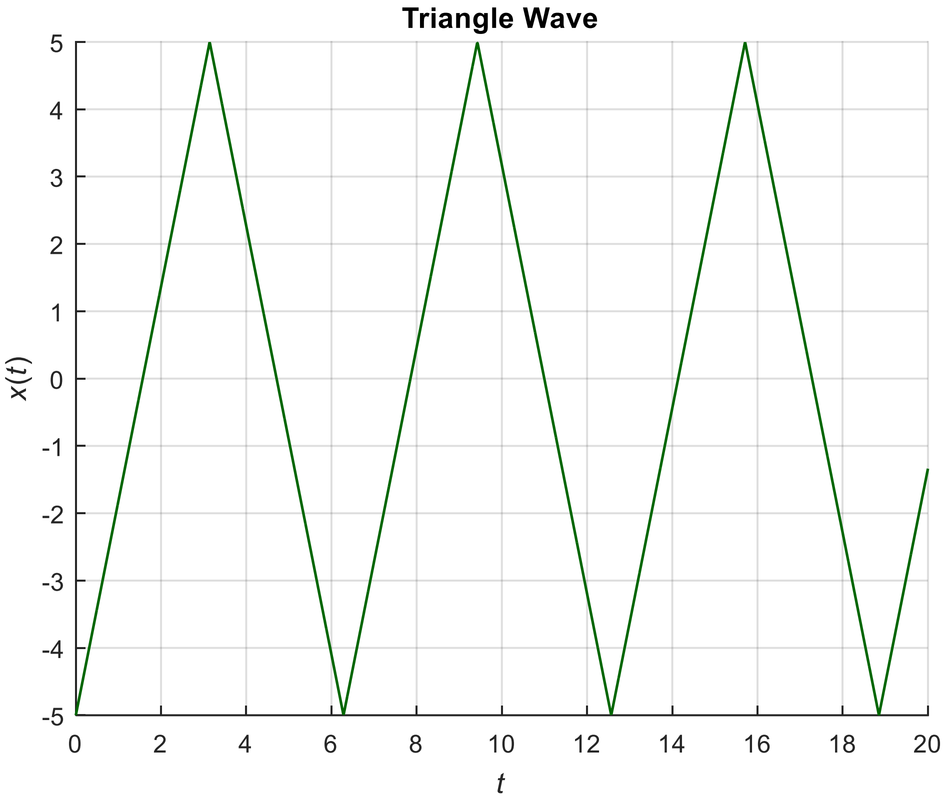
Periodic Signal Examples





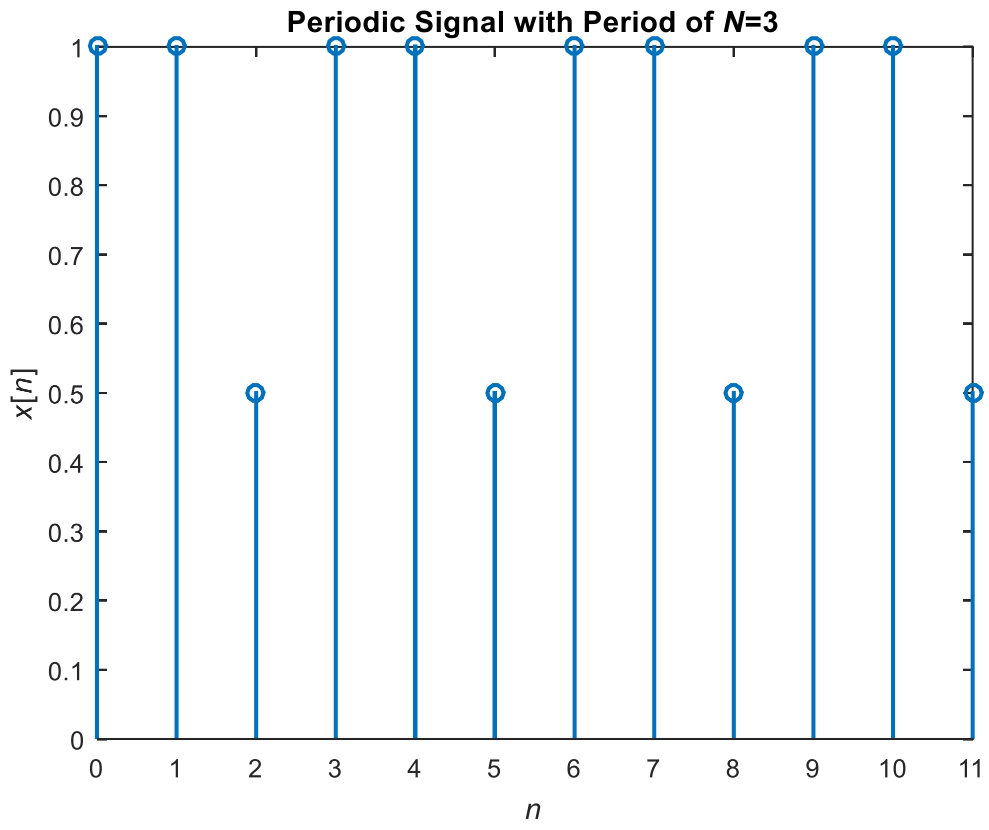
Periodic Signal Examples





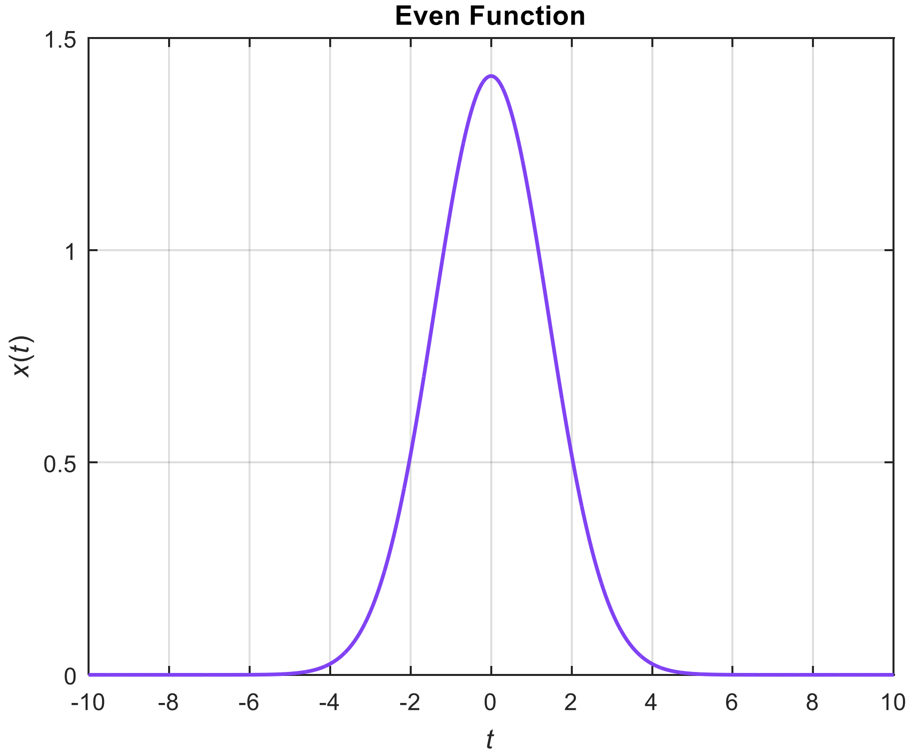
• Periodic Signals

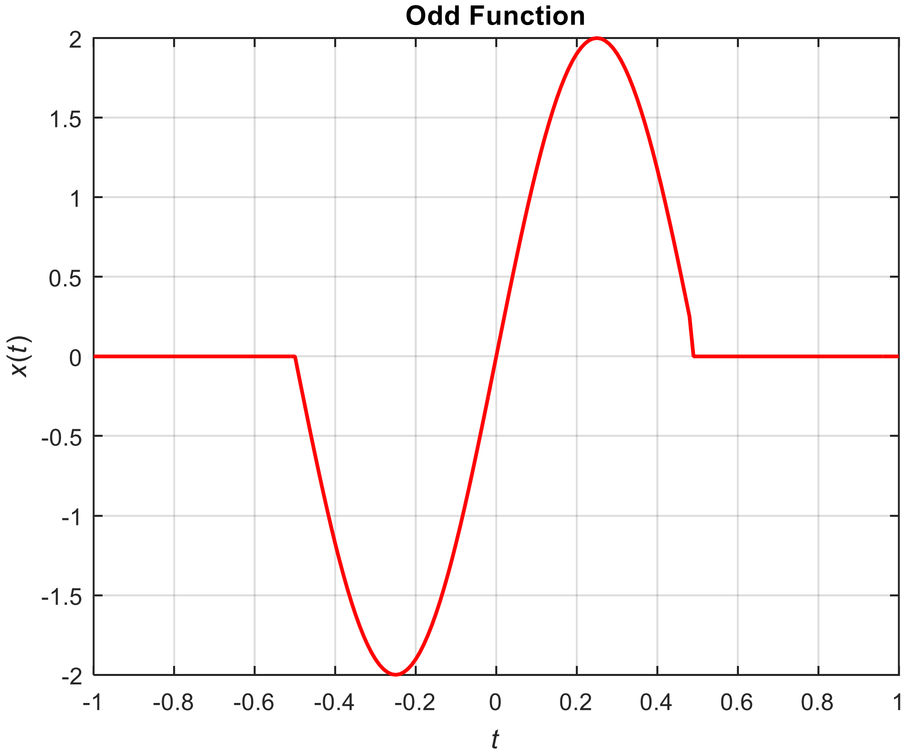
- Discrete-time Periodic Signal: $x[n]=x[n+N]$, N: Period



Even and Odd Signals

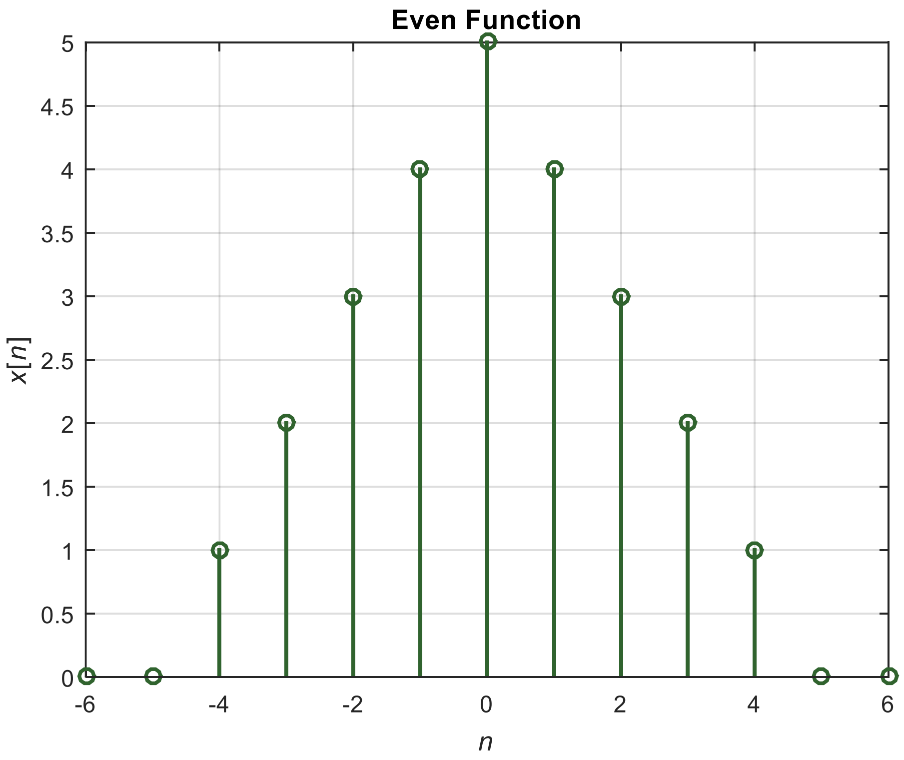
- A continuous-time signal is **even** if $x(-t)=x(t)$
- A continuous-time signal is **odd** if $x(-t)=-x(t)$

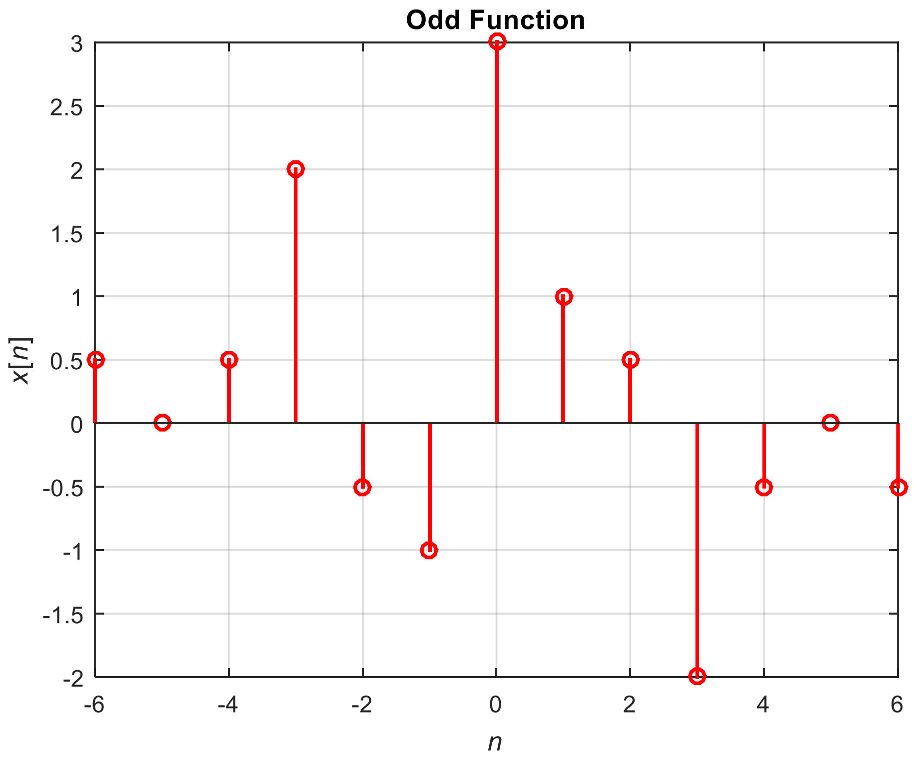




Even and Odd Signals

- A discrete-time signal is **even** if $x[-n]=x[n]$
- A discrete-time signal is **odd** if $x[-n]=-x[n]$





Continuous time

- Even part of signal *x*(*t*)

$$Ev\{x(t)\} = \frac{1}{2}\{x(t) + x(-t)\}$$

- Odd part of signal *x*(*t*)

$$Od\{x(t)\} = \frac{1}{2}\{x(t) - x(-t)\}$$

Discrete time

- Even part of signal *x*[*n*]

$$Ev\{x[n]\} = \frac{1}{2}\{x[n] + x[-n]\}$$

- Odd part of signal *x*[*n*]

$$Od\{x[n]\} = \frac{1}{2}\{x[n] - x[-n]\}$$

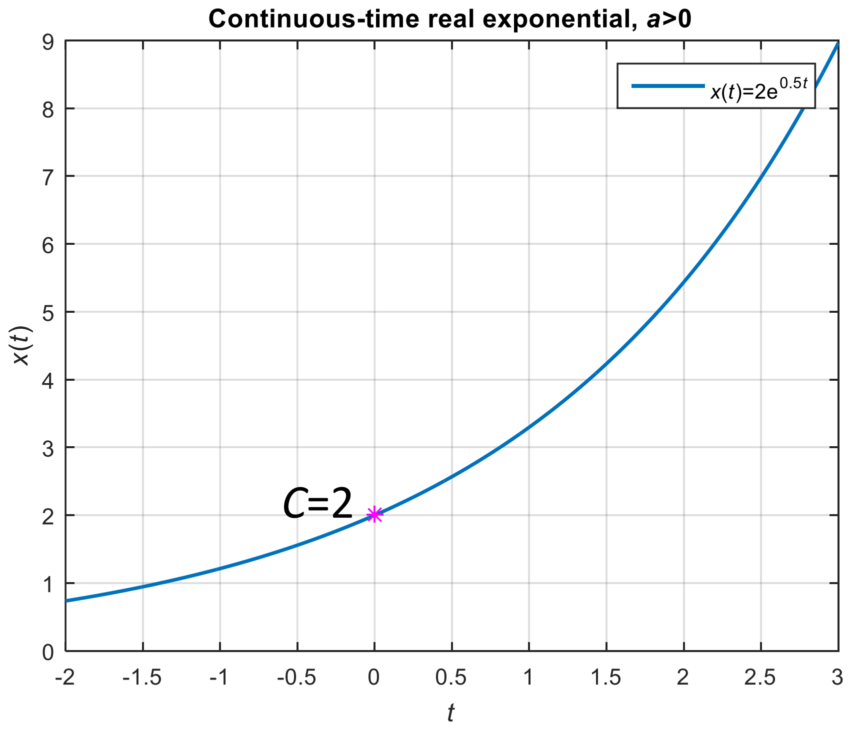
Continuous-Time Complex Exponential and Sinusoidal Signals

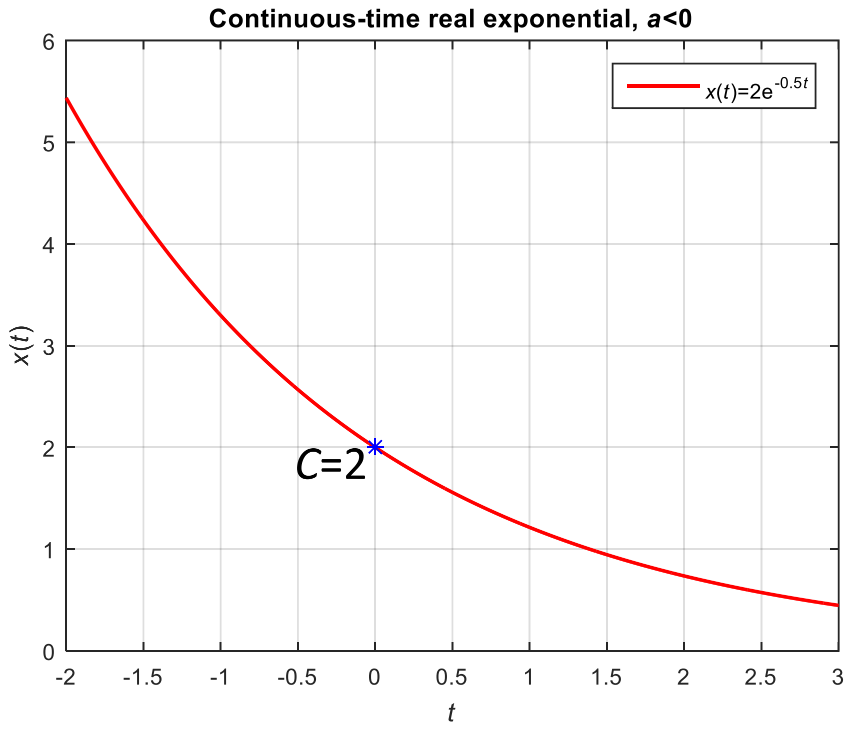
Continuous-Time Complex Exponential Signals

$$x(t) = Ce^{at}$$

where C and a are complex numbers in general.

- **Real exponential signal:** If C and a are real, $x(t)$ is called real exponential signal.





- **Periodic Complex Exponential and Sinusoidal Signals**

Consider $x(t) = e^{j\omega_o t}$ (*a* is purely imaginary)

x(*t*) is periodic, where ω_o is frequency in radian per second (rad/s)

$$\omega_o = \frac{2\pi}{T_o} \quad (T_o \text{ is fundamental period})$$

Since *x*(*t*) is periodic, $e^{j\omega_o t} = e^{j\omega_o (t+T)}$

Therefore, $e^{j\omega_o t} = e^{j\omega_o t} e^{j\omega_o T}$

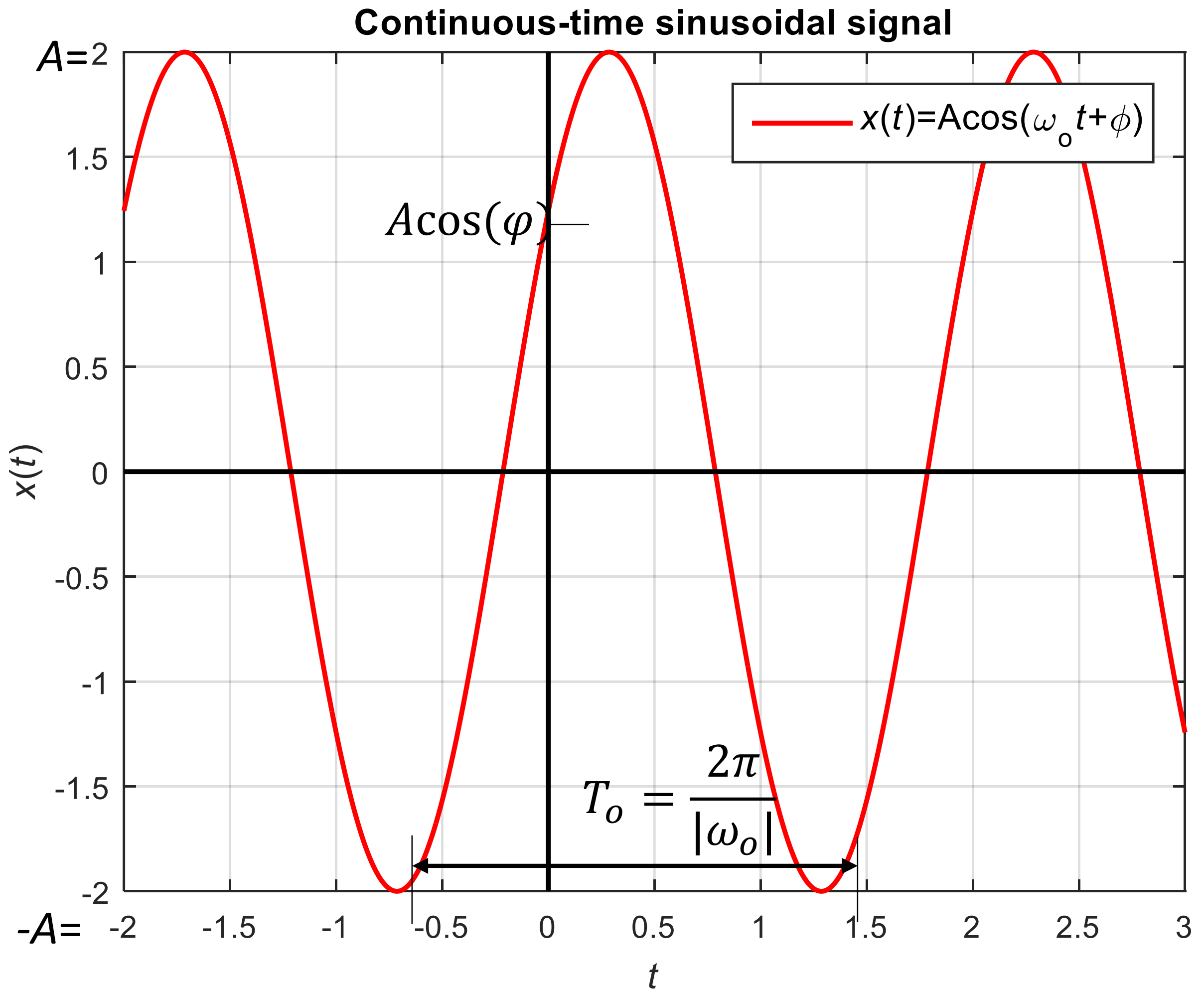
For periodicity, $e^{j\omega_o T} = 1$

In fact, $e^{j\omega_o T} = \cos(\omega_o T) + \mathrm{j}\sin(\omega_o T)$ (Euler's Relation)

When $\omega_o = 0$, then *x*(*t*)=1, which is periodic for any value of *T*. When $\omega_o \neq 0$, the fundamental period T_o (smallest positive value of *T*) of *x*(*t*) is $T_o = \frac{2\pi}{|\omega_o|}$.

- **Sinusoidal signal** is closely related to periodic complex exponential:

$$x(t) = A\cos(\omega_o t + \varphi)$$



- By using Euler's Relation periodic complex exponential can be written in terms of periodic sinusoidals:

$$e^{j\omega_o t} = \cos(\omega_o t) + \mathrm{j}\sin(\omega_o t)$$

- Similarly, sinusoidal signal can be written in terms of periodic complex exponentials:

$$A\cos(\omega_o t + \varphi) = \frac{A}{2} e^{j\varphi} e^{j\omega_o t} + \frac{A}{2} e^{-j\varphi} e^{-j\omega_o t}$$

- Therefore cosine can be expressed as

$$A\cos(\omega_o t + \varphi) = ARe\{e^{j(\omega_o t+\varphi)}\}$$

and sine will be

$$A\sin(\omega_o t + \varphi) = AIm\{e^{j(\omega_o t+\varphi)}\}$$

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab