ELE 321
Linear System Analysis

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Fourier Series Representation of Periodic Signals

ELE321 Linear System Analysis

Lecture 8

Agenda

- Historical Perspective
- Eigenvalues and Eigenfunctions
- Fourier Series Representation of Continuous-Time Periodic Signals
- Convergence of the Fourier Series

Historical Perspective

- Sum of harmonically related sines and cosines to represent periodic signals
- Euler, 1748
- Bernoulli, 1753
- Lagrange, 1759
- Fourier, 1807
- Dirichlet, 1829
- Fourier, 1822

Eigenvalues and Eigenfunctions

- LTI Systems
- $e^{st} --\rightarrow H(s)e^{st}$: continuous time
- $z^{n} \;--\rightarrow H(z)z^{n}$: discrete time
- $H(s), H(z)$: eigenvalues
- e^{st}, z^{n} : eigenfunctions

Fourier Series Representation – Continuous Time

- To represent a periodic signal via linear combination of harmonically related complex exponentials, $e^{j\omega t}$
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$
- k: integer
- a_k: Fourier series coefficients

Fourier Series Coefficients – Continuous Time

- $a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-jk\omega_0 t}dt$
- a_k:spectral coefficients
- $a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)dt$: average value of $x(t)$

Convergence of the Fourier Series

- Dirichlet conditions
  - The periodic signal must be absolutely integrable
  - Number of maxima and minima are finite during any single period
  - Finite number of discontinuities

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab