ELE 321
Linear System Analysis

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Linear Time Invariant Systems

ELE321 Linear System Analysis

Lecture 7

Agenda

- LTI systems
- Properties of LTI systems
- Impulse response
- Convolution Summation
- Convolution Integral

Linear Time-Invariant (LTI) Systems

Discrete-Time LTI Systems

Discrete-Time Input Signal in Terms of Unit Impulses:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Discrete-Time Output Signal in Terms of Unit Impulse Responses:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, h_k[n]$$

where $h_k[n]$ is the response of the linear system to the shifted impulse $\delta[n-k]$.

If $h[n]$ is the output of the LTI system, then the output is

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Equivalently

$$y[n] = x[n] * h[n]$$

where * is convolution operator.

Therefore output is determined using convolutional summation.

Continuous-Time LTI Systems

Continuous-Time Input Signal in Terms of Unit Impulses:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

Continuous-Time Output Signal in Terms of Unit Impulse Responses (Convolutional Integral):

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Equivalently,

$$y(t) = x(t) * h(t)$$

where * is convolution operator.

Properties of LTI Systems

- Commutative Property

$$x[n] * h[n] = h[n] * x[n]$$

$$x(t) * h(t) = h(t) * x(t)$$

- Distributive Property

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

$$x(t) * \left(h_1(t) + h_2(t)\right) = x(t) * h_1(t) + x(t) * h_2(t)$$

- LTI Systems with and without Memory

- Associative Property

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

$$x(t) * \left(h_1(t) * h_2(t)\right) = (x(t) * h_1(t)) * h_2(t)$$

- Invertibility

$$h[n] * h_1[n] = \delta[n]$$

$$h(t) * h_1(t) = \delta(t)$$

- Causality

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$y(t) = \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau$$

- Stability

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

$$\int_{-\infty}^{\infty} |h(\tau)| < \infty$$

Linear Constant-Coefficient Difference Equations

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab