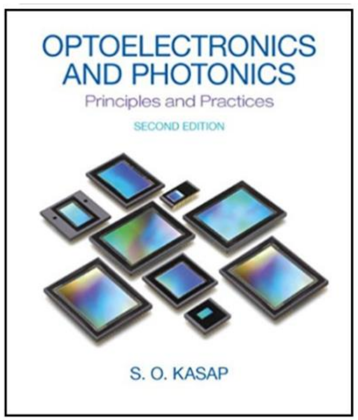
Optoelectronics-I

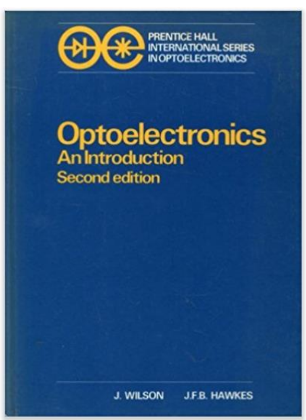
Chapter-7

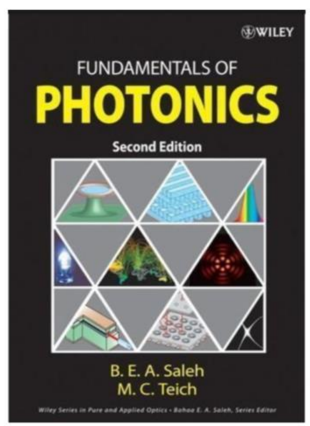
Assoc. Prof. Dr. Isa NAVRUZ

Lecture Notes - 2018

Recommended books







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Refraction & Reflection

Objectives

When you finish this lesson you will be able to:

- ✓ Describe the Reflection and Refraction
- ✓ Define the Snell's Law and critical angle
- ✓ Explain the plane of incidence
- ✓ Define S-Polarization and P-Polarization
- ✓ Explain the Boundary Condition
- ✓ Explain the conversation of energy and define Reflectance (R) and Transmittance (T)

Refraction

Refraction is the bending of light as it passes through a boundary between surfaces with different optical densities.

As described in the previous section, when light enters a medium with a different index of refraction, *the frequency stays the same but the wavelength changes*.

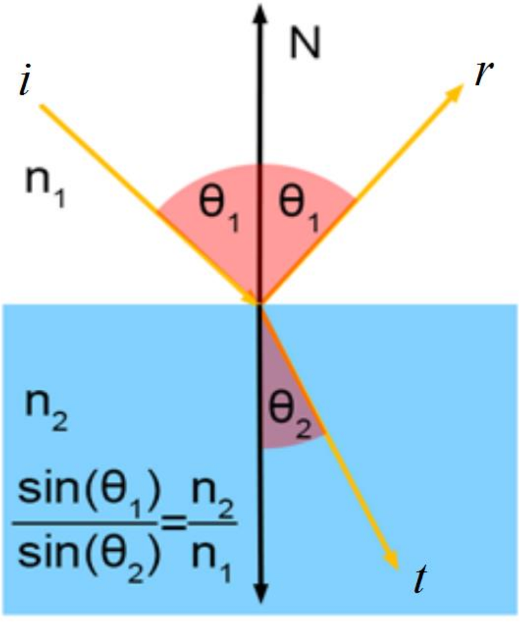
The speed of light depends on the index of refraction, $v=\lambda/n$
So that as the index of refraction goes up, the wavelength goes down and we can write the following formula:

$$\lambda_1 n_1 = \lambda_2 n_2$$

The bending of light when the index of refraction changes is called ***refraction***

Snell' s law describes the **relationship** between the angles of incidence and refraction, when referring to light or other waves passing through a boundary between two different isotropic media.

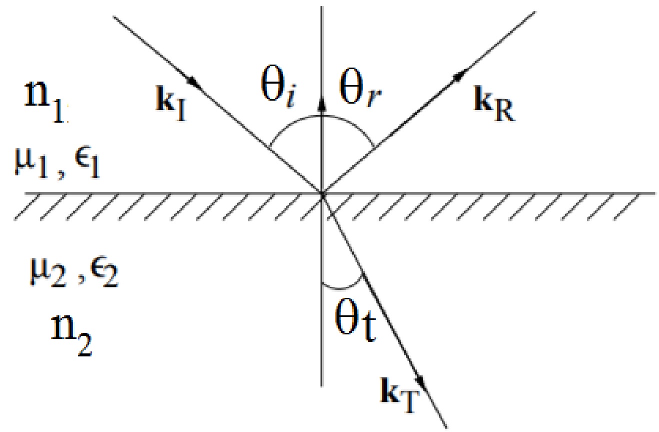
$$\boxed{n_1 \sin(\theta_1)= n_2 \sin(\theta_2)}$$



Refraction

Since n1 is constant for a reflection, λ is the same, therefore the angle of incidence θ_i is equals to the angle of reflection θ_r.

If $n_2 > n_1$,the transmitting light refracts close to its normal direction as shown in the figure.



$$\theta_i = \theta_r \qquad \theta_t = \sin^{-1}[(n_1/n_2)\sin(\theta_i)]$$

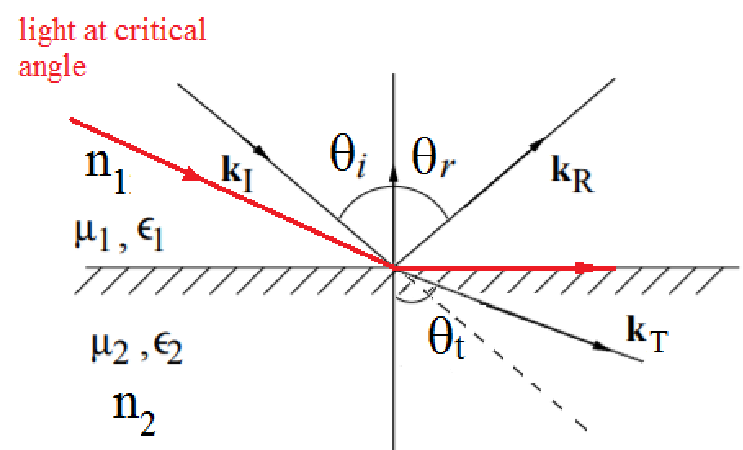
If $n_2 < n_1$,the transmitting light refracts way from the normal direction.

Since the angle cannot be larger than 90° while remaining in the second medium, there is a largest incident angle for refraction when $n_2 < n_1$.

This angle is called **the critical angle**.

If $\theta_t = 90^\circ$ then $\theta_i = \theta_c$

$$\boxed{\theta_c = \sin^{-1}(n_2/n_1)}$$



If $\theta_i > \theta_c$, the transmitted light becomes to zero and all light is reflected back to the incidence medium.

Refraction and Reflection

Snell's law only explains the relationship between reflection and refraction angles of light depending on the refractive index. However we need more information about light propagating on boundary between two materials with different indices.

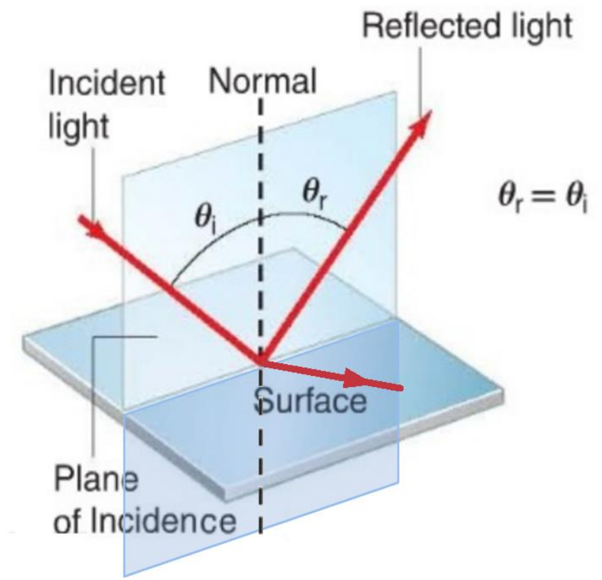
How much of the light from the surface is reflected back? and how much the other medium passes?
How about polarization of light. Does polarization effect the reflection and refraction?

The plane of incidence is the plane containing normal surface and propagation vectors, namely, k_i, k_r and k_t.

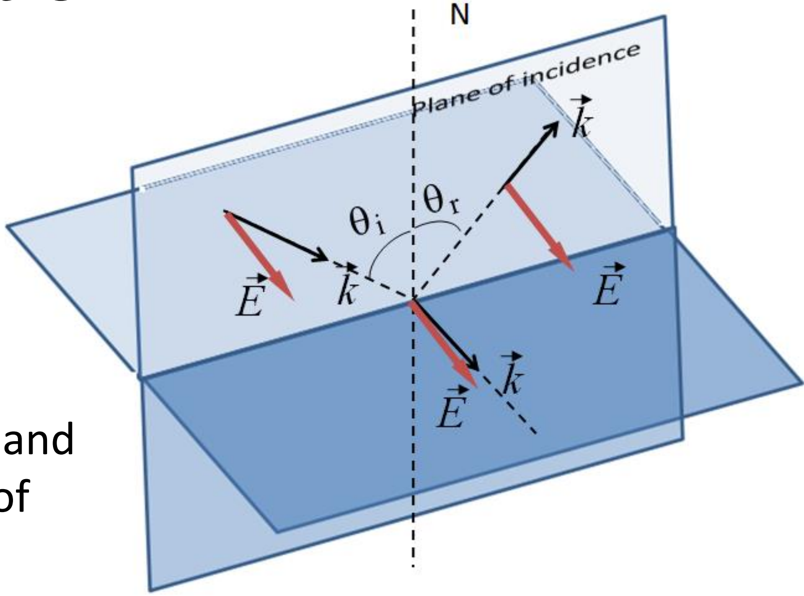
The polarization of light effects reflection and refraction. There are two basic configuration of the polarization.

S-Polarization where the Electric filed vector is perpendicular to plane of incidence.

P-Polarization where the Electric filed vector is parallel to plane of incidence.

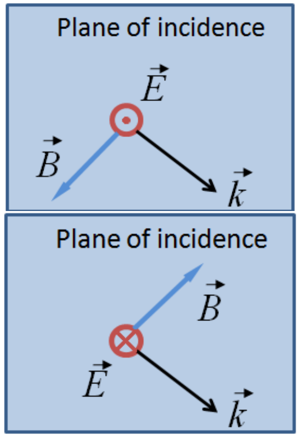


S and P Polarization



S-Polarization (Transverse Electric-TE)

The Electric field vector, E is perpendicular to plane of incidence. In this case magnetic field vector, B is parallel to plane and also perpendicular the E and k vectors.



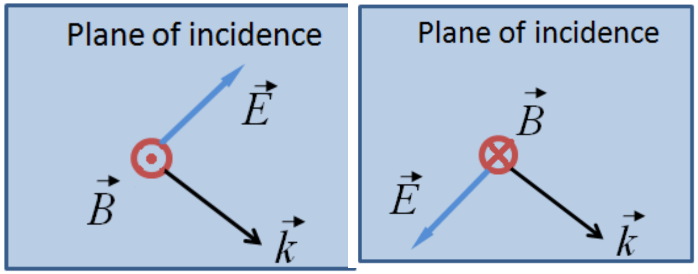
S-Polarization (TE)

The directions of the vectors B and k are located within the plane of incidence.

Now, we can show the electric field vectors propagating from the plane to the outside as a point in the circle. In the case of inverse direction, electric field vectors can represented by symbol of "x"

P-Polarization (Transverse Magnetic- TE)

The Electric field vector, E is parallel to plane of incidence. In this case magnetic field vector, B is perpendicular to plane.



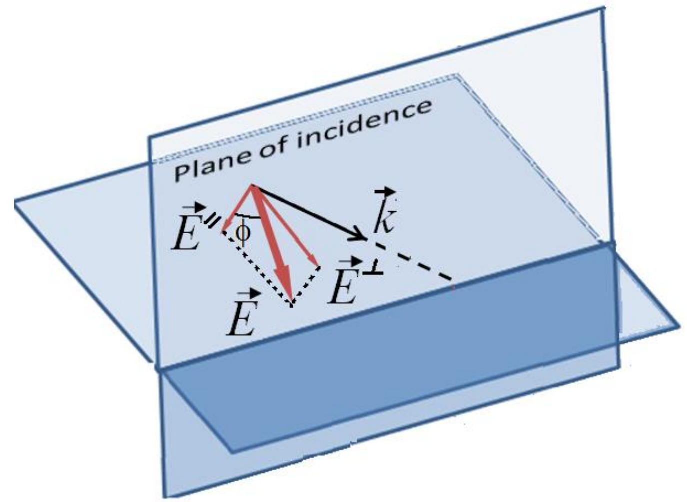
P-Polarization (TM)

S and P Polarization

S-Polarization

The electric field can propagate at any angle of angle with the plane of incidence. In this case, the perpendicular and parallel components of the incident electric field can be calculated as follow:

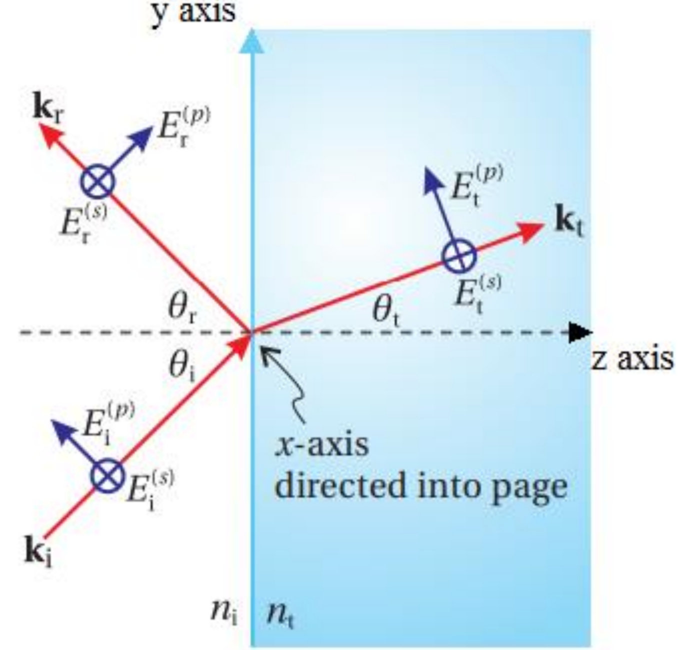
$$\left|\vec{E}_i\right| = \sqrt{\left|\vec{E}_i^{\,\prime\prime}\right|^2 + \left|\vec{E}_i^{\,\perp}\right|^2} \qquad \tan\phi_i = \frac{\left|E_i^{\perp}\right|}{\left|E_i^{\prime\prime}\right|}$$



Example:

Consider a planar boundary between two materials with different indices as given in the figure.

Obtain the electric field vectors $\overrightarrow{E_i}$, $\overrightarrow{E_r}$ and $\overrightarrow{E_t}$



Incident, reflected, and transmitted plane wave fields at a material interface.

S and P Polarization

Solution:

We can write the wave vectors in terms of the $\hat{y}$ and $\hat{z}$ unit vectors:

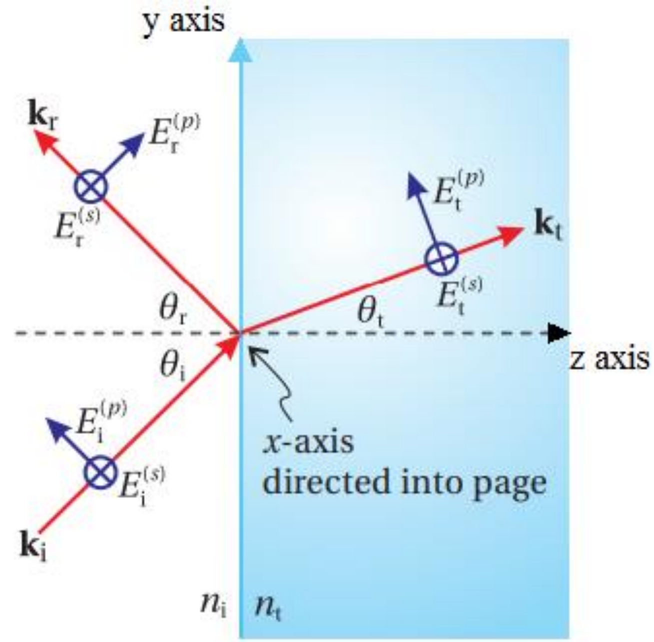
$$\vec{\mathbf{k}}_\mathrm{i} = k_\mathrm{i}\left(\hat{\mathbf{y}}\sin\theta_\mathrm{i} + \hat{\mathbf{z}}\cos\theta_\mathrm{i}\right)$$
$$\vec{\mathbf{k}}_\mathrm{r} = k_\mathrm{r}\left(\hat{\mathbf{y}}\sin\theta_\mathrm{r} - \hat{\mathbf{z}}\cos\theta_\mathrm{r}\right)$$
$$\vec{\mathbf{k}}_\mathrm{t} = k_\mathrm{t}\left(\hat{\mathbf{y}}\sin\theta_\mathrm{t} + \hat{\mathbf{z}}\cos\theta_\mathrm{t}\right)$$

We can write a general formula for electric field vector as follows:

$$\vec{E} = E_o\, e^{i(\vec{k}.\vec{r}-\omega t)}$$

We can write the incident, reflected, andtransmittedfields in terms of $\hat{x}$, $\hat{y}$ and $\hat{z}$

$$\vec{\mathbf{E}}_\mathrm{i} = \left[E_\mathrm{i}^{(p)}\left(\hat{\mathbf{y}}\cos\theta_\mathrm{i} - \hat{\mathbf{z}}\sin\theta_\mathrm{i}\right) + \hat{\mathbf{x}}E_\mathrm{i}^{(s)}\right] e^{i\left[k_\mathrm{i}(y\sin\theta_\mathrm{i} + z\cos\theta_\mathrm{i}) - \omega_\mathrm{i} t\right]}$$
$$\vec{\mathbf{E}}_\mathrm{r} = \left[E_\mathrm{r}^{(p)}\left(\hat{\mathbf{y}}\cos\theta_\mathrm{r} + \hat{\mathbf{z}}\sin\theta_\mathrm{r}\right) + \hat{\mathbf{x}}E_\mathrm{r}^{(s)}\right] e^{i\left[k_\mathrm{r}(y\sin\theta_\mathrm{r} - z\cos\theta_\mathrm{r}) - \omega_\mathrm{r} t\right]}$$
$$\vec{\mathbf{E}}_\mathrm{t} = \left[E_\mathrm{t}^{(p)}\left(\hat{\mathbf{y}}\cos\theta_\mathrm{t} - \hat{\mathbf{z}}\sin\theta_\mathrm{t}\right) + \hat{\mathbf{x}}E_\mathrm{t}^{(s)}\right] e^{i\left[k_\mathrm{t}(y\sin\theta_\mathrm{t} + z\cos\theta_\mathrm{t}) - \omega_\mathrm{t} t\right]}$$



Incident, reflected, and transmitted plane wave fields at a material interface.

Note that the frequency of all waves to be the same:

$$\omega_\mathrm{i} = \omega_\mathrm{r} = \omega_\mathrm{t} \equiv \omega$$

S and P Polarization

Boundary conditions

All the phase factors in the complex exponentials equal to each other

$$k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t$$

$$k_i = k_r = n_i\omega/c \text{ and } k_t = n_t\omega/c$$

✓ The tangential Electric field is continuous.

$$E_i^{(s)} + E_r^{(s)} = E_t^{(s)}$$

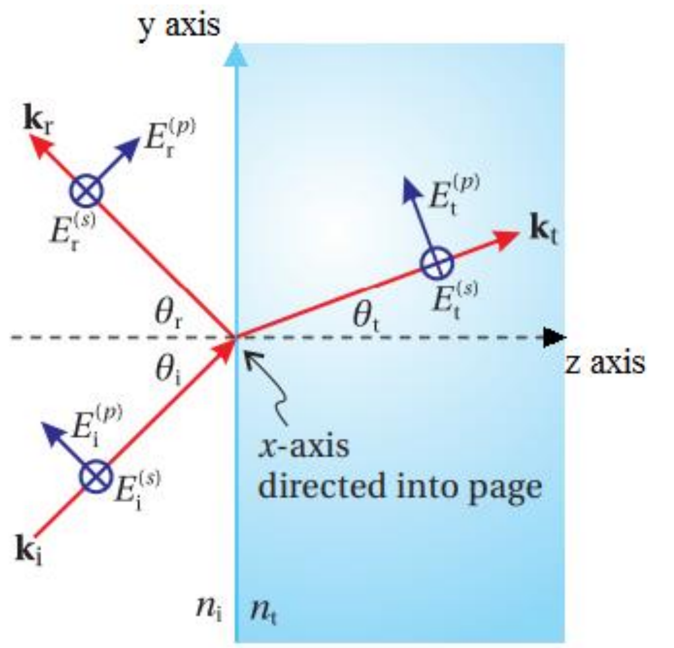
$$\left(E_i^{(p)} + E_r^{(p)}\right)\cos\theta_i = E_t^{(p)}\cos\theta_t$$

✓ The tangential Magnetic field (B/μ) is continuous.

$\mu_i = \mu_r = \mu_t = \mu_0$

$$\frac{1}{\mu_i}\left(B_i^{(p)} + B_r^{(p)}\right) = \frac{1}{\mu_t} B_t^{(p)}$$

$$\frac{1}{\mu_i}\left(B_i^{(s)} + B_r^{(s)}\right)\cos\theta_i = \frac{1}{\mu_t} B_t^{(s)}\cos\theta_t$$



Incident, reflected, and transmitted plane wave fields at a material interface.

S and P Polarization

Boundary conditions

✓ The normal component of Electric field can be write as:

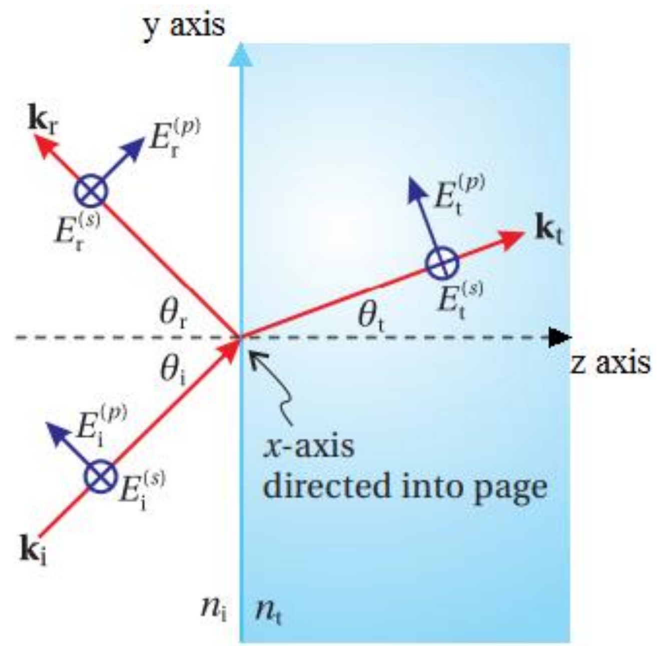
*Only **p** polarization*

$$\boxed{\left[\varepsilon_i(\vec{E}_i+\vec{E}_r)\right]_{normal}=\left[\varepsilon_t(\vec{E}_t)\right]_{normal}}$$

✓ The normal component magnetic field can be write as:

*Only **s** polarization*

$$\boxed{\left[(\vec{B}_i+\vec{B}_r)\right]_{normal}=\left[(\vec{B}_t)\right]_{normal}}$$



Incident, reflected, and transmitted plane wave fields at a material interface.

✓ We need some equations

$$\left|\vec{H}\right| \equiv (\varepsilon/\mu_o)^{1/2}\left|\vec{E}\right|$$

$$\vec{H}=\frac{1}{\mu_o}\left|\vec{B}\right| \Rightarrow \boxed{\left|\vec{B}\right|=\frac{n}{c}\left|\vec{E}\right|}$$

Remember the refractive index can be defined as: $n=\sqrt{\dfrac{\varepsilon}{\varepsilon_o}}$

And speed of light in the free space $c=c_0=\left(\dfrac{1}{\varepsilon_0\mu_0}\right)^{1/2}$

Fresnel Equations

Definition:

The **Fresnel equations** (or **Fresnel** coefficients) describe the reflection and transmission of light (or electromagnetic radiation in general) when incident on an interface between different optical media.

The Fresnel equations describe the behavior of light at optical surfaces, they are relevant to virtually all fields of optical design: lens design, imaging, lasers, optical communication, spectroscopy and holography.

The Fresnel equations inform us about the amplitudes and phases of the waves transmitted and reflected according to their polarization states.

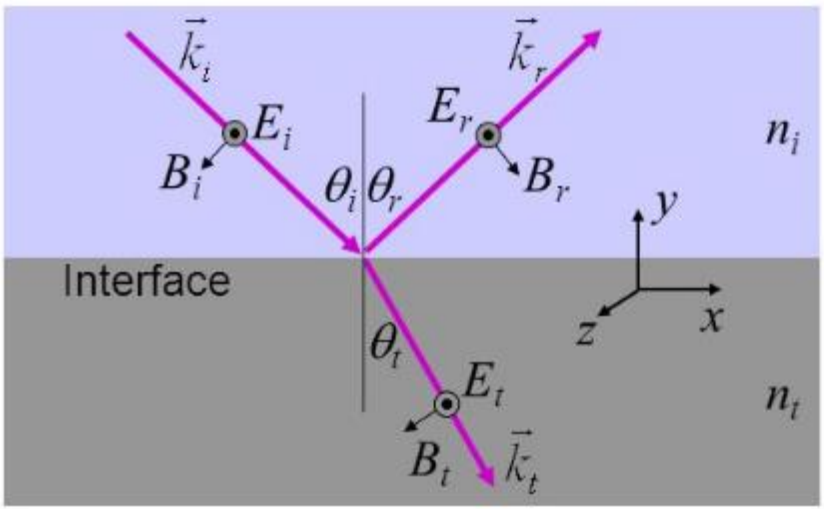
Fresnel coefficients mainly contain reflection and transmission coefficients based on the polarization state of s and p. These are denoted by r_s, r_p, t_s and t_p.

r_s **:** Reflection coefficient for s polarization
r_p **:** Reflection coefficient for p polarization

t_s **:** Transmission coefficient for s polarization
t_p **:** Transmission coefficient for p polarization

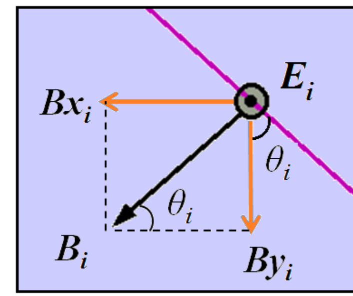
Fresnel Equations

Derivation: Consider two optical media separated by an interface, as shown in figure.



The electric field was polarized as perpendicular. So, The Electric field B is parallel to plane of incidence.

Now, we can decompose the magnetic field vector B into its x and y axis components.



The electric field is already continuous because of perpendicular,

$$E_i + E_r = E_t$$

For the tangential Magnetic field that is continuous, we can write that

$$B_i \cos(\theta_i) - B_r \cos(\theta_r) = B_t \cos(\theta_t)$$

$$\left.\begin{array}{l} |\vec{B}| = \dfrac{n}{c}|\vec{E}| \\ \theta_i = \theta_r \end{array}\right\} \Longrightarrow \quad \frac{n_i}{c} E_i \cos(\theta_i) - \frac{n_i}{c} E_r \cos(\theta_i) = \frac{n_t}{c} E_t \cos(\theta_t)$$

Fresnel Equations

Derivation:

$$\frac{n_i}{c}E_i\cos(\theta_i) - \frac{n_i}{c}E_r\cos(\theta_i) = \frac{n_t}{c}E_t\cos(\theta_t)$$

$$n_iE_i\cos(\theta_i) - n_iE_r\cos(\theta_i) = n_tE_t\cos(\theta_t)$$

$$n_iE_i\cos(\theta_i) - n_iE_r\cos(\theta_i) = n_t(E_i + E_r)\cos(\theta_t)$$

$$E_i(n_icos(\theta_i) - n_tcos(\theta_t)) = E_r(n_tcos(\theta_t) + n_icos(\theta_i))$$

Reflection coefficient for s polarization

$$\boxed{r_s = \frac{E_r}{E_i} = \frac{n_icos\theta_i - n_tcos\theta_t}{n_icos\theta_i + n_tcos\theta_t}}$$

If you calculate the transmission coefficient using the substituting the formula, $E_r = E_t - E_i$

Transmission coefficient for s polarization

$$\boxed{t_s = \frac{E_t}{E_i} = \frac{2n_icos\theta_i}{n_icos\theta_i + n_tcos\theta_t}}$$

Fresnel Equations

Reflection coefficient for s polarization

$$r_s = \frac{E_r}{E_i} = \frac{n_i cos\theta_i - n_t cos\theta_t}{n_i cos\theta_i + n_t cos\theta_t}$$

Transmission coefficient for s polarization

$$t_s = \frac{E_t}{E_i} = \frac{2n_i cos\theta_i}{n_i cos\theta_i + n_t cos\theta_t}$$

For the parallel polarization case, using similar methods, the result are

Reflection coefficient for p polarization

$$r_p = \frac{E_r}{E_i} = \frac{n_t cos\theta_i - n_i cos\theta_t}{n_t cos\theta_i + n_i cos\theta_t}$$

Transmission coefficient for p polarization

$$t_p = \frac{E_t}{E_i} = \frac{2n_i cos\theta_i}{n_i cos\theta_t + n_t cos\theta_i}$$

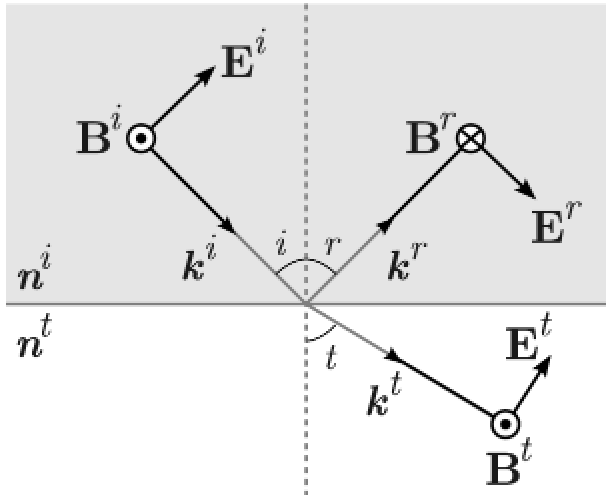
Fresnel Equations

Example:

Prove that the reflection and transmission coefficients for the parallel polarization case can be calculated as follow:

$$r_p = \frac{E_r}{E_i} = \frac{n_t cos\theta_i - n_i cos\theta_t}{n_t cos\theta_i + n_i cos\theta_t}$$

$$t_p = \frac{E_t}{E_i} = \frac{2n_i cos\theta_i}{n_i cos\theta_t + n_t cos\theta_i}$$



Fresnel Equations

Reflectance (R) and Transmittance (T)

The quantities r and t defined and calculated in previous pages are ratios of electric field amplitudes. However, we need to calculate the energy transmitted and transmitted in many electro-optical applications.

The Reflectance (R) and Transmittance (T) coefficient are related to the optical power, indirectly related to the intensity of the light.

Reflectance: $R = \dfrac{P_r}{P_i}$

Transmittance: $T = \dfrac{P_t}{P_i}$

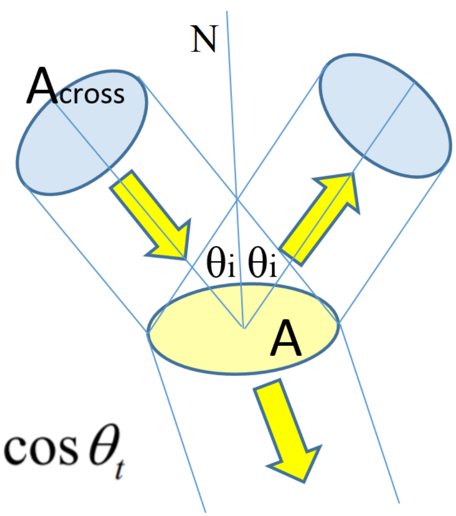
P_i : Incident power

P_r : Reflected power

P_t : Transmitted power

Conservation of energy

$P_i = P_r + P_t$ $\quad (P_r + P_t)/P_i = 1 \Longrightarrow R + T = 1$



$I = \langle S \rangle$ $\quad$ I: Intensity (watt/cm^2) $\quad$ P: Power (watt)

$P = I A_{cross} = I A \cos(\theta_i) \Longrightarrow I_i A \cos\theta_i = I_r A \cos\theta_r + I_t A \cos\theta_t$

Fresnel Equations

Reflectance (R) and Transmittance (T)

$I = <S> \frac{1}{2} n \varepsilon_0 c |E|^2$

$$I_i \cos\theta_i = I_r \cos\theta_r + I_t \cos\theta_t$$

$$\boxed{\frac{1}{2} n_1 \varepsilon_0 c E_i^2 \cos\theta_i = \frac{1}{2} n_1 \varepsilon_0 c E_r^2 \cos\theta_r + \frac{1}{2} n_2 \varepsilon_0 c E_t^2 \cos\theta_t}$$

$$\frac{1}{2} n_1 \varepsilon_0 c E_i^2 \cos\theta_i = \frac{1}{2} n_1 \varepsilon_0 c E_r^2 \cos\theta_r + \frac{1}{2} n_2 \varepsilon_0 c E_t^2 \cos\theta_t$$

$$R = \frac{E_r^2}{E_i^2} = r^2 \qquad T = n\left(\frac{\cos\theta_t}{\cos\theta_i}\right)\frac{E_t^2}{E_i^2} = n\left(\frac{\cos\theta_t}{\cos\theta_i}\right) t^2$$

$$\boxed{\begin{aligned} R &= rr^* = |r|^2 \\ T &= \left(n\frac{\cos\theta_t}{\cos\theta_i}\right) tt^* = \left(n\frac{\cos\theta_t}{\cos\theta_i}\right) |t|^2 \end{aligned}}$$

$$n = \frac{n_2}{n_1} = \frac{n_t}{n_i}$$