Physics 122: Electricity & Magnetism – Lecture 4
Electric Field

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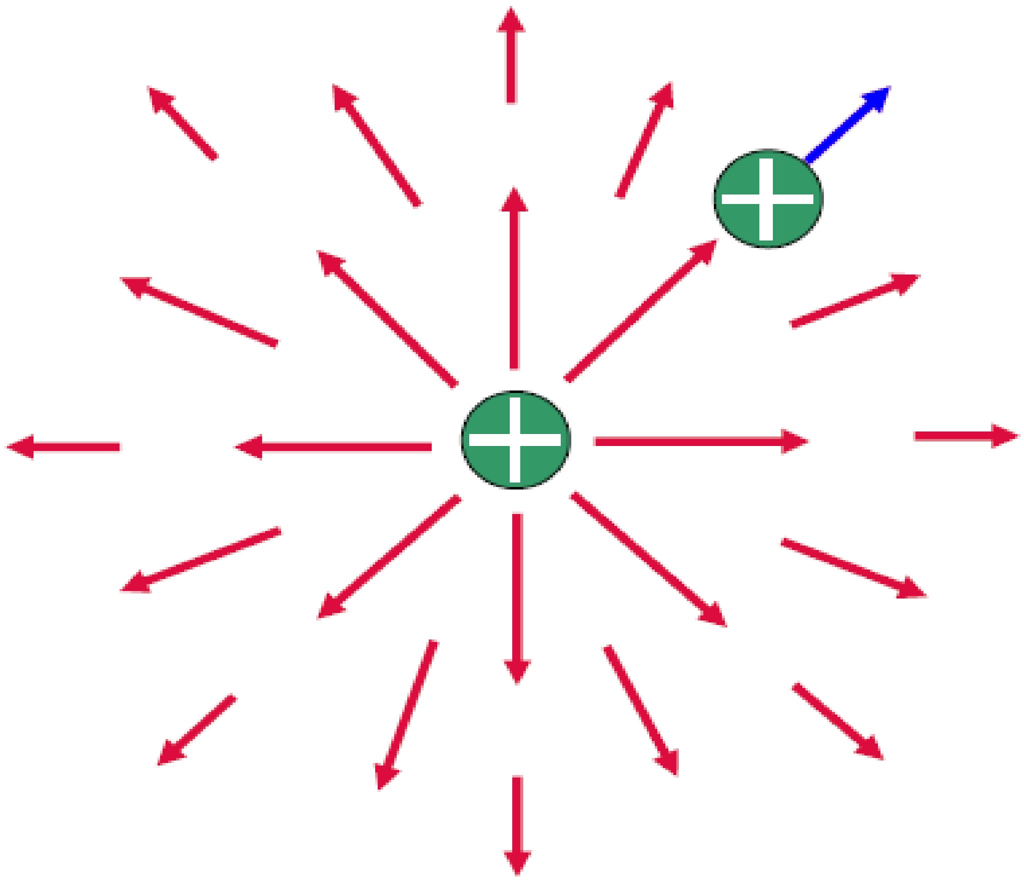
Electric Force and Field Force

- What? -- Action on a distance
- How? – Electric Field
- Why? – Field Force
- Where? – in the space surrounding charges

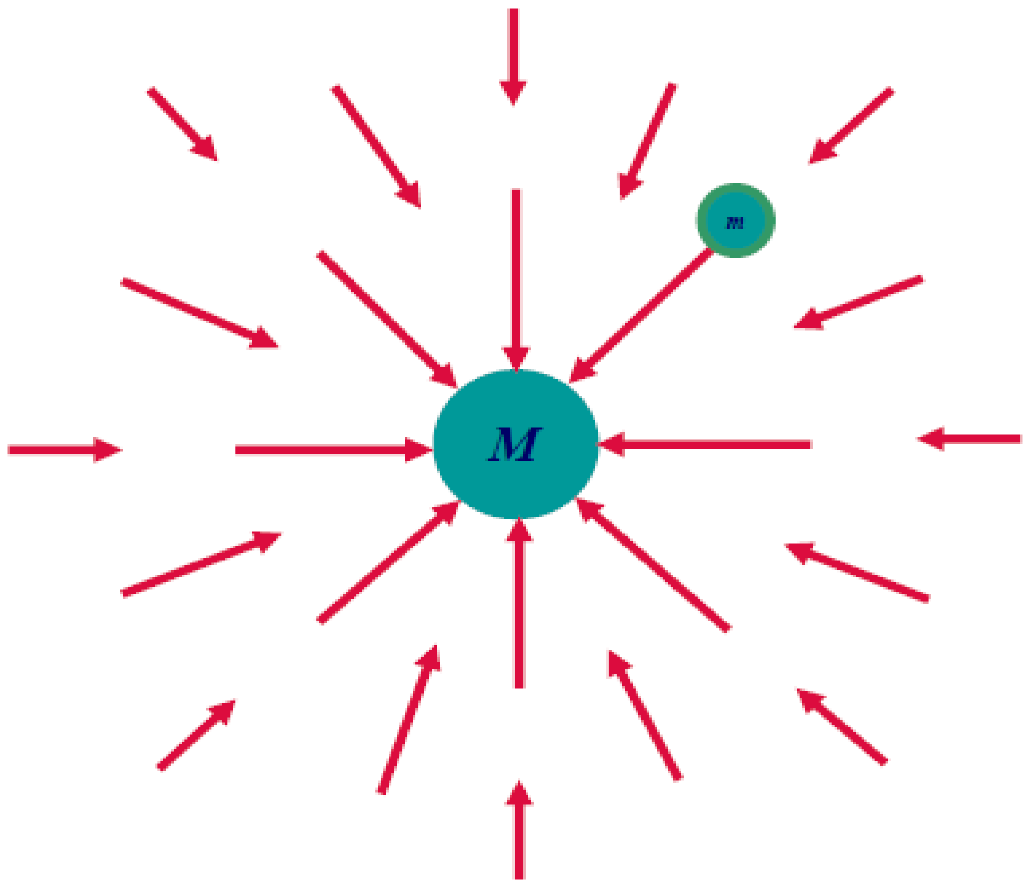


Vector Field Due to Gravity

- When you consider the force of Earth's gravity in space, it points everywhere in the direction of the center of the Earth. But remember that the strength is:

$$\vec{F} = -G\frac{Mm}{r^2}\hat{r}$$

- This is an example of an inverse-square force (proportional to the inverse square of the distance).



Idea of Test Mass

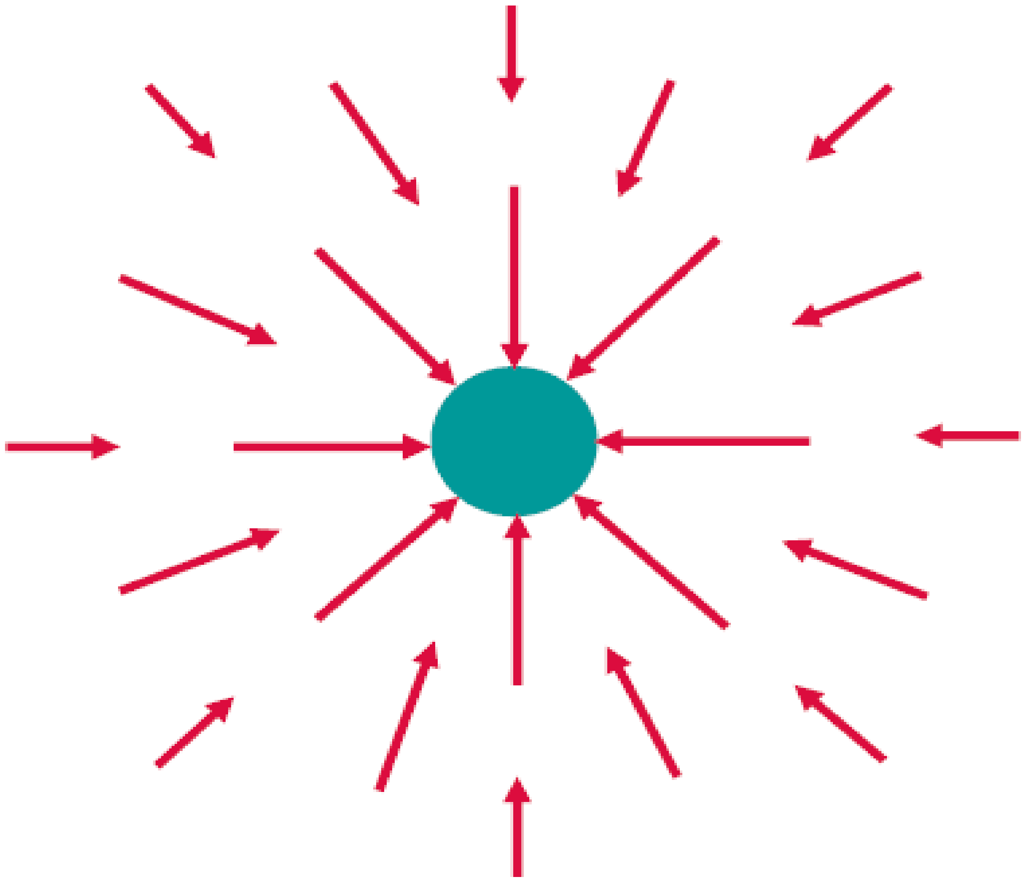
- Notice that the actual amount of force depends on the mass, m:

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}$$

- It is convenient to ask what is the force per unit mass. The idea is to imagine putting a unit test mass near the Earth, and observe the effect on it:

$$\frac{\vec{F}}{m} = -\frac{GM}{r^2}\hat{r} = -g(r)\hat{r}$$

- $g(r)$ is the "gravitational field."

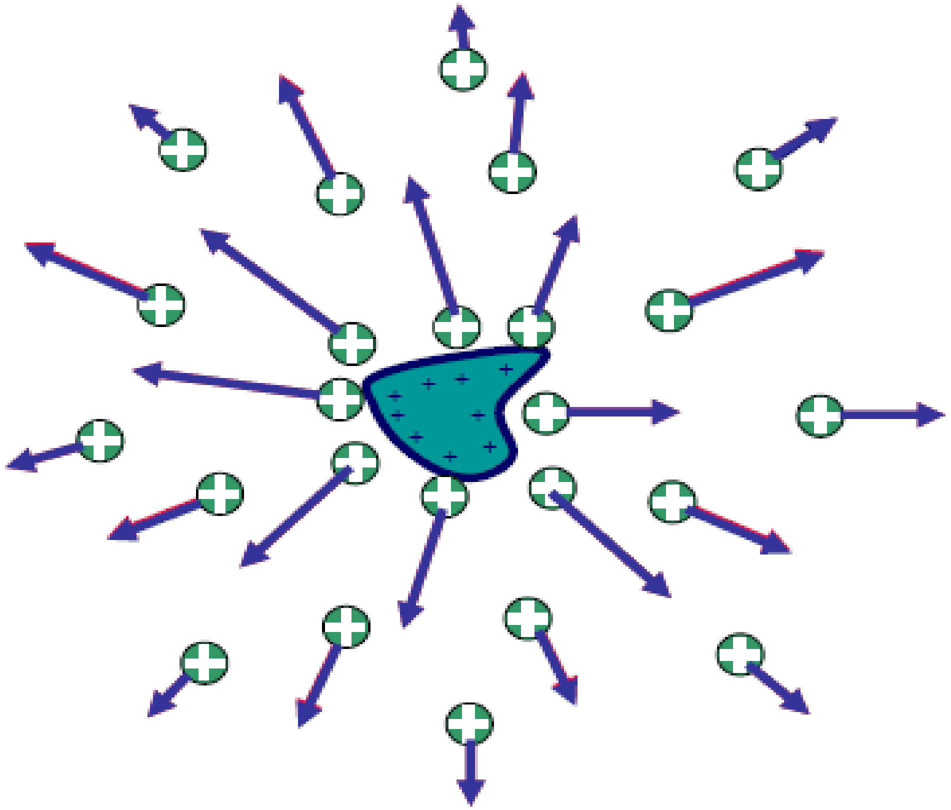


Electric Field

- Electric field is said to exist in the region of space around a charged object: the source charge.
- Concept of test charge:
  - Small and positive
  - Does not affect charge distribution
- Electric field:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

  - Existence of an electric field is a property of its source;
  - Presence of test charge is not necessary for the field to exist;

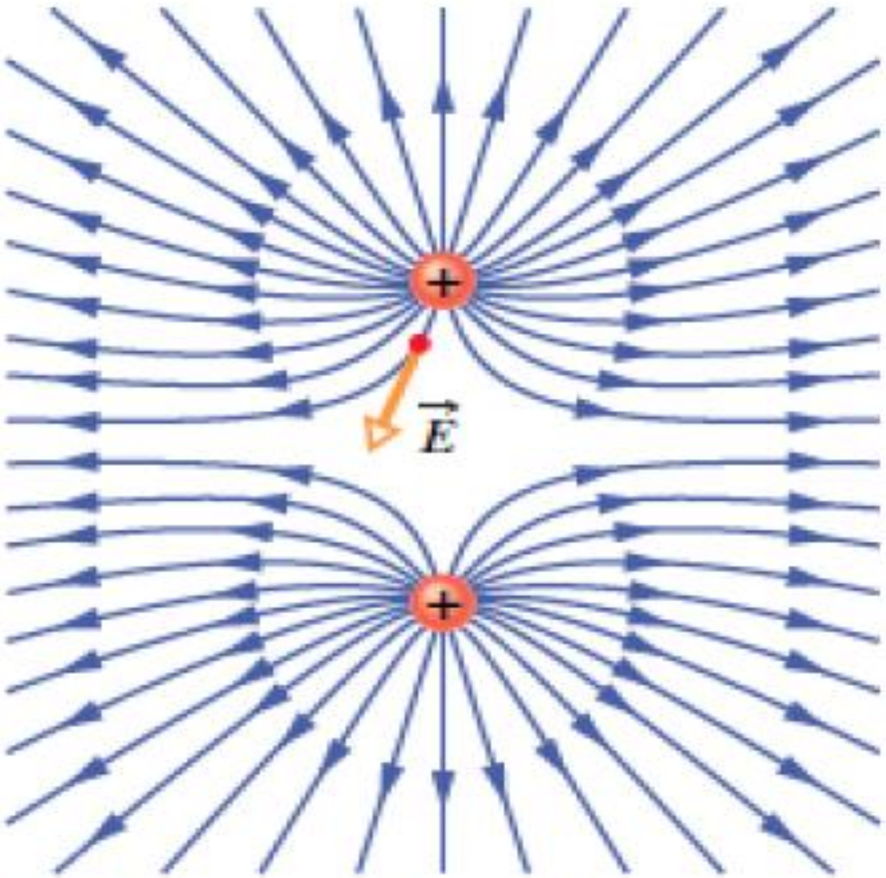


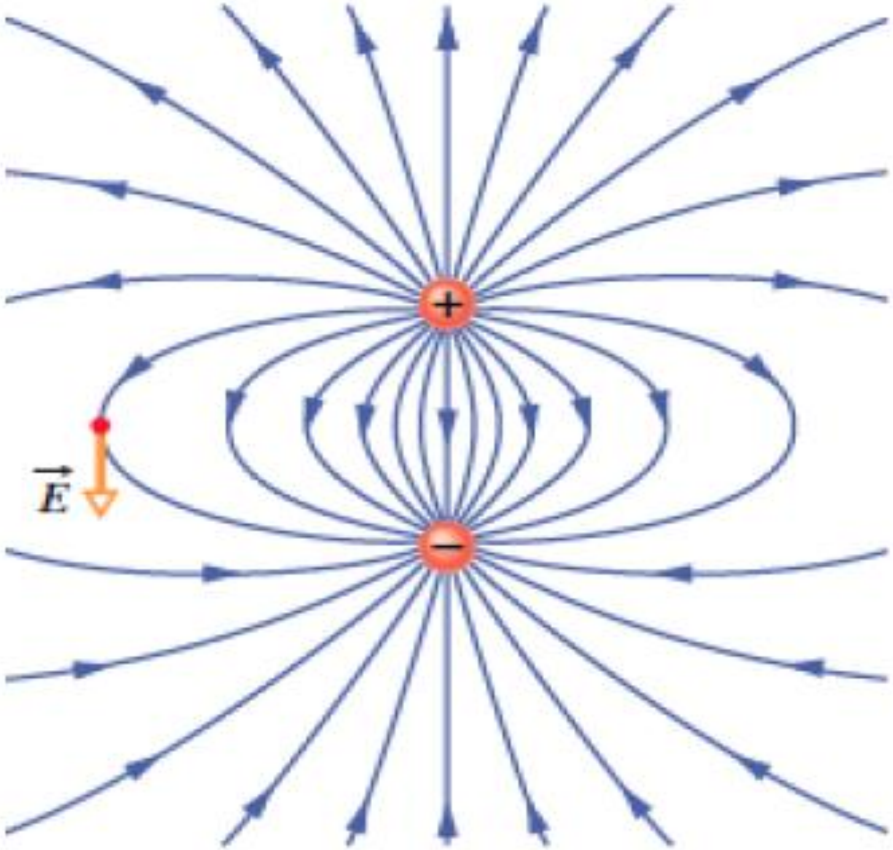
Electric Field

$$\vec{E} = \frac{\vec{F}}{q_0}$$

- Magnitude: $E=F/q_0$
- Direction: is that of the force that acts on the positive test charge
- SI unit: N/C

Situation	Value
Inside a copper wire of household circuits	10^{-2} N/C
Near a charged comb	10^{3} N/C
Inside a TV picture tube	10^{5} N/C
Near the charged drum of a photocopier	10^{5} N/C
Electric breakdown across an air gap	3×10^{6} N/C
At the electron's orbit in a hydrogen atom	5×10^{11} N/C
On the suface of a Uranium nucleus	3×10^{21} N/C

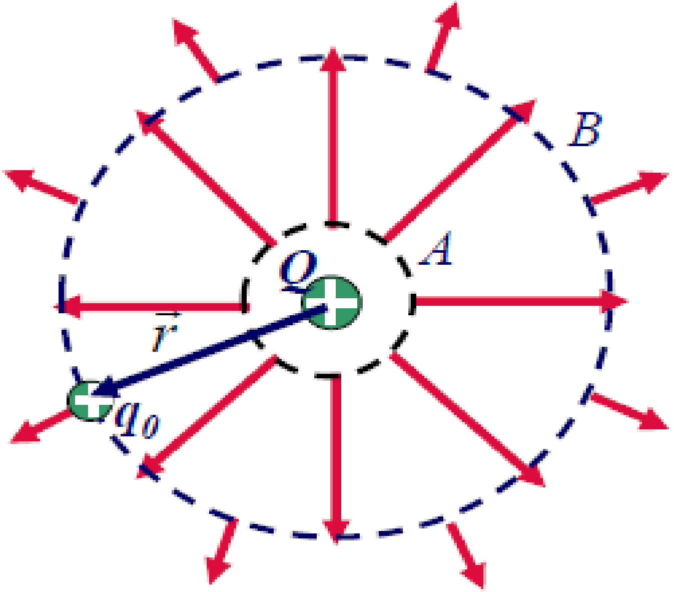

$\vec{E}$


$\vec{E}$

Electric Field due to a Point Charge Q

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

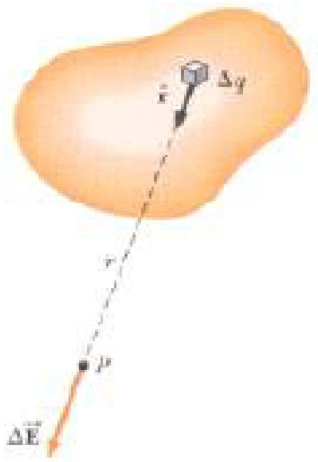


- Direction is radial: outward for +|Q|
  inward for -|Q|
- Magnitude: constant on any spherical shell
- Flux through any shell enclosing Q is the same: $E_A A_A = E_B A_B$

Electric Field of a Continuous Charge Distribution

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r^2}\hat{r}$$

$$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i$$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\lim_{\Delta q \to 0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r}$$

- Find an expression for dq:
  - $dq = \lambda dl$ for a line distribution
  - $dq = \sigma dA$ for a surface distribution
  - $dq = \rho dV$ for a volume distribution
- Represent field contributions at P due to point charges dq located in the distribution. Use symmetry,

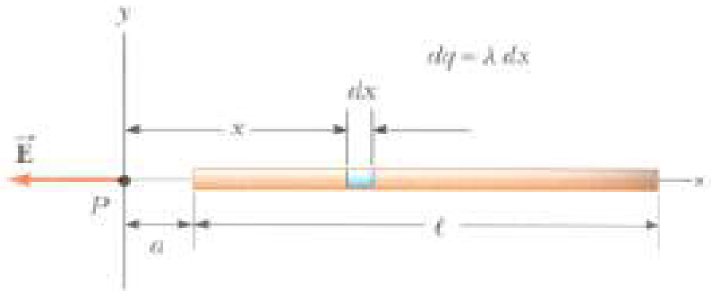
$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

- Add up (integrate the contributions) over the whole distribution, varying the displacement as needed,

$$\vec{E} = \int dE$$

Example: Electric Field Due to a Charged Rod

- A rod of length l has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.



- Start with

$$dq = \lambda dx$$

- then,

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2}$$

- So

$$E = \int_a^{l+a} \frac{\lambda}{4\pi\varepsilon_0}\frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\int_a^{l+a}\frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_a^{l+a}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{l}\left(\frac{1}{a} - \frac{1}{l+a}\right) = \frac{Q}{4\pi\varepsilon_0 a(l+a)}$$

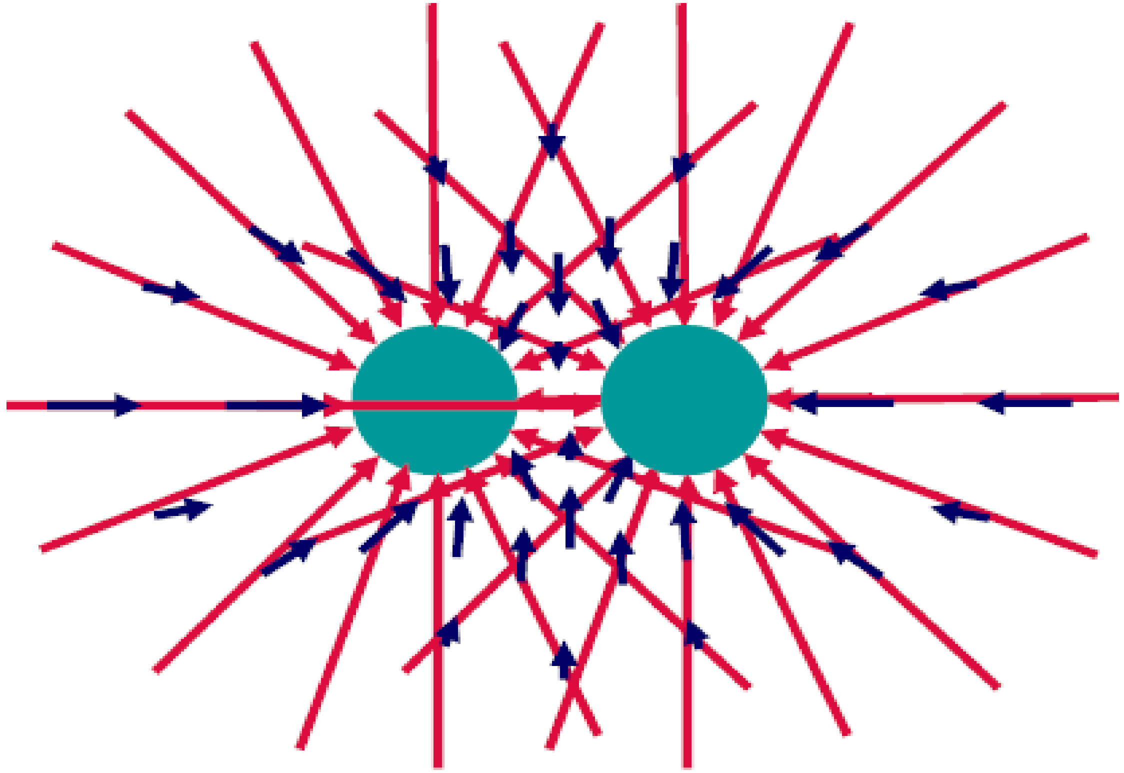
- Finalize
  - *l* => 0 ?
  - *a* >> *l* ?

Electric Field due to a group of individual charge

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \ldots + \vec{F}_{0n}$$

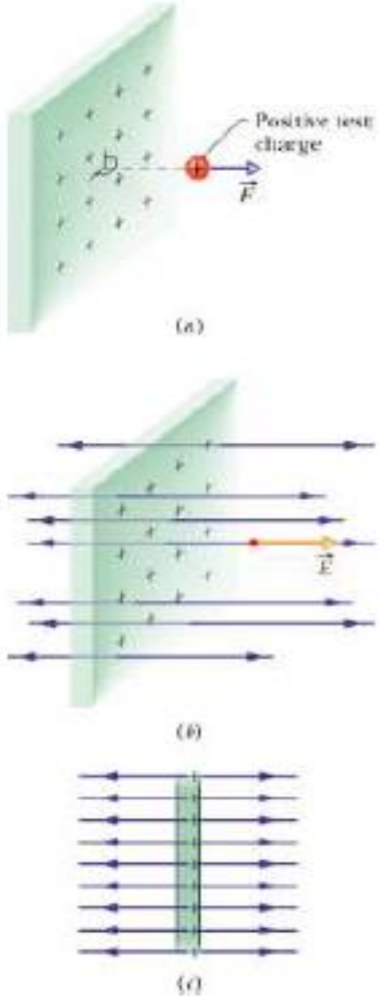
$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \ldots + \frac{\vec{F}_{0n}}{q_0}$$
$$= \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n$$

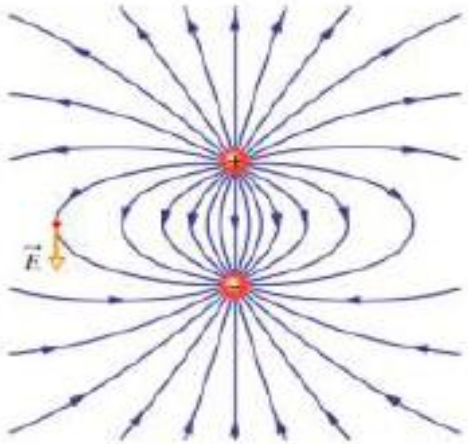
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

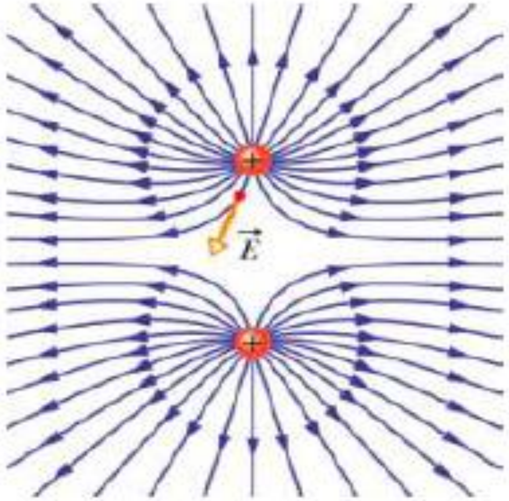


Electric Field Lines

- The lines must begin on a positive charge and terminate on a negative charge. In the case of an excess of one type of charge, some lines will begin or end infinitely far away.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.







Motion of a Charged Particle in a Uniform Electric Field

- If the electric field E is uniform (magnitude and direction), the electric force F on the particle is constant.

- If the particle has a positive charge, its acceleration a and electric force F are in the direction of the electric field E.

- If the particle has a negative charge, its acceleration a and electric force F are in the direction opposite the electric field E.

$$\vec{F} = q\vec{E}$$

$$\vec{F} = q\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{q\vec{E}}{m}$$