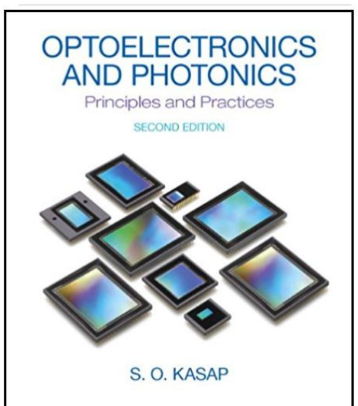
Optoelectronics-I

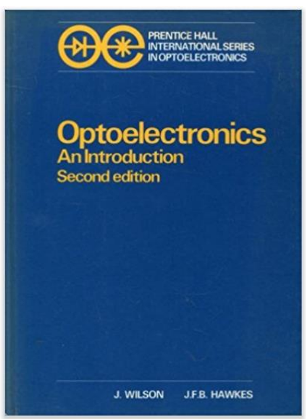
Chapter-5

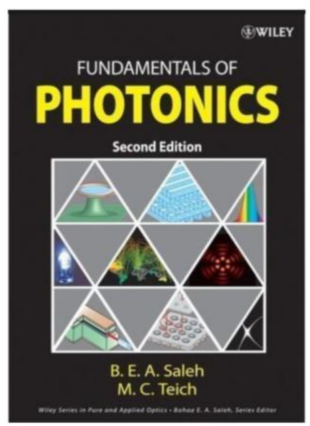
Assoc. Prof. Dr. Isa NAVRUZ

Lecture Notes - 2018

Recommended books







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Light Propagation in Materials

Objectives

When you finish this lesson you will be able to:

- ✓ Describe the phase and group velocities
- ✓ Describe the Maxwell Equations in materials
- ✓ To be able to derive wave equation for materials from Maxwell's equations
- ✓ Define the electrical and magnetic field components of light
- ✓ Light propagation in dielectric materials
- ✓ Light propagation in conductive materials
- ✓ Refractive index in conductive materials
- ✓ Penetration depth

Light Propagation in Materials

Phase Velocity and Group Velocity

For the light waves in the vacuum have the special property, the velocity is exactly the same for all wavelengths. However, this is not true for light propagating in materials. The waves with different wavelengths travel at different velocities in many material such as glass, water or other dielectric materials.

There should be a relationship between frequency and wavelength. A pure waves propagating in the +x direction can be represented as;

$$Y(r,t) = A(x)e^{j(kx-\omega t)} \qquad k = \frac{2\pi}{\lambda}$$

The frequency ω is determined in terms of k or alternatively, the wavelength.

$$\omega = ck$$

So, the pure waves can be written as;

$$Y(r,t) = A(x)e^{jk(x-ct)}$$

Any superposition of these pure waves will always be some function of the form $f(x - ct)$

It is clearly that the velocity of any wave packet like the velocity of the pure waves. So, ω(k) is not simply equal to vk

Light Propagation in Materials

Phase Velocity and Group Velocity

The pure wave with a given k can be represented as;

$$Y(r,t) = A(x)e^{jk\left(x-\frac{\omega(k)}{k}t\right)}$$

So, the velocity is determined in terms of k for pure wave with the wave number k can be represented as;

$$v_{pure}(k) = \frac{\omega(k)}{k}$$

This velocity will be different for different k. The velocity for the pure wave with a predetermined k_0,

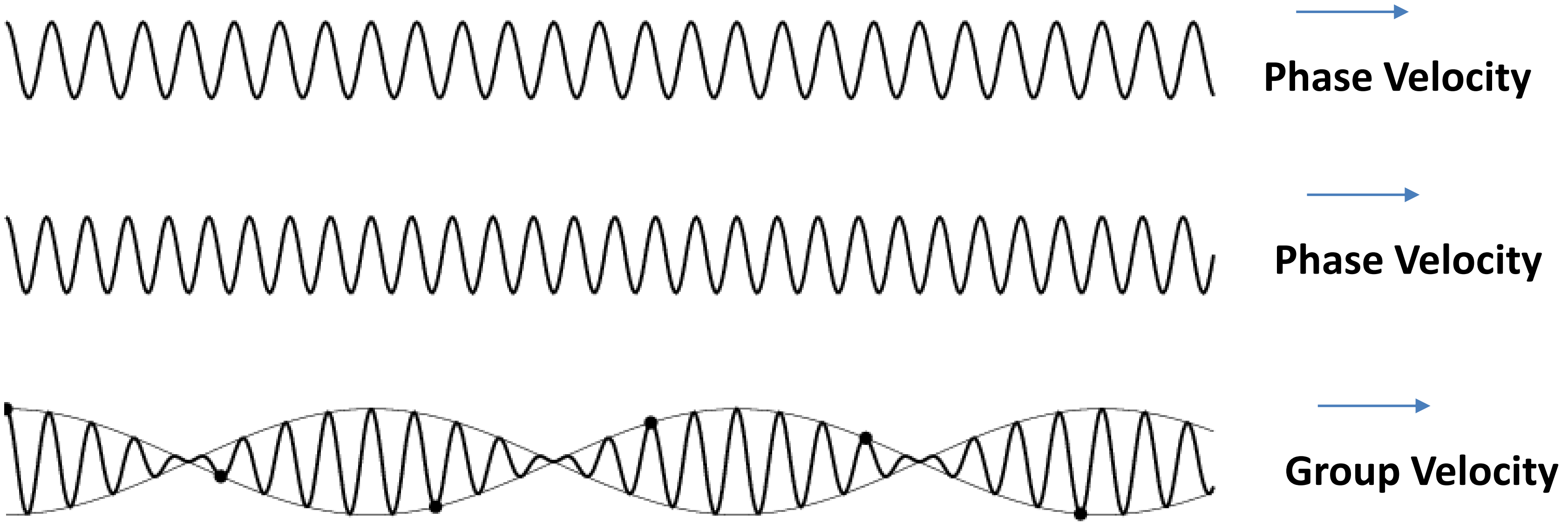
$$v_{phase} = \frac{\omega(k_0)}{k_0}$$

This is known as the **Phase Velocity**. In the case of only pure wave or monochromatic wave, the phase velocity is simply equal to ω/k. On the other hand, the **Group Velocity** is related to how fast the energy in the wave packet propagates. This is the velocity of the whole wavepacket and it can be calculate with,

$$v_{group} = \frac{d\omega}{dk}$$

Light Propagation in Materials

Phase Velocity and Group Velocity



Light Propagation in the Free Space

Phase Velocity and Group Velocity

Exercise:

Suppose that there are two wave of electric field given as,

$$E_1(z,t)=E_o\cos[k_1 z-\omega_1 t)]$$

$$E_2(z,t)=E_o\cos[k_2 z-\omega_2 t)]$$

a) Calculate the phase velocity for each electric field wave.
b) Suppose that the wave packet is built from these waves (E1+E2). Calculate the wave form of wave packet and sketch in time domain. (Use the Matlab programming)
c) Calculate the group velocity.

Light Propagation in Materials

Maxwell's equations for Materials

1. $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_o}$

2. $\vec{\nabla}.\vec{H} = 0$

3. $\vec{\nabla}\times\vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t}$

4. $\vec{\nabla}\times\vec{H} = \varepsilon_o \frac{\partial \vec{E}}{\partial t} + \vec{J}$

For any $\vec{U}$ vector defined in Cartesian coordinates,

Curl operation:

$$\vec{\nabla}\times\vec{U} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ U_x & U_y & U_z \end{vmatrix}$$

Here, the $\vec{U}$ corresponds to the electric field or the magnetic field vector and $\hat{\imath}$, $\hat{\jmath}$ and $\widehat{k}$ represent unit vectors in the axes.

The divergence of a vector field $\longrightarrow$ $\nabla \cdot \vec{U} = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z}$

Light Propagation in Materials

Maxwell's equations for Dielectric Materials

If there are no electrical charges or currents in the medium, The new form of Maxwell' s Equations:

1. $\vec{\nabla}.\vec{E} = \dfrac{\rho}{\varepsilon_o} \longrightarrow \vec{\nabla}.\vec{E} = 0$

2. $\vec{\nabla}.\vec{H} = 0 \longrightarrow \vec{\nabla}.\vec{H} = 0$

3. $\vec{\nabla}\times\vec{E} = -\mu_o \dfrac{\partial\vec{H}}{\partial t} \longrightarrow \vec{\nabla}\times\vec{E} = -\mu_o \dfrac{\partial\vec{H}}{\partial t}$

4. $\vec{\nabla}\times\vec{H} = \varepsilon_o \dfrac{\partial\vec{E}}{\partial t} + \vec{J} \longrightarrow \vec{\nabla}\times\vec{H} = \varepsilon_o \dfrac{\partial\vec{E}}{\partial t}$

Maxwell' s Equation for source-free medium

Light Propagation in Materials

In the medium in which there is no free electric charges or currents, two more vector fields need to be defined-the electric flux density $D(r, t)$ (also called the electric displacement) and the magnetic flux density $B(r, t)$.

There is a relation depending on the electric properties of the medium between the electric flux density and the electric field.

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

Similarly, the relation between the magnetic flux density and the magnetic field is given as,

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

Here, $\vec{P}$ is the polarization density and $\vec{M}$ is the magnetization density.
The polarization density is the macroscopic sum of the electric dipole moments that the electric field induces in a ***dielectric medium***. The magnetization density is similarly defined.

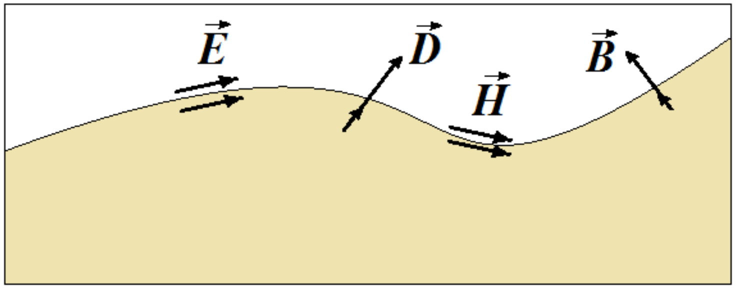
Light Propagation in Materials

For the free space medium,

$$\vec{P} = \vec{M} = 0 \quad \Longrightarrow \quad \vec{D} = \varepsilon_0 \vec{E} \quad \text{and} \quad \vec{B} = \mu_0 \vec{H}$$

Boundary Conditions

In a homogeneous medium, all components of the fields $\vec{D}$, $\vec{B}$, $\vec{E}$ and $\vec{H}$ continuous functions of position. At the boundary between two dielectric media the tangential components of the electric and magnetic fields and the normal components of the electric and magnetic flux densities must be continuous.



Intensity and Power

The flow of electromagnetic power is governed by the vector

$$\vec{\mathbf{S}} = \vec{E} \times \vec{H}$$

known as the Poynting vector.

Light Propagation in Materials

Intensity and Power

The optical intensity I (power flow across a unit area normal to magnitude of the Poynting vector $\mathbf{S}$) is the magnitude of time averaged Poynting vector.

$$I = < \mathbf{S} >$$

The average is taken over times that optical cycle, but short compared to other times of interest.

Dielectric Medium

A dielectric medium is exhibited in the relation between the polarization density *P* and the electric field *E*, called the medium equation.

$$E(r,t) \longrightarrow \boxed{\text{Dielectric Medium}} \longrightarrow P(r,t)$$

Linear
Nondispersive
homogeneous
isotropic.

The average dipole moment per unit volume is called as Polarization Vector P. The polarization vector is a vector field. As a result, the direction and magnitude of the Polarization vector can change as function of position (r).

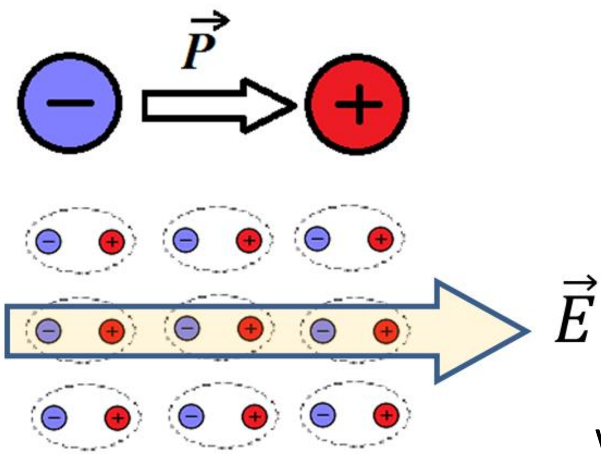
Light Propagation in Materials

Dielectric Materials

Polarization Vector

$$\vec{P} = \frac{\sum \vec{\mu}_{atom}}{V}$$

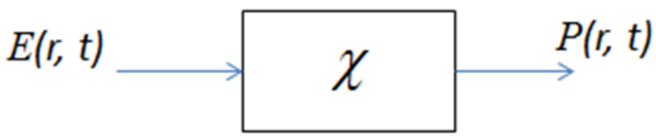
μ_{atom} is the atomic dipole moment
V is the volume



The relationship between the polarization density and the electric field can be expressed with the following formula.

$$\vec{P}(\vec{E}) = \vec{P}_o + \varepsilon_o \chi^{(1)} \vec{E} + \varepsilon_o \chi^{(2)} \vec{E}^2 + \varepsilon_o \chi^{(3)} \vec{E}^3 + \ldots$$

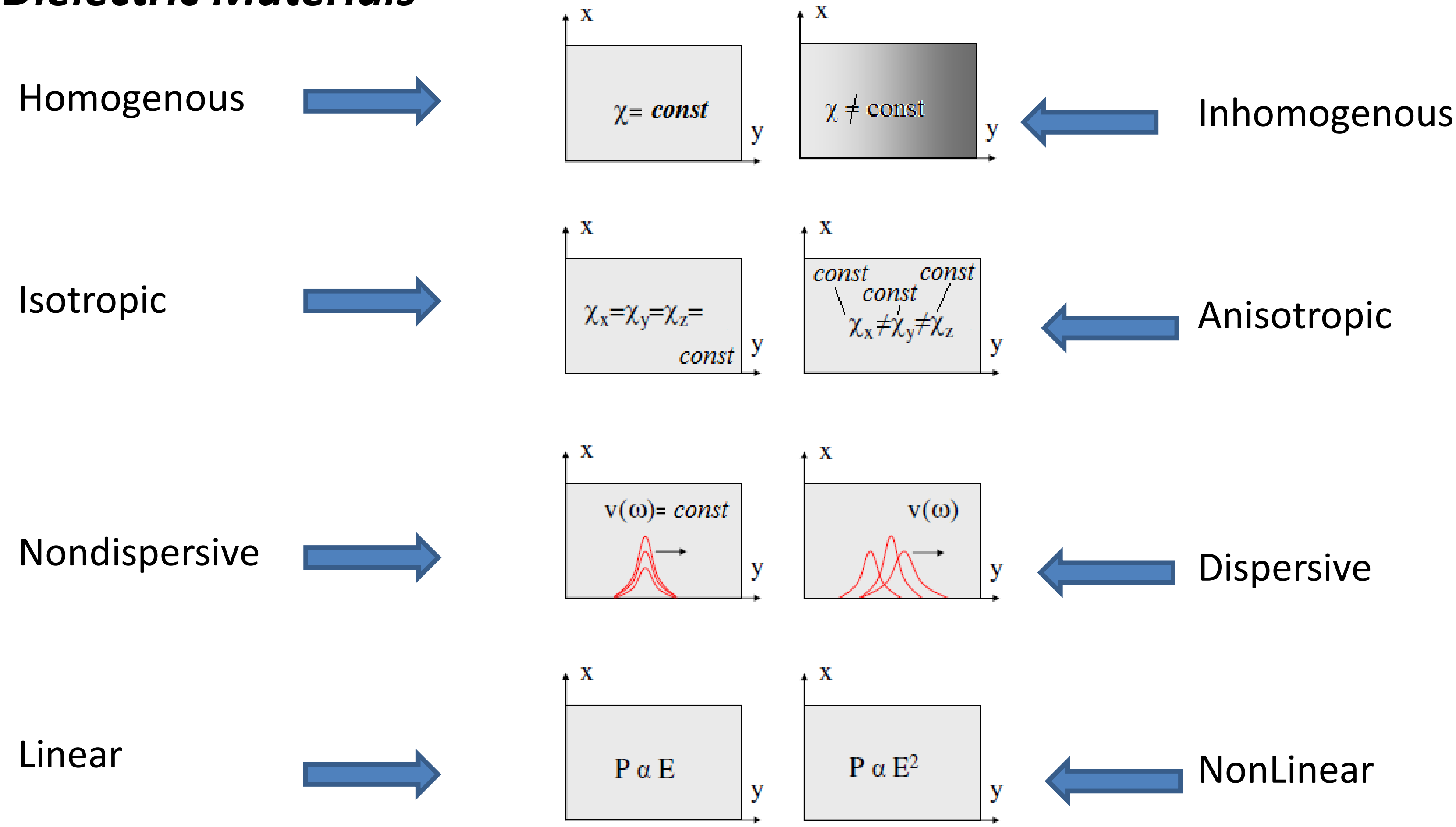
where χ is a scalar constant called as the ***electric susceptibility***.



Here P_0 must be permanently polarized. Some materials have permanent polarization such as quartz (SiO_2)

Light Propagation in Materials

Dielectric Materials



The simplest case of the dielectric medium is a linear, non-dispersive, homogeneous and isotropic media.

Light Propagation in Materials

Dielectric Materials

For a linear, non-dispersive, homogeneous and isotropic media, the polarization density vector can be easily calculated .The vectors P and E are parallel and proportional at the any time and position. So,

$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

$$\vec{\mathrm{D}} = \varepsilon \vec{E}$$

$$\varepsilon = \varepsilon_0 (1 + \chi)$$

$$c = \frac{c_0}{n}$$

The light travels with velocity of $c = 1/(\varepsilon \mu_0)^{1/2}$. Therefore,

$$\boxed{n = \left(\frac{\varepsilon}{\varepsilon_o}\right)^{1/2} = (1 + \chi)^{1/2}}$$

Remember that

$$c_0 = \left(\frac{1}{\varepsilon_0 \mu_0}\right)^{1/2}$$

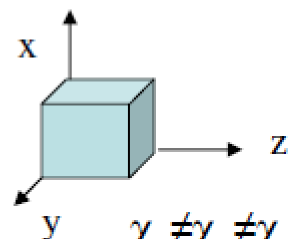
Light Propagation in Materials

Dielectric Materials

Because the directional properties of crystal materials, the electrical *susceptibility* of this material is expressed in the most general case by a tensor. In such materials, the E and P vectors may not always be parallel to each other.

In this case, the following statement is used to define the polarization density separately for each axis.

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \varepsilon_o \begin{bmatrix} \chi_{xx} & \chi_{xy} & \chi_{xz} \\ \chi_{yx} & \chi_{yy} & \chi_{yz} \\ \chi_{zx} & \chi_{zy} & \chi_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$



$\chi_x \neq \chi_y \neq \chi_z$

linear,
non-dispersive,
homogeneous ,
anisotropic
Material

$$\vec{D} = \varepsilon \vec{E} \qquad \varepsilon \equiv \varepsilon_o (1 + \chi^{(1)}) \qquad \vec{\nabla} \times \vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t} \qquad \nabla^2 \vec{E} = \varepsilon \mu_o \frac{\partial^2 \vec{E}}{\partial t^2}$$

Similarly, we can write some equations for magnetic filed. However, most optical materials do not show magnetic properties, So, it can be written as,

$$\boldsymbol{B} = \mu_o \boldsymbol{H} \Longrightarrow \mu \approx \mu_o \qquad \boldsymbol{B} = \mu \boldsymbol{H} \qquad \vec{\nabla} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

Light Propagation in Materials

The Refractive Index in Dielectric Medium

$$n = \left(\frac{\varepsilon}{\varepsilon_o}\right)^{1/2} \qquad \frac{c_o}{\mathrm{v}_{mat}} = n \qquad \mathrm{v}_{mat} = \frac{\omega}{k_{mat}} \qquad \lambda_{mat}\nu = \mathrm{v}_{mat}$$

The wavelength in Dielectric Medium

$$\lambda_{mat} = \frac{\lambda_o}{n}$$

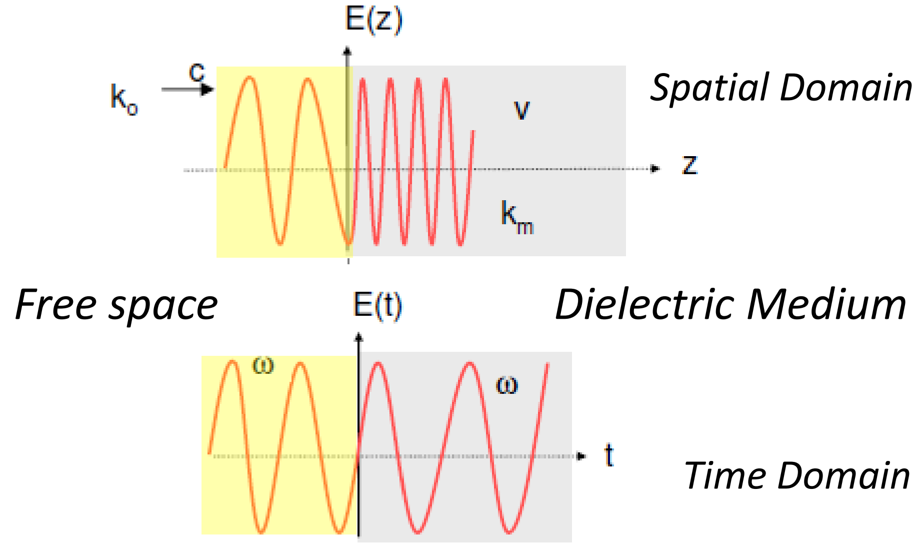
The angular frequency in Dielectric Medium

$$\omega_{mat} = \omega_0 = 2\pi\nu$$

Unchanging !!!

The wavenumber in Dielectric Medium

$$k_{mat} = \frac{2\pi}{\lambda_{mat}} = \frac{n2\pi}{\lambda_o} = nk_o$$



Light Propagation in Materials

Maxwell's equations for Conductive Materials

1. $\vec{\nabla}.\vec{E} = \dfrac{\rho}{\varepsilon_o}$ $\Rightarrow$ $\vec{\nabla}.\vec{E} = 0$

2. $\vec{\nabla}.\vec{H} = 0$ $\Rightarrow$ $\vec{\nabla}.\vec{H} = 0$

3. $\vec{\nabla}\times\vec{E} = -\mu_o \dfrac{\partial \vec{H}}{\partial t}$ $\Rightarrow$ $\vec{\nabla}\times\vec{E} = -\mu_o \dfrac{\partial \vec{H}}{\partial t}$

4. $\vec{\nabla}\times\vec{H} = \varepsilon_o \dfrac{\partial \vec{E}}{\partial t} + \vec{J}$ $\Rightarrow$ $\vec{\nabla}\times\vec{H} = \varepsilon_o \dfrac{\partial \vec{E}}{\partial t} + \vec{J}$

As a result of free electrons in conductive materials, the current density cannot be zero. The relation between the current density and the electric field is defined by the law of ohm.

$\vec{J} = \sigma \vec{E}$ σ is the conductivity and it equals to $1/\rho$

ρ is the resistivity.

From the 4th equation of Maxwell, it can be write as:

$$\vec{\nabla}\times\vec{H} = \varepsilon_o \frac{\partial E}{\partial t} + \vec{J} \quad \Rightarrow \quad \vec{\nabla}\times\vec{H} = \varepsilon_o \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E}$$

Light Propagation in Materials

Conductive Materials

$$\vec{\nabla}\times\vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t} + \sigma\vec{E}$$

$$\nabla^2\vec{E} = \mu_o\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} + \mu_o\sigma\frac{\partial\vec{E}}{\partial t}$$

The wave equations in the conductive medium can be expressed as:

$$\vec{E}(r,t) = \vec{E}_o e^{i(\vec{k}_{mat}.\vec{r}-\omega t+\phi)}$$

$$\vec{H}(r,t) = \vec{H}_o e^{i(\vec{k}_{mat}.\vec{r}-\omega t+\phi)}$$

$$\vec{\nabla}\times\vec{E} = -\mu_o\frac{\partial\vec{H}}{\partial t} \Rightarrow \vec{\nabla}\times\vec{E} = +i\omega\vec{H}$$

$$\vec{\nabla}\times\vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t} + \sigma\vec{E} \Rightarrow \vec{\nabla}\times\vec{H} = \sigma\vec{E} + \varepsilon(-i\omega)\vec{E}$$

$$= \left[\frac{\sigma}{(-i\omega)} + \varepsilon\right](-i\omega)\vec{E}$$

$$= \hat{\varepsilon}(-i\omega)\vec{E}$$

$$\vec{\nabla}\times\vec{H} = \hat{\varepsilon}(-i\omega)\vec{E}$$

$$\hat{\varepsilon} \equiv \frac{\sigma}{-i\omega} + \varepsilon = \varepsilon + i\frac{\sigma}{\omega}$$

Light Propagation in Materials

Conductive Materials

Remember the refractive index can be defined as: $n = \sqrt{\dfrac{\varepsilon}{\varepsilon_o}}$

$$\boxed{\hat{\varepsilon} \equiv \varepsilon + i\frac{\sigma}{\omega}}$$

So, we can obtain the refractive index as: $\hat{n} = \sqrt{\dfrac{\hat{\varepsilon}}{\varepsilon_o}}$

Now, we have a complex refractive index for conductive medium. The complex refractive index can be shown as:

$$\boxed{\hat{n} = n + iK}$$

Here, n and K are the real and imaginary components of the refractive index, respectively.

Light Propagation in Materials

Conductive Materials

The Electric Field wave equations in the free space

$$\boxed{\nabla^2 \vec{E} = \mu_o \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2}}$$

The Permittivity of free space (ε_o) is a real number equal to 8.854×10^{-12} F/m.

$$c_o = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

Light Speed

The Electric Field wave equations in the conductive materials

$$\boxed{\nabla^2 \vec{E} = \mu_o \hat{\varepsilon} \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\hat{\varepsilon} = \varepsilon + i\frac{\sigma}{\omega}$$

The Permittivity of conductive materials ($\hat{\varepsilon}$) is a complex number.

$$\mathrm{v} = \frac{1}{\sqrt{\hat{\varepsilon}\mu_o}} = \frac{\omega}{k_{mat}}$$

Refractive Index

$$n = \sqrt{\frac{\varepsilon}{\varepsilon_o}} \Rightarrow \hat{n} = \sqrt{\frac{\hat{\varepsilon}}{\varepsilon_o}}$$

$$\boxed{\hat{n} = n + iK}$$

K is the imaginer term and it represents to absorption

Light Propagation in Materials

Conductive Materials

Wave Number

$$\hat{k}_{mat} = \frac{\omega}{c}\hat{n} \quad \Rightarrow \quad \boxed{\hat{k}_{mat} = \frac{\omega}{c}(n + iK)}$$

Suppose that The electromagnetic wave is travelling on z direction in the conductive material.

$$\vec{E}(z,t) = \vec{E}_o e^{i(\hat{k}_{mat}.z - \omega t + \phi)}$$

$$\boxed{\vec{E}(z,t) = \underbrace{\vec{E}_o e^{-\frac{\omega}{c}Kz}}_{\text{Amplitude decreases exponentially}} e^{i\left[\frac{\omega}{c}nz - \omega t + \phi\right]} = E(z)e^{i\left[\frac{\omega}{c}nz - \omega t + \phi\right]}}$$

Light Propagation in Materials

Conductive Materials

<u>Intensity</u>

$$I = \langle \mathbf{S} \rangle \qquad \vec{\mathbf{S}} = \vec{E} \times \vec{H} \quad \Rightarrow \quad I = \varepsilon_o c \left\langle \left| \vec{E} \right|^2 \right\rangle$$

At the z=0 position,

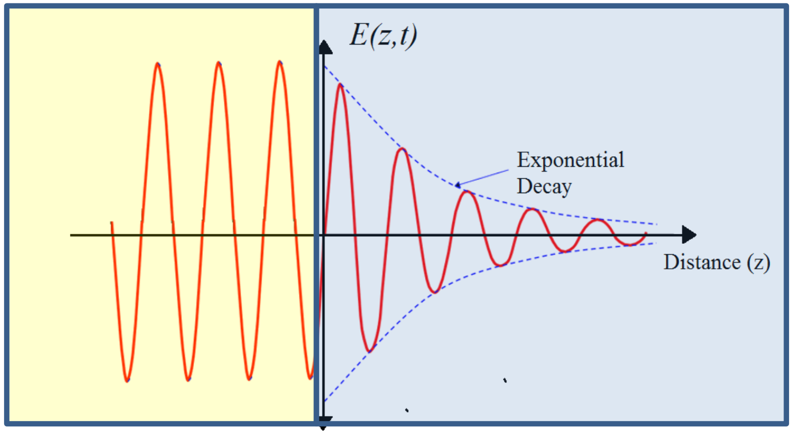
$$\boxed{\vec{E}(0,t) = \vec{E}_o \, e^{i\left[\frac{\omega}{c} nz - \omega t + \phi\right]}} \qquad \boxed{I = I_o}$$

At any position of z,

$$\boxed{\vec{E}(z,t) = \vec{E}_o e^{-\frac{\omega}{c} Kz} e^{i\left[\frac{\omega}{c} nz - \omega t + \phi\right]}} \qquad \boxed{I(z) = I_o e^{-\alpha z}}$$

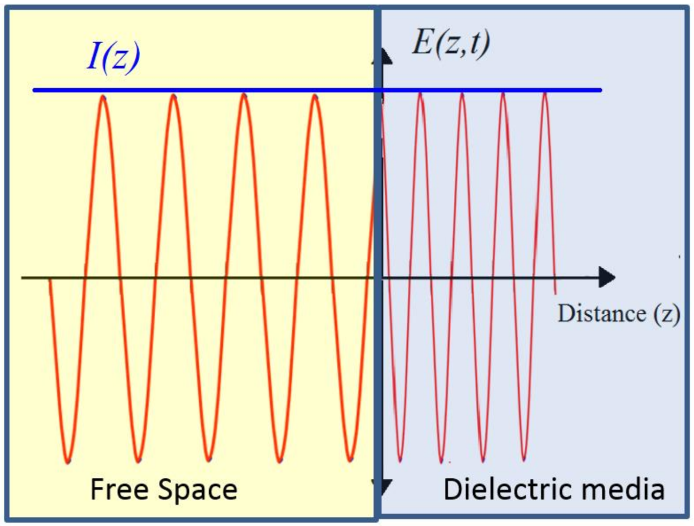
α is the absorption coefficient and is bigger than zero in the lossy media

$$\boxed{\alpha = \frac{2\omega}{c} K}$$



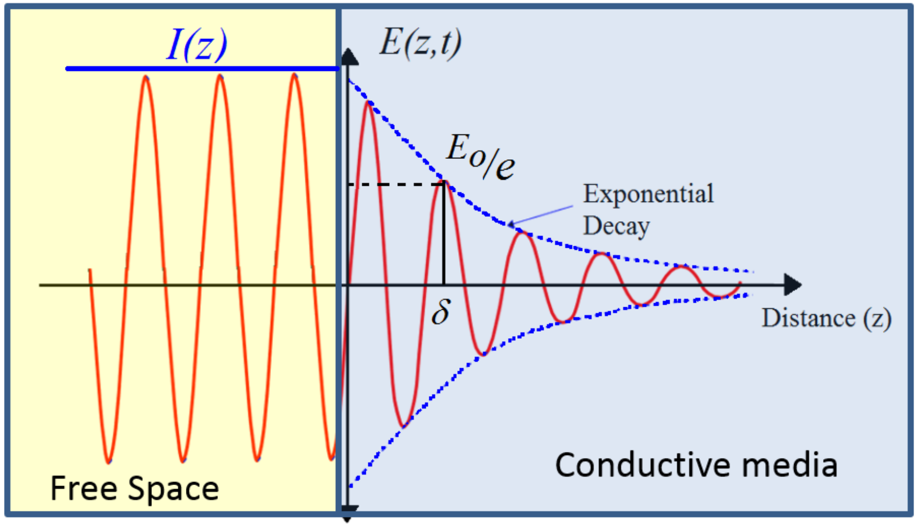
Light Propagation in Materials

Dielectric Material



n: Real

Conductive Material



$\hat{n}$: Complex

Penetration Depth

Penetration depth (δ) is a measure of how deep light or any electromagnetic radiation can penetrate into a material at which the amplitude of the Electric Field inside the material falls to **1/e** .

For incident radiation in the ultraviolet range (~ 100nm), the penetration depth of copper is 0.6nm, For radiation in the infrared range (l_0 ~ 10,000 nm) is 6nm.