EEE 321
Signals and Systems

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Fourier Series Representation of Discrete-Time Periodic Signals

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Lecture 10

Agenda

- Harmonically Related Complex Exponentials
- Discrete-Time Fourier Series Representation
- Properties of Discrete-Time Fourier Series

Harmonically Related Complex Exponentials

- $x[n] = x[n+N]$: periodic discrete-time signal
- $\emptyset_k[n] = e^{jk\omega_0 n}$
- $\emptyset_k[n] = \emptyset_{k+rN}[n] = e^{jk\omega_0 n} . e^{jk\left(\frac{2\pi}{N}\right)rN}$

Discrete-Time Fourier Series Representation

- $x[n] = \sum_{k=0}^{N-1} a_k e^{jk\omega_0 n} = \sum_{k=<N>} a_k e^{jk\omega_0 n}$
- $a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n}$
- a_k: spectral coefficients of $x[n]$
- $a_k = a_{k+N}$

Linearity

- $x[n] \longleftrightarrow a_k$ and $y[n] \longleftrightarrow b_k$
- $z[n] = Ax[n] + By[n] \longleftrightarrow c_k = Aa_k + Bb_k$

Time Shifting

- $x[n] \longleftrightarrow a_k$
- $x[n-n_0] \longleftrightarrow e^{-jk\omega_0 n_0} a_k$

Time Reversal

- $x[n] \longleftrightarrow a_k$
- $x[-n] \longleftrightarrow a_{-k}$

Multiplication

- $x[n] \longleftrightarrow a_k$ and $y[n] \longleftrightarrow b_k$
- $x[n]y[n] \longleftrightarrow c_k = \sum_{l=0}^{N-1} a_l b_{k-l}$

Conjugation and Conjugate Symmetry

- $x[n] \longleftrightarrow a_k$
- $x^*[n] \longleftrightarrow a^*_{-k}$

First Difference

- $x[n] \longleftrightarrow a_k$
- $x[n] - x[n-1] \longleftrightarrow \left(1 - e^{-jk\omega_0}\right)a_k$

Parseval's Relation for DT Periodic Signals

- $\frac{1}{N}\sum_{n=<N>}|x[n]|^2 = \sum_{n=<N>}|a_k|^2$

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab