ELE 321
Linear System Analysis

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Power and Energy Signals

ELE321 Linear System Analysis

Lecture 4

Agenda

- Power
- Energy
- Harmonically related complex exponentials

Power and Energy of Periodic Signals

- Periodic signals are examples of signals with infinite total energy, but finite average power.
- Consider periodic complex exponential signal $e^{j\omega_o t}$.

  Total energy and average power of the signal over one period are

$$E_{T_o} = \int_0^{T_o} |e^{j\omega_o t}|^2 dt$$

$$= \int_0^{T_o} 1 dt = T_o$$

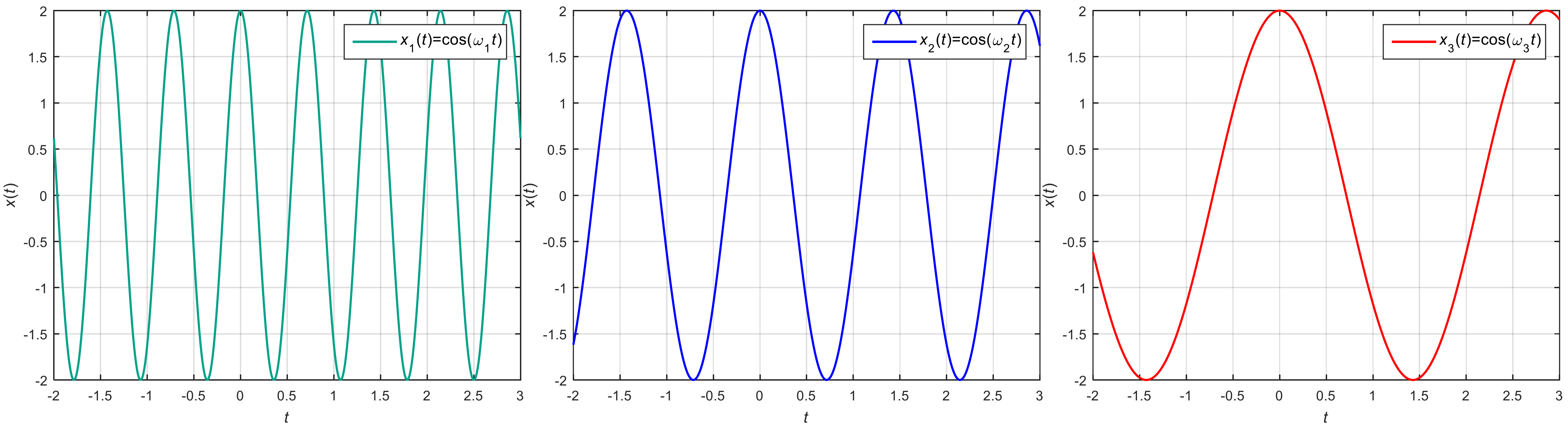
$$P_{T_o} = \frac{1}{T_o} E_{T_o} = 1$$

It is also clear that $E_\infty = \infty$

and

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |e^{j\omega_o t}|^2 dt = 1$$

- Relationship between the fundamental frequency and period for continuous-time signals



$$\omega_1>\omega_2>\omega_3 \Leftrightarrow T_1<T_2<T_3$$

Sets of harmonically related complex exponentials

$$\phi_k(t) = e^{jk\omega_o t}, k = 0, \pm 1, \pm 2, \ldots$$

Since $e^{j\omega T_o} = 1$ implies that $\omega T_o = 2\pi k, k = 0, \pm 1, \pm 2, \ldots$ (ωT_o is multiple of πk)

$$\omega_o = \frac{2\pi}{T_o}$$

For k=0, $\phi_k(t)$ is a constant, while for other values of k, $\phi_k(t)$ is periodic with fundamental frequency $|k|\omega_o$ and fundamental period

$$\frac{2\pi}{|k|\omega_o} = \frac{T_o}{|k|}$$

The k^{th} harmonic $\phi_k(t)$ is still periodic with period T_o as well, as it goes through exactly $|k|$ of its fundamental periods during any time interval of length T_o.

Example *

Plotting the magnitude of the signal

$$x(t) = e^{j2t} + e^{j3t}$$

It can be rewritten as

$$x(t) = e^{j2.5t}(e^{-j0.5t} + e^{j0.5t})$$

Using Euler's relation

$$x(t) = 2e^{j2.5t}\cos(0.5t)$$

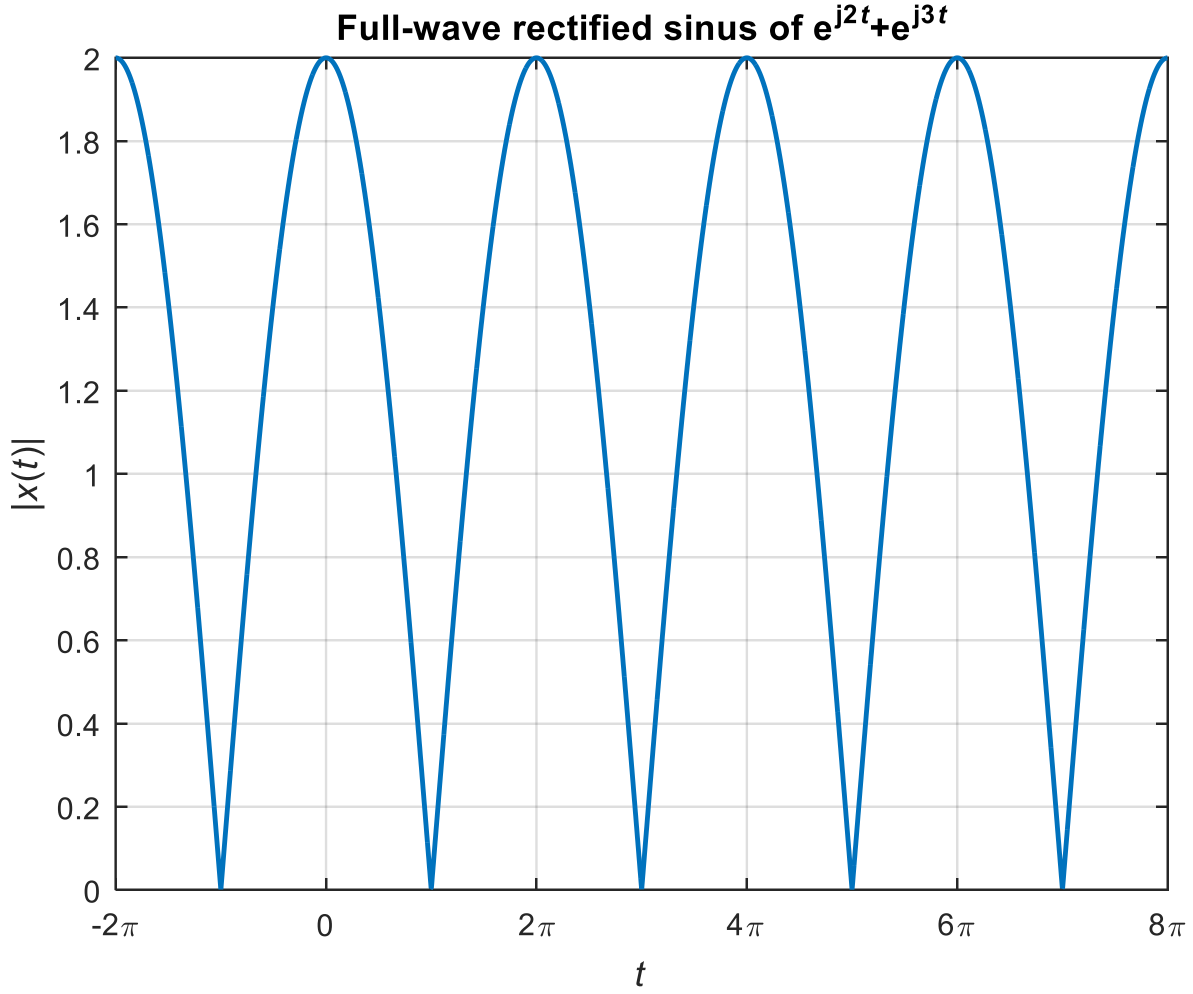
Therefore

$$|x(t)| = 2|\cos(0.5t)|$$

Note that $|e^{j2.5t}|$=1.

* Example 1.5. Signals and Systems, A.V. Oppenheim, A. S. Willsky with S. H. Nawab

Example (cont.)*



* Example 1.5. Signals and Systems, A.V. Oppenheim, A. S. Willsky with S. H. Nawab

General Complex Exponential Signals

$$x(t) = Ce^{at}$$

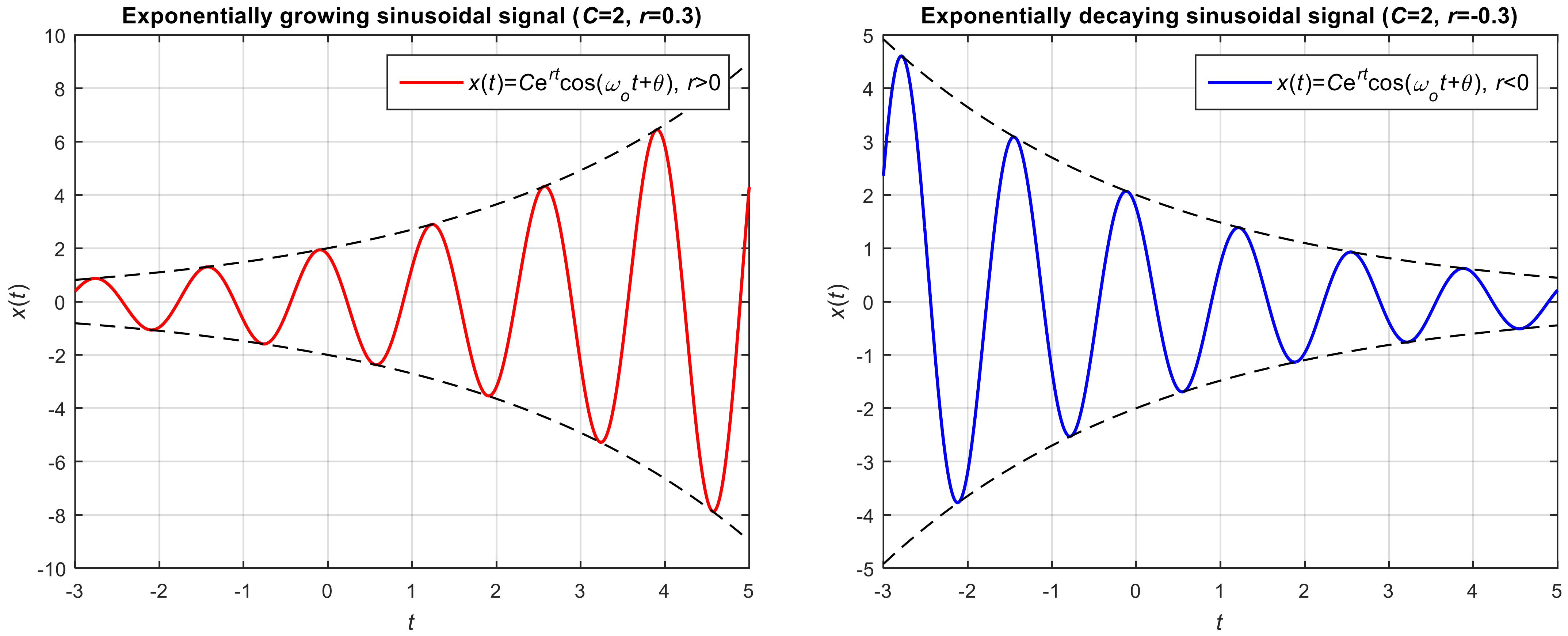
If *C* and *a* are expressed in polar and rectengular form, respectively

$$C = |C|e^{j\theta}$$
$$a = r + j\omega_o$$

Then $Ce^{at} = |C|e^{j\theta}e^{(r+j\omega_o)t} = |C|e^{rt}e^{j(\omega_o t+\theta)}$

By using Euler's equation, $Ce^{at} = |C|e^{rt}\cos(\omega_o t + \theta) + j|C|e^{rt}\sin(\omega_o t + \theta)$

Therefore, for *r*=0, the real and imaginary parts of a complex exponential are sinusoidal. For *r*>0, the signal is sinusoidal multiplied by growing exponential. If *r*<0, the signal is sinusoidal multiplied by decaying exponential.


Exponentially growing sinusoidal signal (C=2, r=0.3)
x(t)=Ce^rt cos(ω_o t+θ), r>0
x(t)
t
10
8
6
4
2
0
-2
-4
-6
-8
-10
-3
-2
-1
0
1
2
3
4
5
Exponentially decaying sinusoidal signal (C=2, r=-0.3)
x(t)=Ce^rt cos(ω_o t+θ), r<0
x(t)
t
5
4
3
2
1
0
-1
-2
-3
-4
-5
-3
-2
-1
0
1
2
3
4
5

Discrete-Time Complex Exponential and Sinusoidal Signals

Discrete-Time Complex Exponential Signals

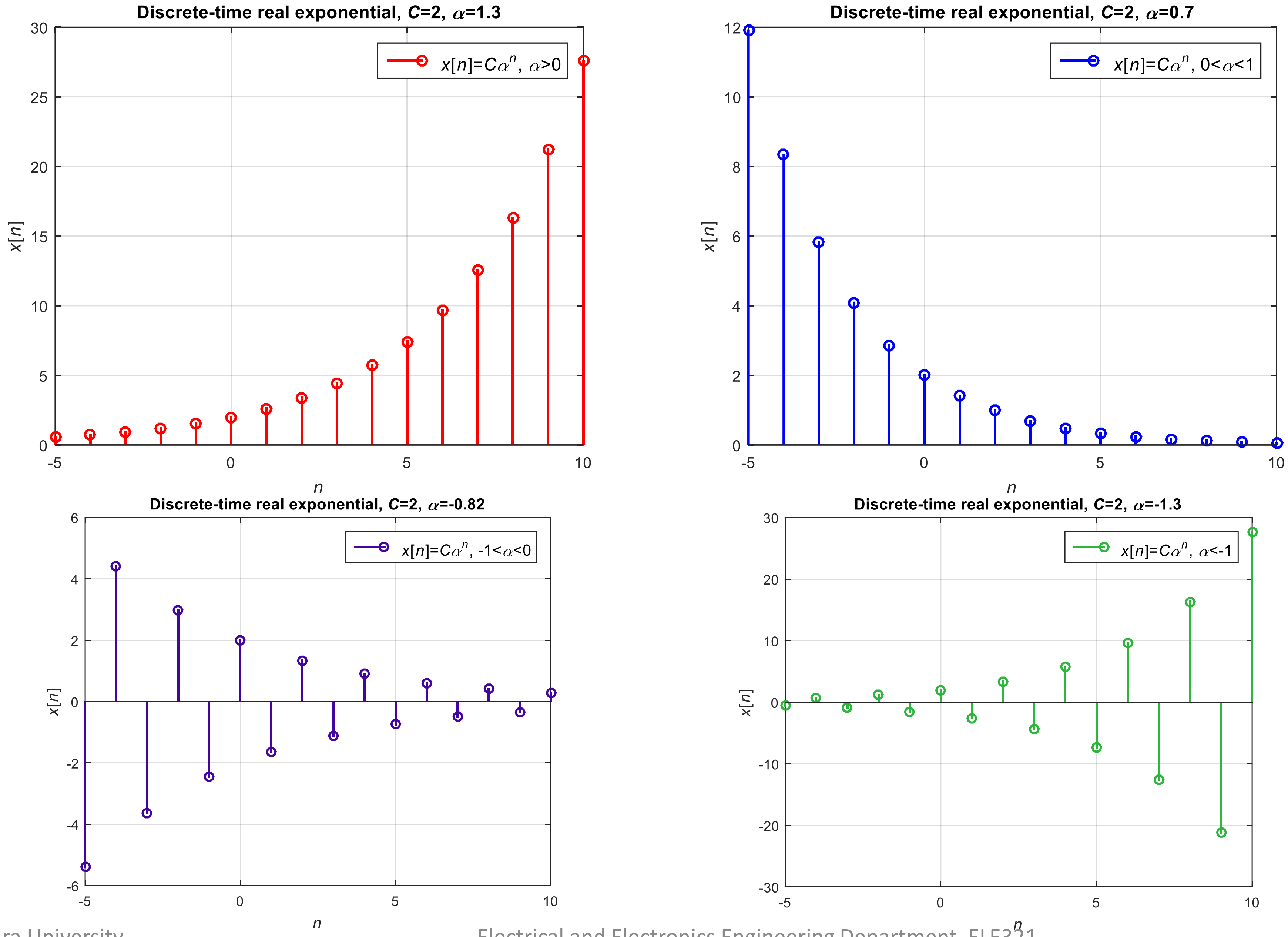
$$x[n] = C\alpha^n$$

where C and α are complex numbers in general.

If α=e^{β} , then $x[n] = Ce^{\beta n}$

Real-exponential signals: If C and α are real $x[n]$ is called real exponential signal.

* If α is 1, then $x[n]$ is constant, if α is than $x[n]$ alternates between $-C$ and $+C$.


Discrete-time real exponential, C=2, α=1.3
x[n]=Cα^n, α>0
Discrete-time real exponential, C=2, α=0.7
x[n]=Cα^n, 0<α<1
Discrete-time real exponential, C=2, α=-0.82
x[n]=Cα^n, -1<α<0
Discrete-time real exponential, C=2, α=-1.3
x[n]=Cα^n, α<-1
x[n]
n

- **Discrete-Time Sinusoidal Signals:**

If β is purely imaginary than $|\alpha|$=1. Specifically, consider

$$x[n] = e^{j\omega_o n}$$

which has infitine total energy, but finite average power.

As in continuous-time case this is related to

$$x[n] = A\cos(\omega_o n + \varphi)$$

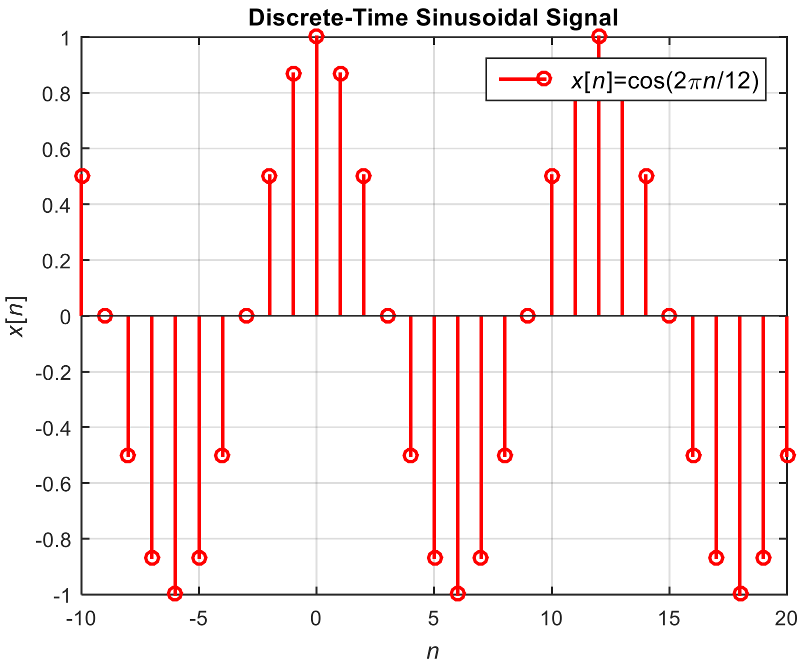
which also has infinite total energy, but finite average power.

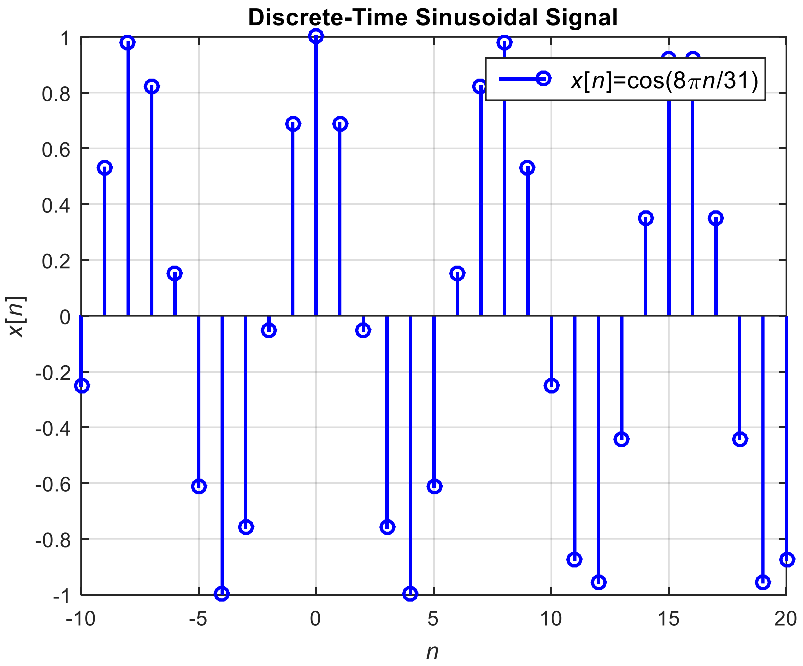
Since n is dimensionless, both ω_o and φ will have units of radians.

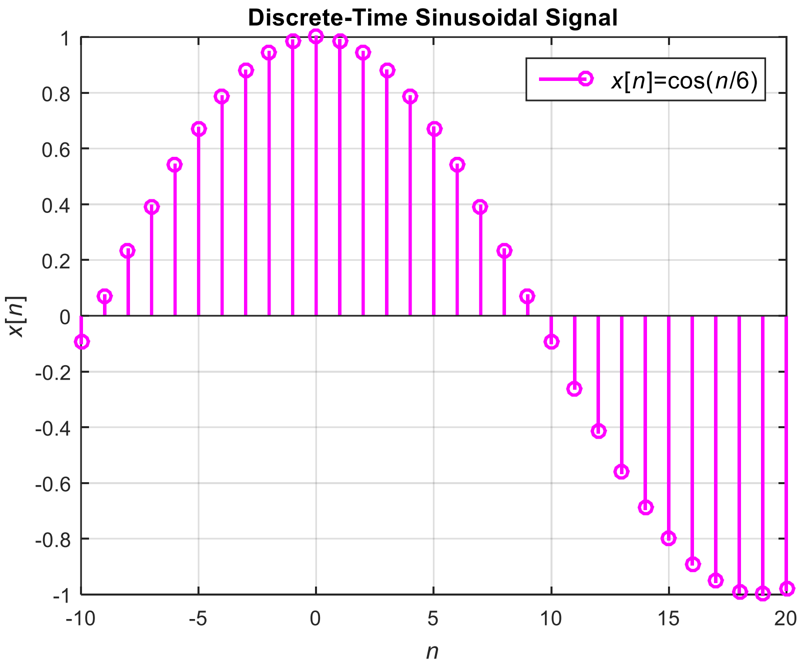
Also Euler's equation is $e^{j\omega_o n} = \cos(\omega_o n) + jsin(\omega_o n)$

and therefore

$$A\cos(\omega_o n + \varphi) = \frac{A}{2} e^{j\varphi} e^{j\omega_o n} + \frac{A}{2} e^{-j\varphi} e^{-j\omega_o n}$$


Discrete-Time Sinusoidal Signal
x[n]=cos(2πn/12)
x[n]
n


Discrete-Time Sinusoidal Signal
x[n]=cos(8πn/31)
x[n]
n


Discrete-Time Sinusoidal Signal
x[n]=cos(n/6)
x[n]
n

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab