

EEE 321

Signals and Systems

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Periodic Signals

EEE321 Signals and Systems

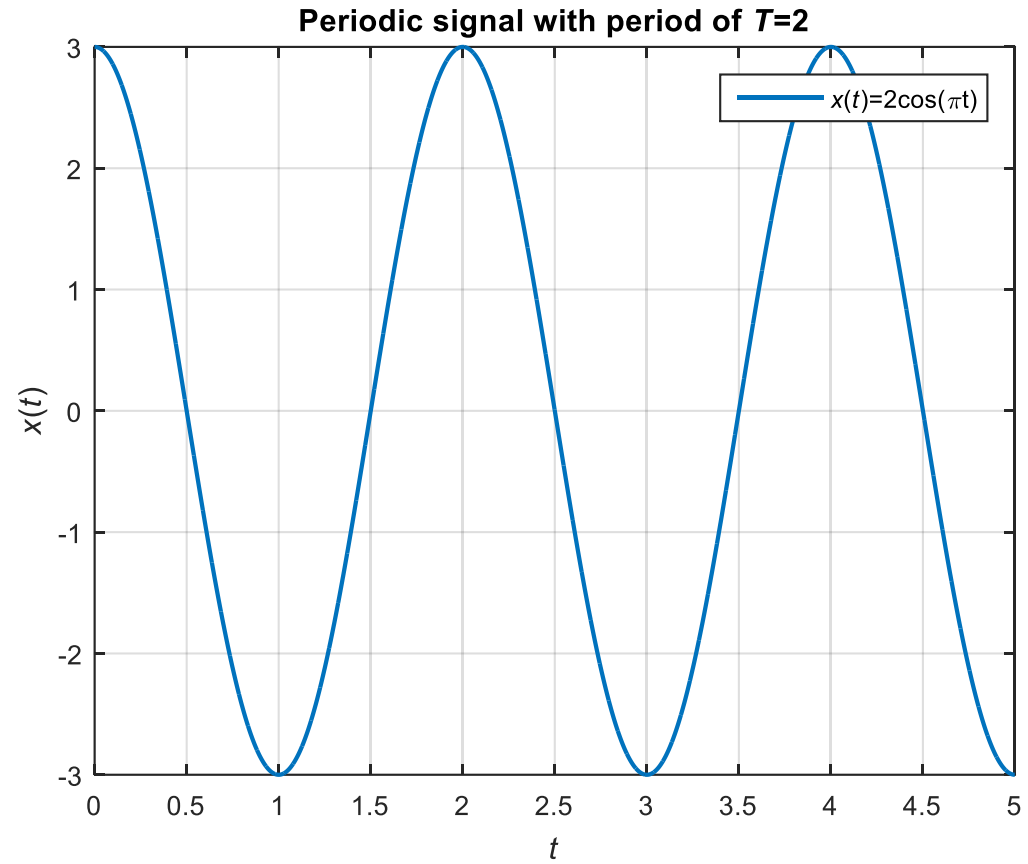
Lecture 3

Agenda

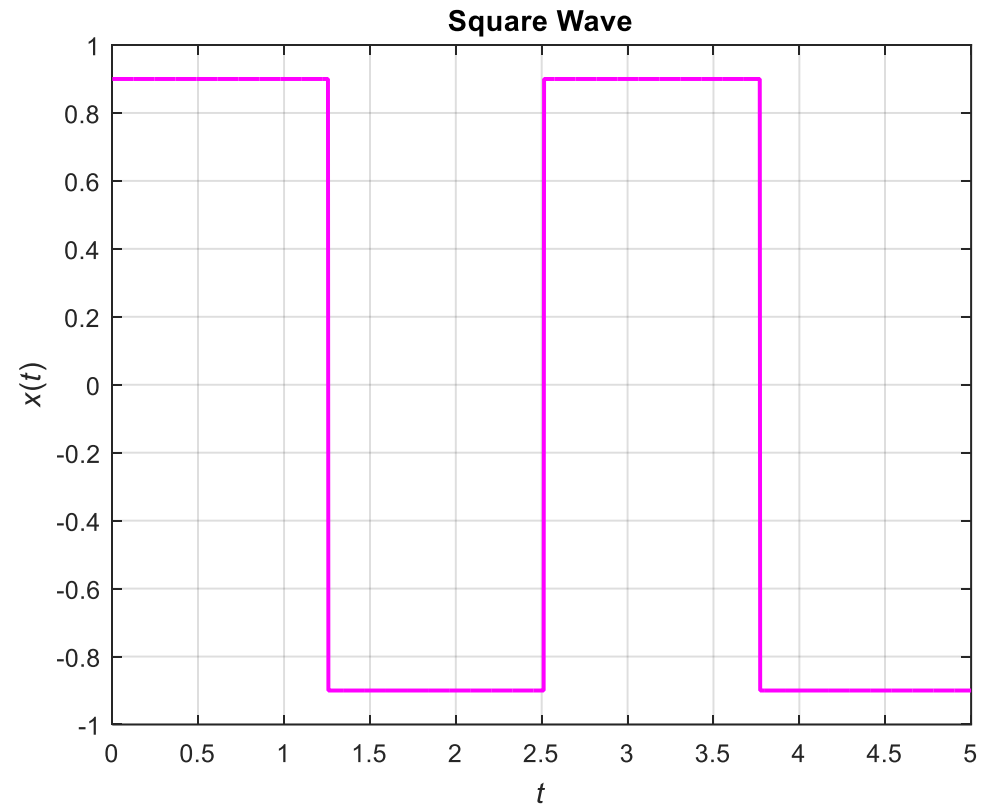
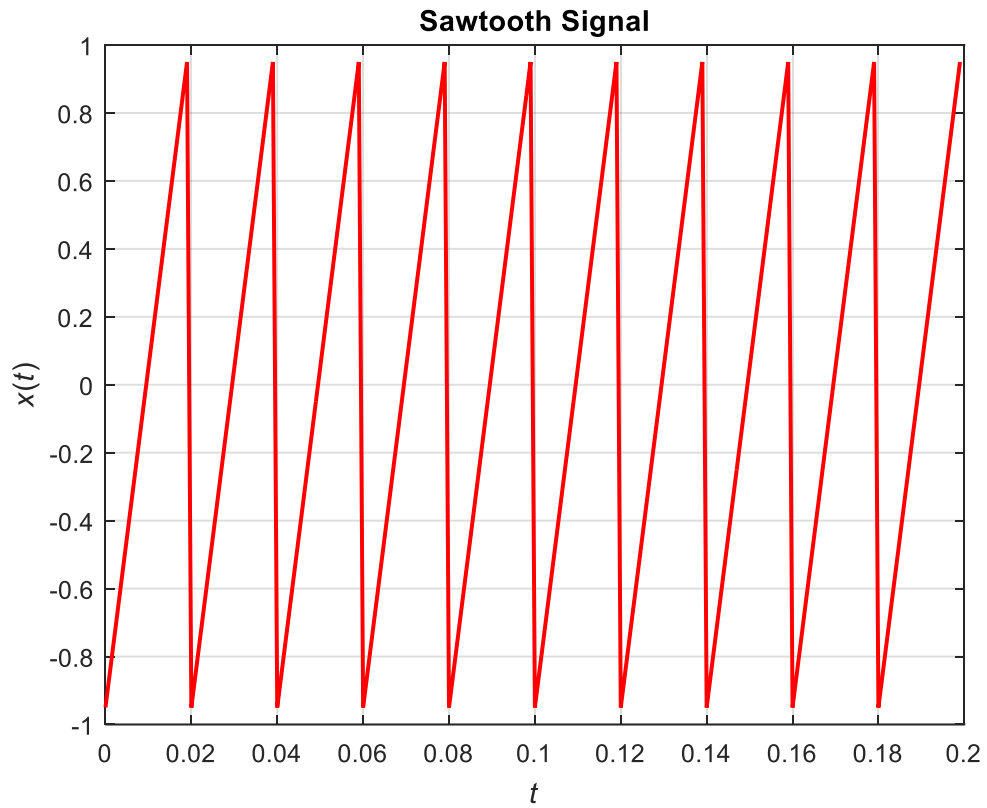
- Periodic signals
- Even and odd signals
- Complex exponentials
- Sinusoidal signals

Periodic Signals

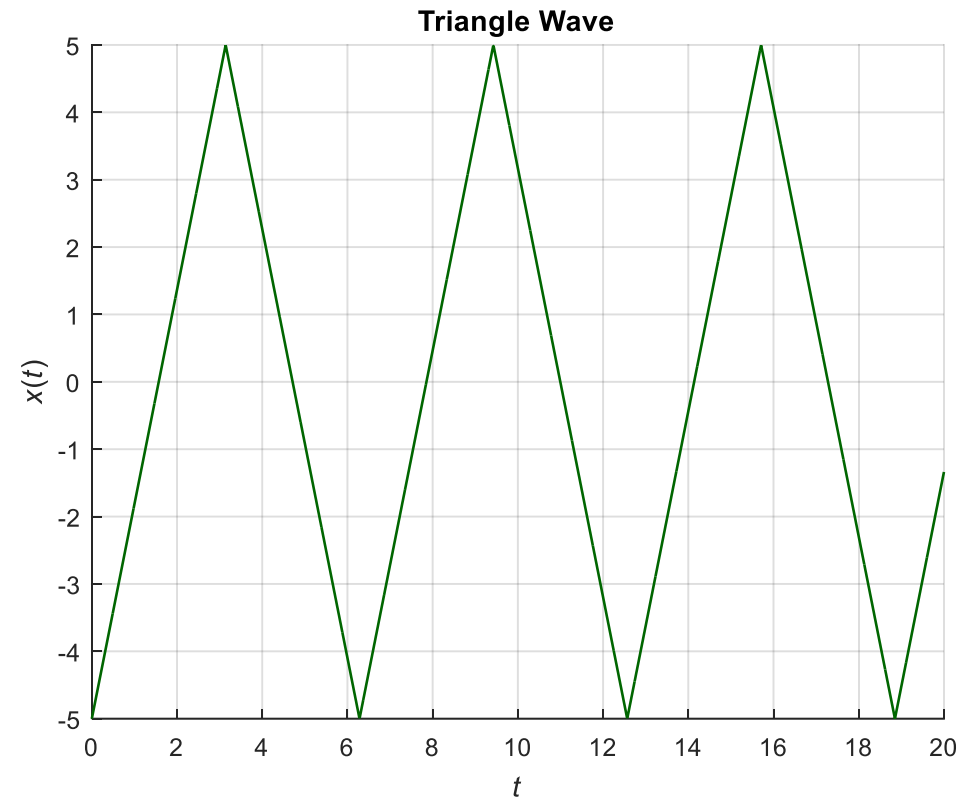
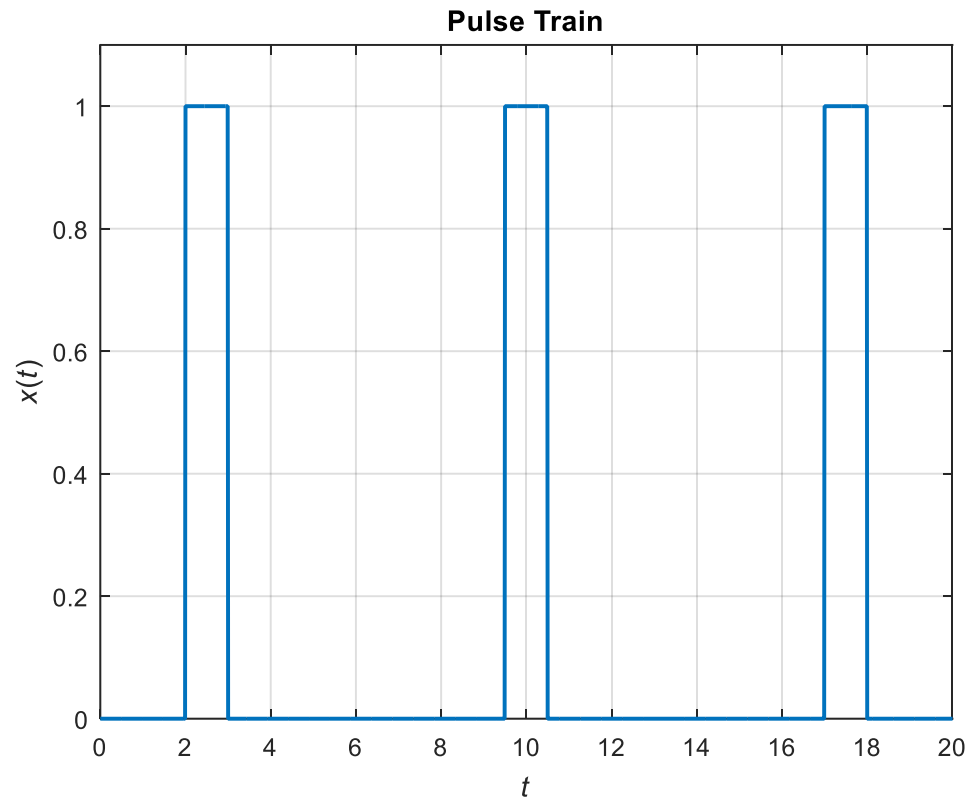
- Continuous-time Periodic Signal: $x(t)=x(t+T)$, T : Period



Periodic Signal Examples

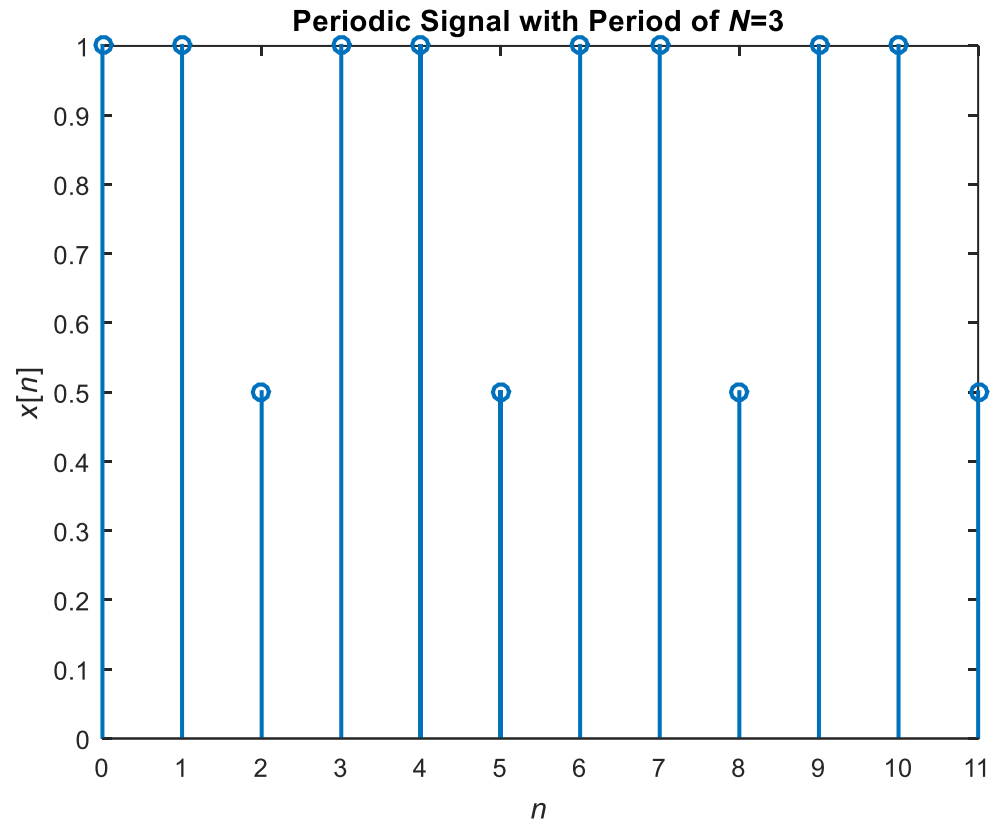


Periodic Signal Examples



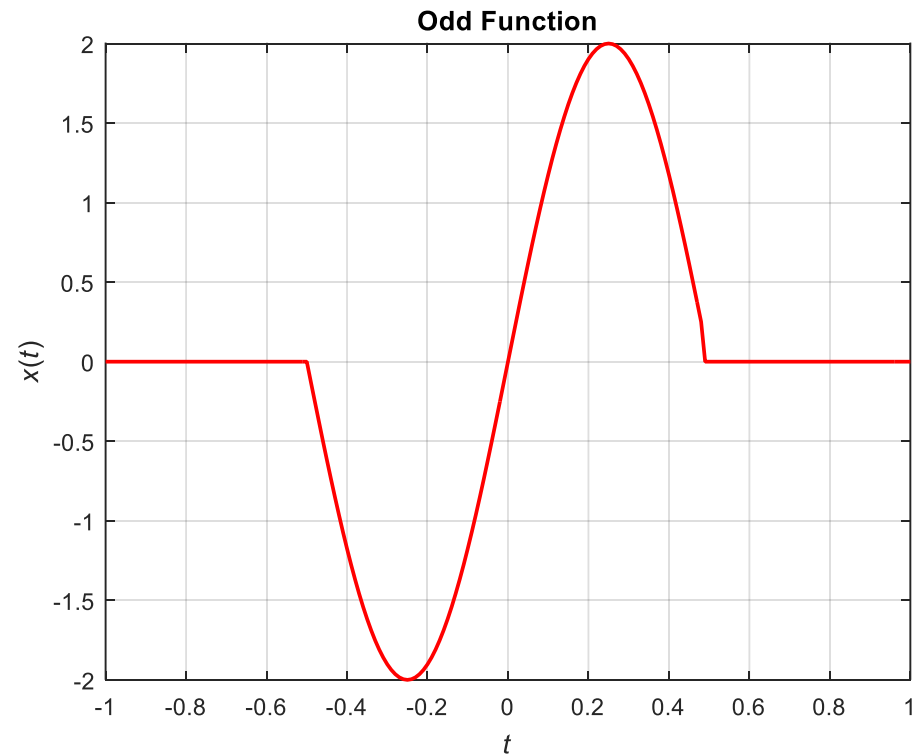
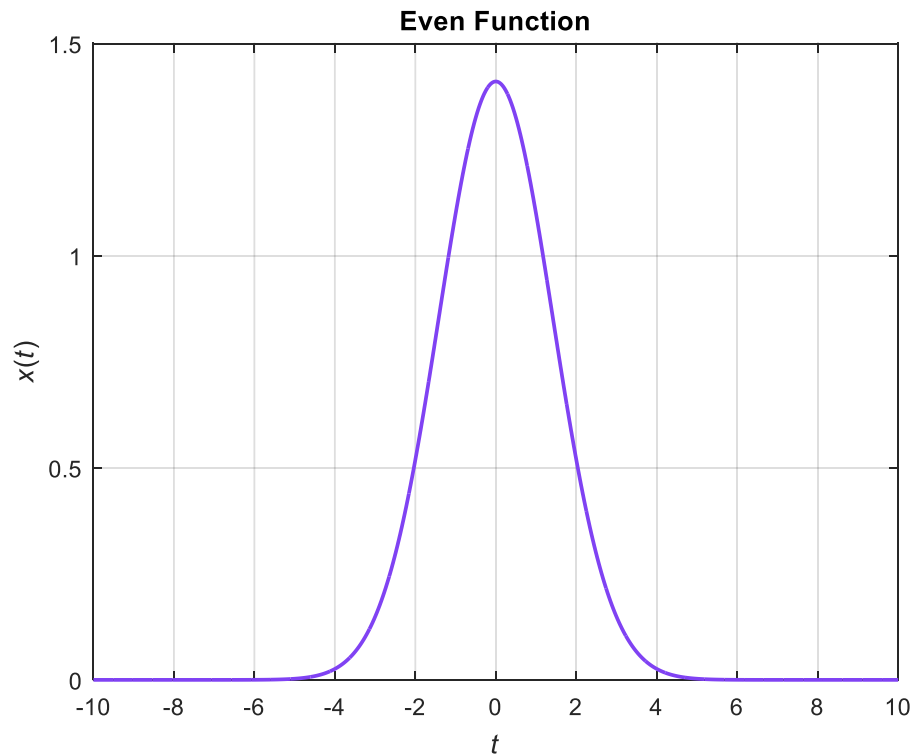
- Periodic Signals

- Discrete-time Periodic Signal: $x[n]=x[n+N]$, N : Period



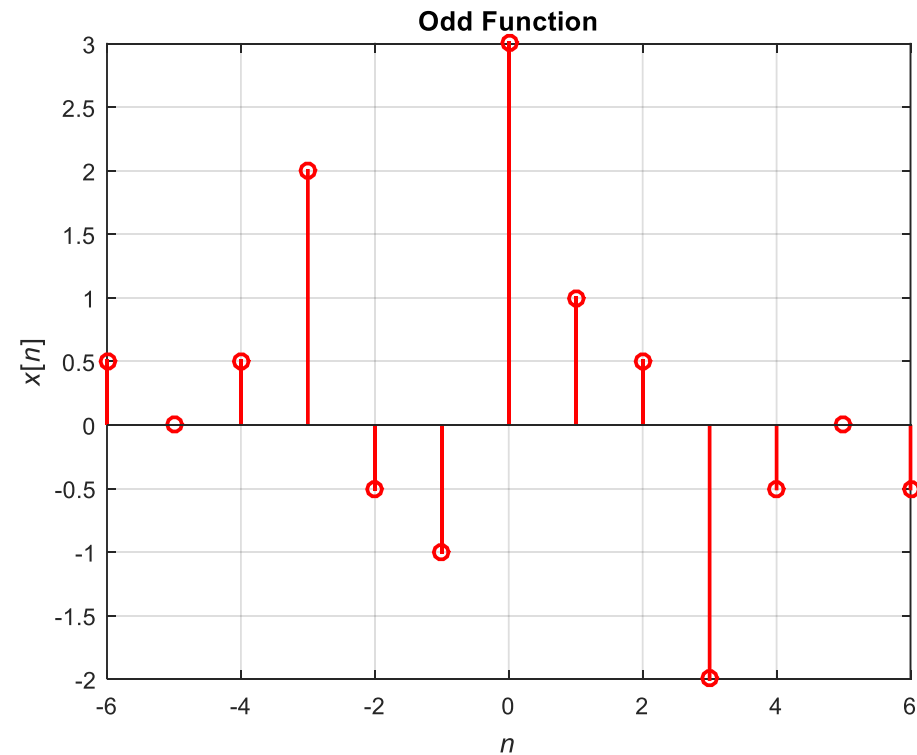
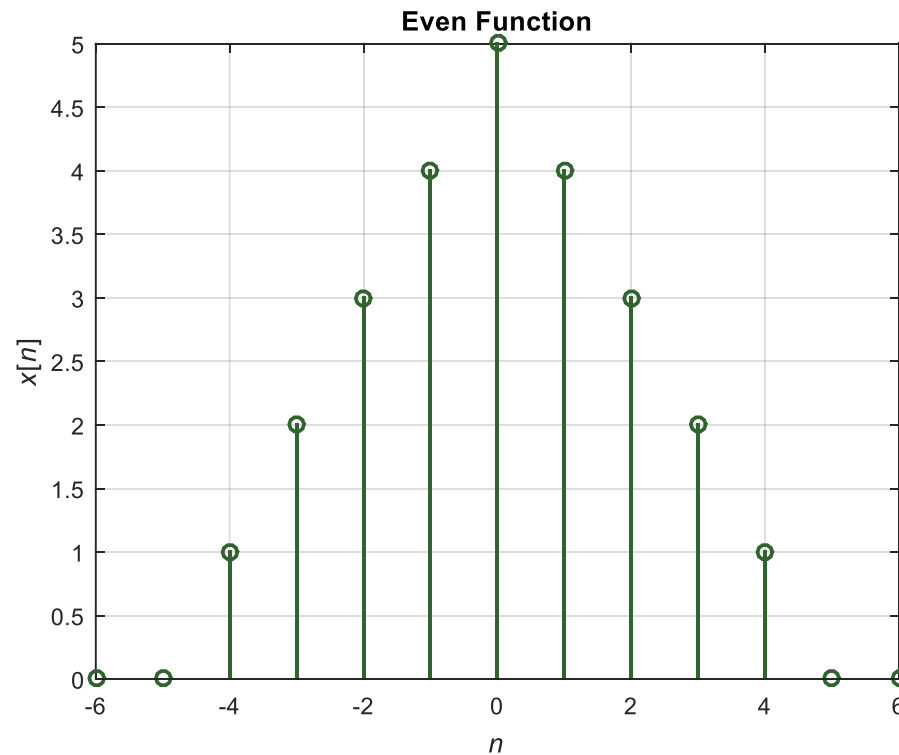
Even and Odd Signals

- A continuous-time signal is **even** if $x(-t)=x(t)$
- A continuous-time signal is **odd** if $x(-t)=-x(t)$



Even and Odd Signals

- A discrete-time signal is **even** if $x[-n]=x[n]$
- A discrete-time signal is **odd** if $x[-n]=-x[n]$



Continuous time

- Even part of signal $x(t)$

$$Ev\{x(t)\} = \frac{1}{2}\{x(t) + x(-t)\}$$

- Odd part of signal $x(t)$

$$Od\{x(t)\} = \frac{1}{2}\{x(t) - x(-t)\}$$

Discrete time

- Even part of signal $x[n]$

$$Ev\{x[n]\} = \frac{1}{2}\{x[n] + x[-n]\}$$

- Odd part of signal $x[n]$

$$Od\{x[n]\} = \frac{1}{2}\{x[n] - x[-n]\}$$

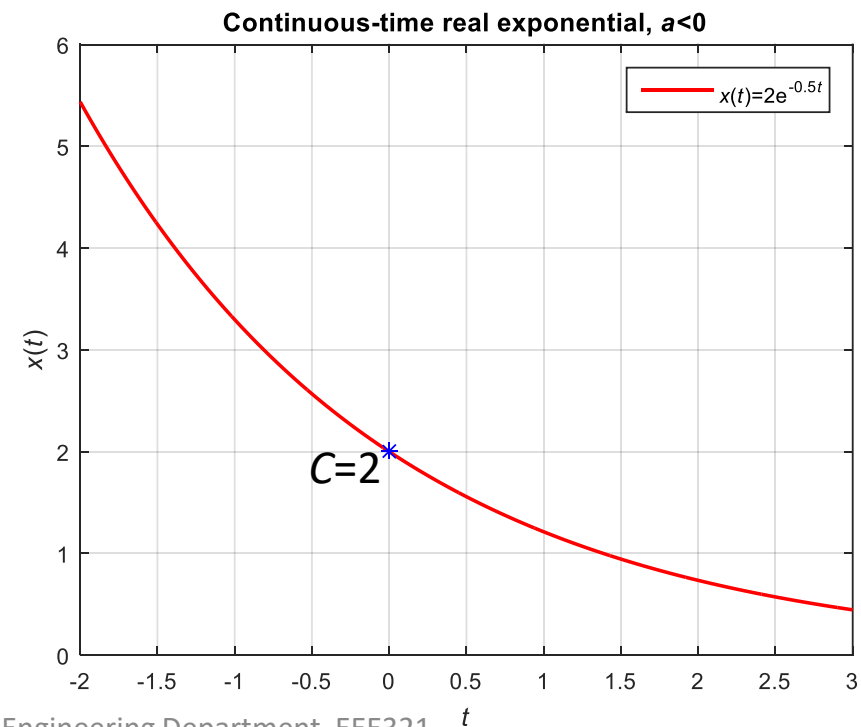
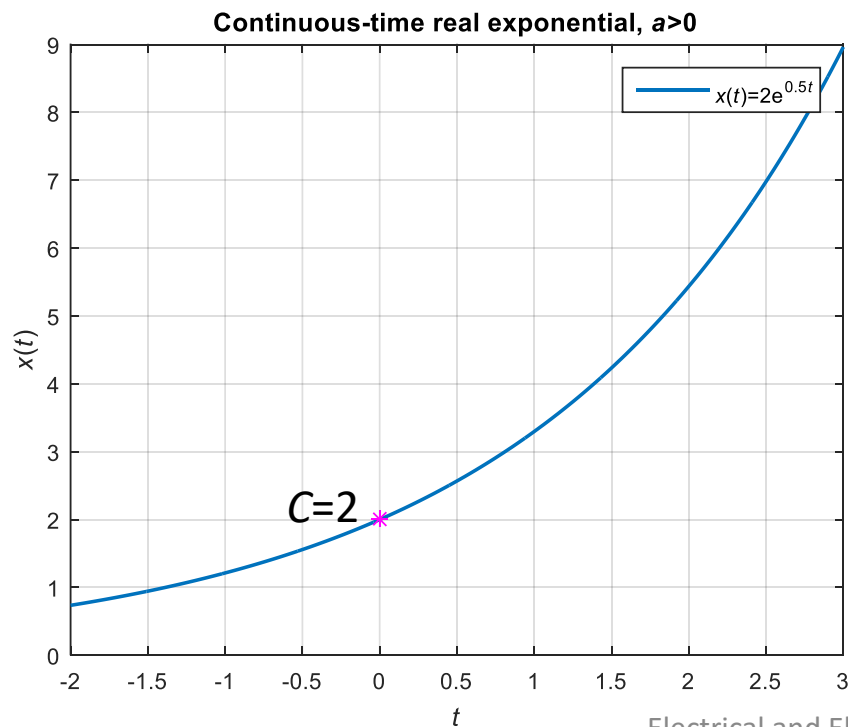
Continuous-Time Complex Exponential and Sinusoidal Signals

Continuous-Time Complex Exponential Signals

$$x(t) = Ce^{at}$$

where C and a are complex numbers in general.

- **Real exponential signal:** If C and a are real, $x(t)$ is called real exponential signal.



- **Periodic Complex Exponential and Sinusoidal Signals**

Consider $x(t) = e^{j\omega_0 t}$ (a is purely imaginary)

$x(t)$ is periodic, where ω_0 is frequency in radian per second (rad/s)

$$\omega_0 = \frac{2\pi}{T_0} \quad (T_0 \text{ is fundamental period})$$

Since $x(t)$ is periodic, $e^{j\omega_0 t} = e^{j\omega_0(t+T)}$

Therefore, $e^{j\omega_0 t} = e^{j\omega_0 t} e^{j\omega_0 T}$

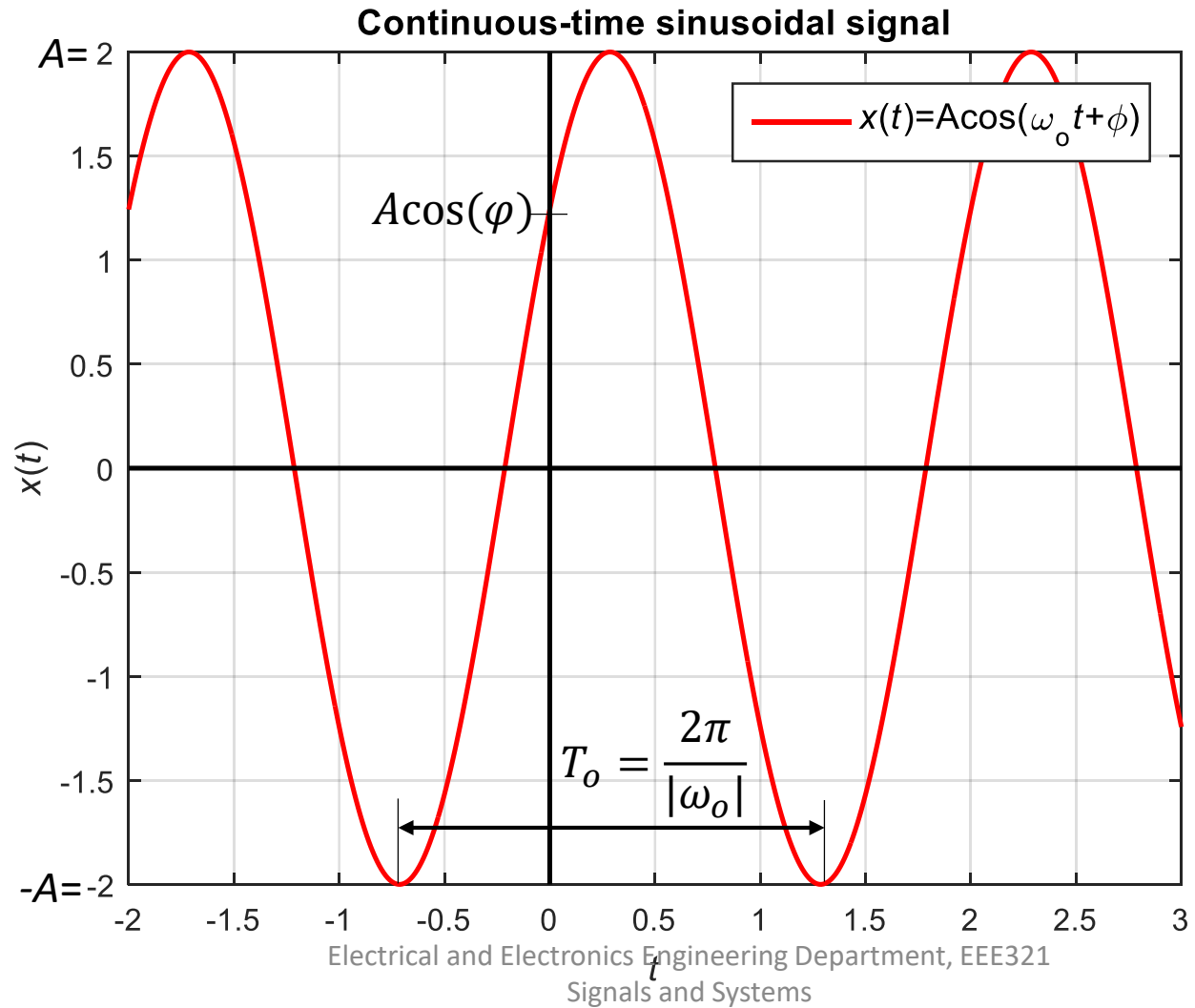
For periodicity, $e^{j\omega_0 T} = 1$

In fact, $e^{j\omega_0 T} = \cos(\omega_0 T) + j\sin(\omega_0 T)$ (Euler's Relation)

When $\omega_0 = 0$, then $x(t)=1$, which is periodic for any value of T . When $\omega_0 \neq 0$, the fundamental period T_0 (smallest positive value of T) of $x(t)$ is $T_0 = \frac{2\pi}{|\omega_0|}$.

- **Sinusoidal signal** is closely related to periodic complex exponential:

$$x(t) = A\cos(\omega_0 t + \phi)$$



- By using Euler's Relation periodic complex exponential can be written in terms of periodic sinusoidals:

$$e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

- Similarly, sinusoidal signal can be written in terms of periodic complex exponentials:

$$A\cos(\omega_0 t + \varphi) = \frac{A}{2} e^{j\varphi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\varphi} e^{-j\omega_0 t}$$

- Therefore cosine can be expressed as

$$A\cos(\omega_0 t + \varphi) = A\operatorname{Re}\{e^{j(\omega_0 t + \varphi)}\}$$

and sine will be

$$A\sin(\omega_0 t + \varphi) = A\operatorname{Im}\{e^{j(\omega_0 t + \varphi)}\}$$

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab